

## Resonance Tunneling Through GaN/AlGa<sub>N</sub> Superlattice System

Ridha H. Risan, Auday H. Shaban, Moafak C. Abdulrida

Nanoscience Devices Group, College of Education (Ibn Al-Haitham), University of Baghdad,  
Adamiyah, Baghdad, Iraq

### Abstract

The purpose of our work is to report a theoretical study of electrons tunneling through semiconductor superlattice (SSL). The (SSL) that we have considered is (GaN/AlGa<sub>N</sub>) system within the energy range of  $\varepsilon < V_0$ ,  $\varepsilon = V_0$  and  $\varepsilon > V_0$ , where  $V_0$  is the potential barrier height. The transmission coefficient ( $T_N$ ) was determined using the transfer matrix method. The resonant energies are obtained from the  $T(E)$  relation. From such system, we obtained two allowed quasi-levels energy bands for  $\varepsilon < V_0$  and one band for  $\varepsilon \geq V_0$ .

### Keywords

Resonance Tunneling  
Nanoscience

### Article info

Received: June 2009

Accepted: Sep. 2009

Published: Dec. 2009

## التنفق الرنيني لنظام الشبكة الفائقة GaN/AlGa<sub>N</sub>

رضا حزام رسن، عدي حاتم شعبان، موفق كاظم عبد الرضا

مجموعة النبايط النانومترية، كلية التربية ابن الهيثم، جامعة بغداد، الأعظمية، بغداد، العراق.

### الخلاصة

الهدف من هذا العمل هو تقديم دراسة نظرية للتنفق الإلكتروني في الشبكات الفائقة لاشباه الموصلات. ان الشبكة الفائقة التي أخذت بنظر الإعتبار هي منظومة (GaN/AlGa<sub>N</sub>) ضمن المدى الطاقي  $\varepsilon < V_0$  و  $\varepsilon = V_0$  و  $\varepsilon > V_0$  حيث  $V_0$  تمثل ارتفاع حاجز الجهد داخل المنظومة. تم حساب معامل الانتقال ( $T_N$ ) باستخدام نظرية مصفوفة الانتقال. و تم الحصول على الطاقات الرنينية من علاقة ( $T_N$ ). ومن هكذا نظام فقد حصلنا على حزم طاقة المستويات المؤقتة المسموحة لحالة  $\varepsilon < V_0$  وحزمة واحدة لـ  $\varepsilon \geq V_0$ .

### Introduction

The interest of tunneling and electrical conduction in semiconductor multibarrier heterostructures system date back to early 1970 [1]. Since then, with the help of progress in molecular-beam-epitaxy growth techniques, there has been intensive concentration on such system, both theoretically and experimentally [2]. Most of the theoretical

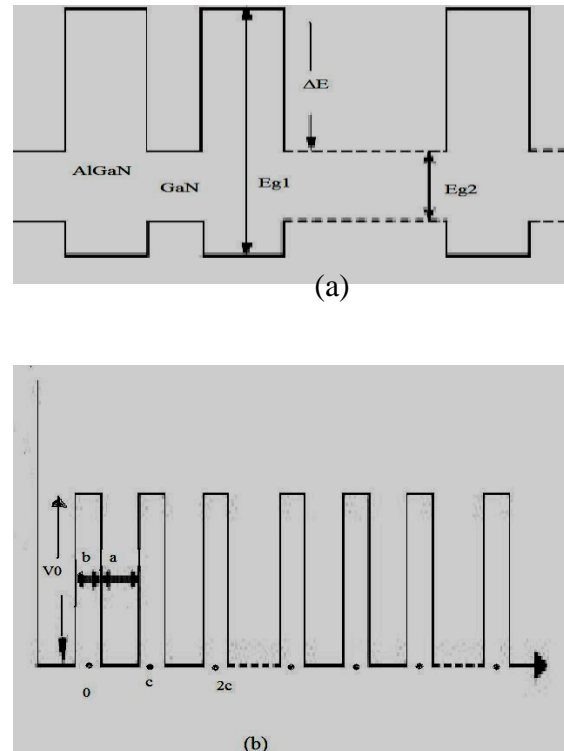
approaches in resonant tunneling through semiconductor multibarrier heterostructures have been limited to the calculation of the transmission coefficient through multibarrier system (MBS). In MBS, the transmission coefficient is the relative probability of an incident electron crossing the multiple barriers. Resonant tunneling in the MBS corresponds to unit transmission coefficient across the structure [3].

A new cutting-edge class of electronic and optoelectronic devices has emerged to a great extent due to III-V semiconductor compounds. Specifically, GaN and AlGaN are the basis for a number of well-established commercial technologies due to their large electron and saturation velocity, high thermal stability and large band gap. All of these characters make them ideal candidates for high power, high temperature and high frequency electronic applications like laser diodes, light emitting diodes, electro-optic modulators, high electron mobility transistor and heterostructure bipolar transistors [4-7]. Some basic properties of GaN/AlGaN still remain poorly studied [7-10]. It is worth noting that GaN/AlGaN system is considered to be highly promising for tomorrow technology [11].

In this paper, we report a study of tunneling of electrons through this MBS. We have obtained the transmission coefficient for a very general form of MBS and carried out quantitative analysis of diverse features of tunneling through the above mentioned MBS with special attention to resonant tunneling which plays an important role for high speed devices [12].

### Theoretical Model

In this model, the multibarrier structure is obtained by alternatively stacking layers of a low-gap and a high-gap semiconductor material having similar band structure. The small gap material GaN forms the well, while the large gap material  $\text{Al}_y\text{Ga}_{1-y}\text{N}$  forms the barrier of the MBS. The schematic energy diagram for the stacking layers is shown in Fig. 1(a). The MBS with well and barrier regions, originated from the band offset is shown in Fig. 1(b). The barrier height is assumed [3,13] to be 88% of the difference between the band gaps of two materials. Our model consists of  $N$  barriers of thickness ( $b$ ), and  $N-1$  wells of thickness ( $a$ ). Thus, the superlattice has a period ( $c$ ), where  $c = (a+b)$ . The height of the potential barrier is considered as  $V_0$ .



**Fig.1 (a): Energy band diagram of stacking layers of GaN /  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{N}$ . (b) The multibarrier heterostructure with well and barrier regions originating.**

### 1. Boundary conditions

The boundary conditions for the wave function ( $\psi$ ) at points of potential discontinuity like P in figure 1(a) are:

- (i) continuity of ( $\psi$ ).
- (ii) continuity of  $1/m\{d\psi/dx\}$ , where ( $m$ ) is the mass of electron at  $x$ .

The points (i) and (ii) are necessary for keeping the probability density and the current density continuous [3, 12].

### 2. Transmission coefficient

The transmission coefficient,  $T$ , across the  $N$  barrier superlattice is formulated on the basis of transfer-matrix method using the one-dimensional time-independent Schrödinger equation, specifically the BenDaniel\_Duke equation for the electron in the potential  $V(x)$ . Effective mass dependent boundary conditions are used at the interfaces of the barrier and well

regions. The transmission coefficient  $T_N$  across  $N$  barriers can be obtained as [3,14]:

$$T_N = \frac{1}{|(W_N)_{11}|^2} = \frac{1}{1 + |(W_N)_{12}|^2} \dots\dots\dots (1)$$

$$|(W_N)_{12}|^2 = \begin{cases} |(M_1)_{12}|^2 \left| \frac{\sin N\theta}{\sin \theta} \right|^2 & \dots\dots\dots (2) \\ |(M_1)_{12}|^2 N^2 \\ |(M_1)_{12}|^2 \left| \frac{\sinh N\theta}{\sinh \theta} \right|^2 \end{cases}$$

$$(M_1)_{12} = (M_1)_{21}^* = \begin{cases} \frac{k_1^2 + k_2^2 f^2}{2i k_1 k_2 f} \sinh k_2 b & \text{for } \varepsilon < V_0 \\ \frac{k_1 b}{2i f} & \text{for } \varepsilon = V_0 \\ \frac{k_1^2 - k_2^2 f^2}{2i k_1 k_2 f} \sin k_3 b & \text{for } \varepsilon > V_0 \end{cases} \dots\dots (3)$$

$$G_{tr} = \begin{cases} 2 \left[ \cos k_1 a \cosh k_2 b + \frac{k_2^2 f - k_1^2}{2k_1 k_2 f} \sin k_1 a \sinh k_2 b \right] & \text{for } \varepsilon < V_0 \\ 2 \left[ \cos k_1 a - \frac{k_1 b}{2f} \sin k_1 a \right] & \text{for } \varepsilon = V_0 \\ 2 \left[ \cos k_1 a \cos k_3 b - \frac{k_2^2 f - k_1^2}{2k_1 k_2 f} \sin k_1 a \sin k_3 b \right] & \text{for } \varepsilon > V_0 \end{cases} \dots\dots\dots (4)$$

$$\lambda_1 = \frac{1}{\lambda_2} = e^{i\theta}, \theta = \cos^{-1} \left( \frac{G_{tr}}{2} \right) \text{ for } G_{tr} < 2,$$

$$\lambda_1 = \lambda_2 = 1, \text{ for } G_{tr} = 2 \dots\dots\dots 5$$

$$\lambda_1 = \frac{1}{\lambda_2} = e^\theta, \theta = \cosh^{-1} \left( \frac{G_{tr}}{2} \right), \text{ for } G_{tr} > 2$$

$$f = \frac{m_w^*}{m_b^*}, \quad k_1^2 = \frac{2m_w^* \varepsilon}{h^2}$$

$$k_2^2 = \frac{2m_b^* (V_0 - \varepsilon)}{h^2}, \quad k_3^2 = \frac{2m_b^* (\varepsilon - V_0)}{h^2}$$

$m_w^*$  and  $m_b^*$  are the effective masses of the well and the barrier region materials of the superlattice,  $a$  and  $b$  are the well-width and barrier-width respectively and  $V_0$  is the height of the potential barrier.

### 3. Condition for resonant tunneling

If the resonant tunneling occurs, the

transmission coefficient,  $T_N$ , becomes unity. For  $G_{tr} \geq 2$  the resonant tunneling can never occur, this is because  $|M_{12}|^2 \left| \frac{\sinh N\theta}{\sinh \theta} \right|^2$  is always greater than zero. Resonant tunneling can occur only according to  $T_N = \frac{1}{1 + |(M_1)_{12}|^2 \left| \frac{\sin N\theta}{\sin \theta} \right|^2}$  which

corresponds to the transmission coefficient for  $G_{tr} < 2$ . The possibility of resonant tunneling arises only when  $N \geq 2$  [15]. The occurrence of resonant tunneling in the  $N$ -barrier system under consideration corresponds to the following equation

$$\frac{\sin N\theta}{\sin \theta} = 0 \dots\dots\dots (6)$$

The energies which satisfy (6) and, hence, correspond to resonant tunneling, are those which conform to the criterion  $G_{tr} < 2$ :

$$\cos \theta = \frac{G_{tr}}{2} \text{ For } G_{tr} < 2 \dots\dots\dots (7)$$

The criterion  $G_{tr} < 2$  corresponds to energies lying in the allowed bands of an infinite crystal consisting of rectangular, barrier-type potentials of  $N$ -barrier system, with periodicity  $(a+b)$ , while  $G_{tr} \geq 2$  corresponds to forbidden energies in such a system [16, 17].

### Numerical Analysis

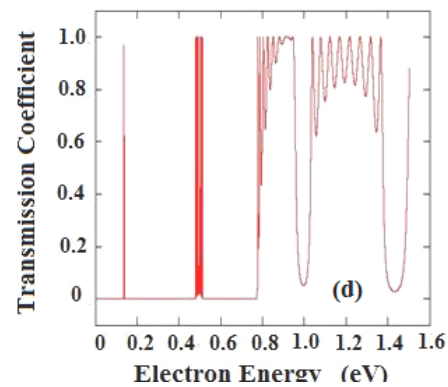
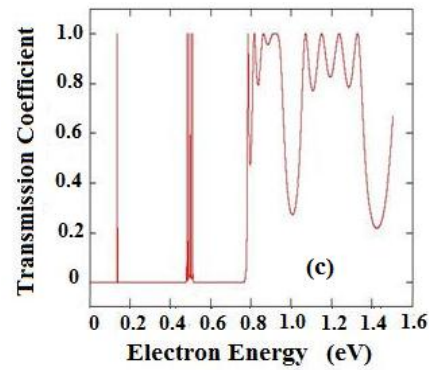
The numerical analysis is basically concerned with the transmission coefficient across multibarrier systems for incident energies  $\varepsilon < V_0$ ,  $\varepsilon = V_0$  and  $\varepsilon > V_0$ . We have chosen the GaN/Al<sub>y</sub>Ga<sub>1-y</sub>N ( $y < 0.45$ ) superlattice with the values of various parameters as follows:

$a$  is the well width =  $ncw \times (a_w)$ , where  $ncw$  is the number of cells in the well material in each well slab and  $a_w$  is the lattice constant of the well material GaN.  $a_w = 5.185$  eV.  $b$  is the barrier width =  $ncb \times a_b$ , and  $ncb$  is the number of unit cells of the barrier material in each barrier slab and  $a_b$  is the lattice constant of the barrier material Al<sub>0.3</sub>Ga<sub>0.7</sub>As.  $a_b = 5.124$  Å,  $m_w^*$  and  $m_b^*$  = the effective masses of the well (GaN) and the barrier Al<sub>0.3</sub>Ga<sub>0.7</sub>N region materials of the superlattice  $m_w^* = 0.2 m_0$  and  $m_b^* = 0.222 m_0$ ;  $m_0$  is the free electron

mass.  $E_{g1}$  and  $E_{g2}$  = energy band gap in the well and barrier Materials.  $E_{g1}$ = 3.5 eV and  $E_{g2}$  = 4.25 eV.  $V_o$  is the height of the potential barrier [0.88 of  $(E_{g2} - E_{g1})$ ] = 0.660 eV.

**Results and Discussion**

The transmission coefficient  $T_N$  across the N barrier system for the three different situations corresponding to the incident energy  $\epsilon < V_o$ ,  $\epsilon = V_o$  and  $\epsilon > V_o$  can be obtained from Eq. (1) in combination with Eqs. (2) and (3). Fig (2) depicts the transmission coefficient versus incident energy. It is show the variation of  $T \sim \epsilon$  for systems with  $n_{cw}=5$ ,  $n_{cb} = 5$  having 2,3,5 and 9 barriers. The  $T \sim \epsilon$  curve for 9 barrier systems with  $n_{cw} = 5$ ,  $n_{cb} = 4$  is presented in graphs in 3(a) and that for  $n_{cw} = 4$  and  $n_{cb}=5$  in graph 3(b). The graphs clearly show that the transmission coefficient varies rapidly and attains the value of unity for certain incident energies. These energies are referred as resonant energies both for  $\epsilon < V_o$  and  $\epsilon > V_o$ .



**Fig.2 Transmission coefficient versus electron energy for GaN / Al<sub>0.3</sub>Ga<sub>0.7</sub>N superlattice by varying number of barriers ‘N’,  $n_{cw} = n_{cb} = 5$  for all the value of N. (a) N = 2, (b) N = 3, (c) N = 5 and (d) N=9.**

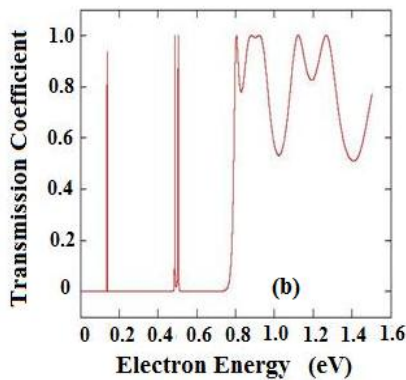
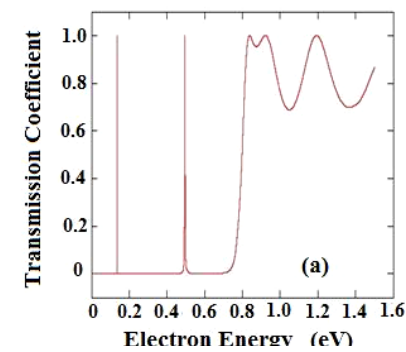
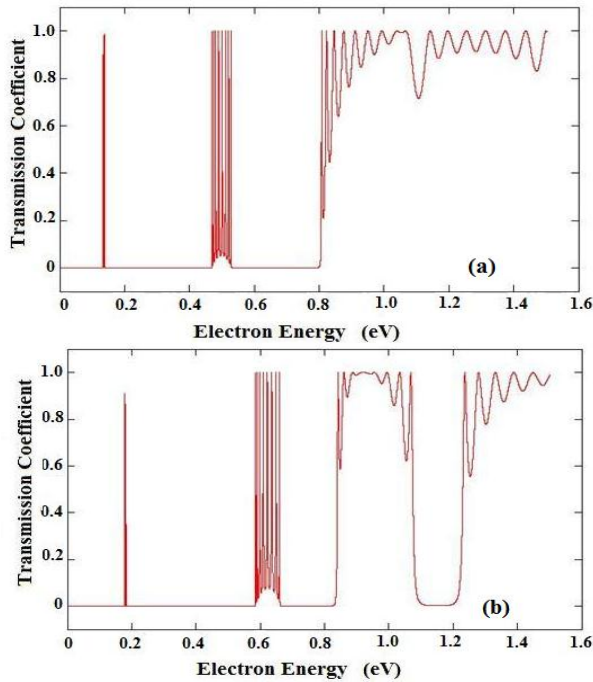


Fig.3 shows that the resonant energy peak is found to be dependent on the number of cells in the well region and in the barrier region. For all the cases T varies in the range zero to one and shows resonant peaks. In MBS the resonant tunneling corresponds to unit T across the structure. The incident energies for which the resonant tunneling occurs are termed as resonant tunneling energies.



**Fig.3 Transmission coefficient versus electron energy for GaN / Al<sub>0.3</sub>Ga<sub>0.7</sub>N superlattice by varying N , ncw , ncb . (a) N= 9, ncw=5, ncb=4 (b) N=9, ncw=4, ncb=5.**

One of the most striking features of the multi-barrier systems is the occurrence of quasi-level resonant tunneling energy states. Incident electrons on the MBS with energies equal to any one of these quasi-level resonant energy states suffer resonant tunneling i.e. electrons incident on the MBS with energies equal to resonant state energies tunnel out without any significant attenuation in their intensity [3, 11, 18]. These quasi-level resonant energy states group themselves into tunneling energy bands separated by forbidden gaps. The number of resonant energy levels in each band in all MBS is found to be (N-1) which is equal to the number of wells in the MBS. It is worth pointing out that a miniband in a superlattice with (N-1) periods contains (N-1) energy levels. In the forbidden region  $T_N$  remains almost zero. For our system in all these graphs, we have obtained two allowed quasi-level energy bands for  $\epsilon < V_0$ , and one band for  $\epsilon \geq V_0$ . The incident electron resonates at the quantum well bound state, and the allowed

tunneling bands are the same number in each multi-barrier system or single finite well. With increasing of N the resonant energies move from central region to the edge of the band and become zero at the maxima. Hence, for resonant energies the variation of the transmission coefficient is not rapid in the higher band. For the present case the number of bound states in the well comes to be three for  $\epsilon < 1\text{eV}$ , and the third and higher tunneling bands correspond to energies,  $\epsilon > V_0$ . The T, for  $\epsilon > V_0$ , oscillates around unity with the above-barrier resonance peaks being less sharp having large widths correspond to the bands when,  $\epsilon < V_0$  [19].

### Conclusions

Transmission coefficient for GaN/Al<sub>y</sub>Ga<sub>1-y</sub>N (with  $y < 0.45$ ) has been computed using the transfer matrix method along with the application of mass dependent boundary condition. The resonant energy values are found to be dependent on the number of barriers, number of cells in the well region, and number of cells in the barrier region. It is observed that the resonant energy state group themselves in to allowed and forbidden energy bands within the entire energy range of  $\epsilon < V_0$  and  $\epsilon > V_0$ . The results indicate that the number of resonant tunneling peaks increases generally with N and at the same time, such peaks become shaper for larger N. From such system we obtained two allowed quasi-level energy bands for  $\epsilon < V_0$ , and one band for  $\epsilon \geq V_0$ .

### Acknowledgement

We acknowledge Prof. Dr. Abdl Jabbar A. Mukhlus, , Dr. Ahlam H. Jaffar Al-Mousawy and Dr. Nidhal Moosa Abdul-Ameer for their useful and fruitful discussion.

### References

- [1] R. Tsu, L. Esaki, Appl. Phys.Lett., 22, (1973), 562.
- [2] Gyungock Kim, Gerald B. Arnold, Physical Review B, 38, (1988),

- 3252.
- [3] J. Nanda, p. k. Mahapatra, C. L. Roy, *Physica B*, 383 (2006), 232.
- [4] B. Van Daele, G. van Tendeloo, K. Jacobs, I. Moerman, M. r. Leys, *Appl. Phys. Lett.*, 85 (2004), 4379.
- [5] R. J. Kaplar, S.R. Kurtz, D.D. Koleske, *Appl. Phys. Lett.*, 85 (2004), 5436.
- [6] A. Y. Polyakov, N.B. Smirnov, A.V. Govorkov, S.J. Pearton, J.M. Zavada, *J. Appl. Phys.*, 94, (2003), 3069.
- [7] E.P. Samuel, K. Talele, U. Zope, D.S. Patil, *Optoelectronics and Advanced Materials-Rapid Communications*, 1, (2007), 221.
- [8] Gmachl C., D.L. Sivco, R.f. Colombelli, F. Capasso, A.Y. Cho, *Letters to Nature*, 415, (2002), 883.
- [9] Mora-Ramos M.E., R. Perez-Alvarez, V.R. Velasco, *Progress in electromagnetic Research Letters*, 1 (2008), 27.
- [10] E.P. Samuel, D.s. Patil, *Progress in Electromagnetics Research Letters*, 1 (2008), 119.
- [11] P.K. Mahapatra, P. Panchadhyayee, S.P. Bhattacharya, Arif Khan, *Physica B*, 403, (2008), 2780.
- [12] C.L. Roy, *Mater. Sci.*, 25, (2002) 469.
- [13] D. Mukherjee, B.R. Nag, *Phys. Rev. B*, 12, (1975), 4338.
- [14] J. Nanda, p. k. Mahapatra, C.L. Roy, *AIP. Conf. Proc.*, 1063, (2008), 45.
- [15] C. L. Roy, Arif Khan, *Phys. Stat. Sol. (b)*, 176, (1993), 101.
- [16] E. Merzbacher, *Quantum Mechanics*, Wiley, New York, 1970.
- [17] C. L. Roy, Arif Khan, *Phys. Stat. Sol. (b)*, 176, (1993), K47.
- [18] Santanu Sinha, S.P. Bhattacharjee, P.K. Mahapatra, *Journal of Physical Sciences*, 11, (2007), 99.
- [19] M. Gluck, A.R. Kolovsky, H.J. Korsh, F. Zimmer, *Phys. Rev. B*, 65 (2002), 115302.