Calculation of ^{22}Na(n,α)^{19}F Cross Sections from the ^{19}F(α,n)^{22}Na Cross Sections by Using the Principle of Detailed Balance

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Abstract

The cross sections of ^{22}Na(n,α)^{19}F reaction were measured through the ^{19}F(α,n)^{22}Na cross sections available in the literature using the principle of detailed balance in the energy range of α-particles (0.9 -6.65) MeV. This work covers for the first time a wide energy range, and a considerable correspondence observed within the resonances of both direct and inverse reactions.

(keywords: ^{22}Na(n,α)^{19}F, Cross Sections, Principle of Detailed Balance)

1. Introduction

Neutron-induced helium-production cross sections play a significant role in the astrophysical processes of nucleosynthesis of light and medium mass nuclei, and it is important for reactor design technology, in particular, for estimating radiation damage, nuclear heating, and induced activity in structural materials. The investigation of the (n,α) reactions is of considerable interest to study reaction mechanisms and nuclear structure, and also they are needed in medical and biological fields to estimate biological effects of neutrons. The inverse of the ^{19}F(α,n)^{22}Na reaction may be an important destruction mechanism for ^{22}Na in neutron-rich, high-temperature or explosive nucleosynthesis [1]. At energies between threshold to 10 MeV, the cross section data of the ^{22}Na(n,α)^{19}F reaction is rather scanty and are not available due to the instability of ^{22}Na. So, in this paper the latest cross sections of ^{19}F(α,n)^{22}Na reaction were interpolated and recalculated in fine steps, from which the cross sections of ^{22}Na(n,α)^{19}F were calculated by means of the principle of detailed balance, covering the mentioned energy range.

2. Theory

A. Cross Section Of ^{19}f(A,N)^{22}na

The data of cross section declared by reference [2] have been recalculated in steps of 50 keV up to 10 MeV of the incident α-particle energy. As well as, the plot of neutron yields (in units of n/α) against α-energy measured by Nakasima et.al. [3] were used to calculate the cross sections in similar fine steps by means of the formula [4]:

$$\sigma(E) = \left[\frac{Y(E_2) - Y(E_1)}{E_2 - E_1}\right]\frac{dE}{dx} \quad \text{------ (1)}$$

where E is the mean of the two close energies E_1 and E_2, and (dE/dx) is the stopping power of α-particles given by Norman et.al. [5].

The measured cross sections (with their percentage of error) from both references are tabulated in Table (1).

B. Cross Section Of ^{22}na(N,A)^{19}f Reaction:

The principle of detailed balance allows us to calculate the cross section of a reaction from its inverse. The relation between the cross section of the reaction

$1 + 2 \longrightarrow 3 + 4$ and its inverse is given by [6, 7]:

$$\frac{\sigma_{34\to12}}{\sigma_{12\to34}} = \frac{(1+\delta_{34})g_1 g_2 P_{12}}{(1+\delta_{12})g_3 g_4 P_{34}} \quad \text{------ (2)}$$

where δ_{34}, δ_{12} are the kronecker delta, which accounts for the possibility that the two particles in the exit and entrance channels might be initiated, g_i is the spin multiplicity of the ith particle, and ($g_i = 2\,I_i + 1$, where I_i is the spin of the ith particle), and P_i is the momentum of the ith particle, $P_i^2 = 2\,m_i\,T_i$

Applying this principle to our reactions,

^{19}F + α $\longrightarrow$ ^{22}Na + n direct reaction
^{22}Na + n $\longrightarrow$ ^{19}F + α inverse reaction

we obtain:

$$\frac{\sigma_{n,\alpha}}{\sigma_{\alpha,n}} = \frac{P_\alpha^{\,2}}{P_n^{\,2}}\,\frac{(2I_\alpha+1)(2I_{^{19}F}+1)}{(2I_n+1)(2I_{^{22}Na}+1)} \quad \text{-----(3)}$$

$$= \frac{m_\alpha E_\alpha}{m_n E_n}\,\frac{(2\times0+1)(2\times\frac{1}{2}+1)}{(2\times\frac{1}{2}+1)(2\times3+1)}$$

Then $$\sigma_{n,\alpha} = 0.56689\,\frac{E_\alpha}{E_n}\,\sigma_{\alpha,n} \quad \text{------(4)}$$

The value of E_n in the inverse reaction ^{22}Na(n,α)^{19}F corresponding to E_α in the direct reaction is given approximately by [8]:

$$E_n(lab.) \cong \frac{M(^{19}F)}{M(^{22}Na)}(E_\alpha - E_{thr}) \quad \text{------(5)}$$

where E_{thr} = 2.36 MeV for the ^{19}F(α, n)^{22}Na reaction,

Using eqs. (4) & (5) we obtain the cross sections of the inverse reaction ^{22}Na(n,α)^{19}F as tabulated in Table (2).

3. Results And Discussion:

The reproduced cross sections of ^{19}F(α,n)^{22}Na reaction in the energy range (2.4-10 MeV) are listed in Table (1). The corresponding cross sections of ^{22}Na(n,α)^{19}F reaction calculated using the principle of detailed balance in the energy range (0.9 – 6.65 MeV) are listed in Table (2) and plotted in Figure (1). The lower energy regions, near threshold, were not presented due to the disagreement between the cross section data of the ^{19}F(α,n)^{22}Na reaction for the two references in this energy range. These results and especially in this energy range were not done previously except the work of Wrean [1] in the energy range (0 – 700 keV); so there is no available data for comparison.

Through Tables (1 and 2) and Figure (1), the detailed correspondence between the reaction ^{19}F(α,n)^{22}Na and its inverse ^{22}Na(n,α)^{19}F are observed revealing a good approve of our calculations using the reciprocity theorem.

5. Conclusions:

The cross section measurement of nuclear reactions for the unstable target nuclei which are incapable experimentally can be performed using the principle of detailed balance. This method reveals its high adequacy in the calculation of ^{22}Na(n,α)^{19}F cross section from the ^{19}F(α,n)^{22}Na reaction cross sections.

Table (2) : The adopted Cross Section of ^{22}Na(n,□)^{19}F Reaction Calculated from the ^{19}F(□,n)^{22}Na Reaction.

E$_{n□}$(MeV)	**(□$_{n,□}$ in mb)**		
	Nakasima et.al. 1992 [3] (10%)	**Shibata and Matsunobu 2003 [2] (10%)**	**Weighted Average ± Error**
0.9	54.96100 ± 5.496100	48.08090 ± 4.808090	51.0636 ± 3.6188
0.95	57.33550 ± 5.733550	54.49060 ± 5.449060	55.8407 ± 3.9498
1	59.11440 ± 5.911440	57.97890 ± 5.797890	58.5356 ± 4.1393
1.05	61.20040 ± 6.120040	64.38110 ± 6.438110	62.7102 ± 4.4357
1.1	62.96310 ± 6.296310	68.23800 ± 6.823800	65.3888 ± 4.6274
1.15	66.05170 ± 6.605170	72.64660 ± 7.264660	69.0363 ± 4.8871
1.2	69.98470 ± 6.998470	78.41030 ± 7.841030	73.7206 ± 5.2212
1.25	74.65630 ± 7.465630	83.58690 ± 8.358690	78.6192 ± 5.5681
1.3	77.61680 ± 7.761680	88.13650 ± 8.813650	82.2117 ± 5.8249
1.35	81.65120 ± 8.165120	93.32690 ± 9.332690	86.7134 ± 6.1452
1.4	85.78260 ± 8.578260	99.39290 ± 9.939290	91.5928 ± 6.4941
1.45	90.36180 ± 9.036180	103.2189 ± 10.32189	95.9402 ± 6.7989
1.5	94.66890 ± 9.466890	106.9966 ± 10.69966	100.082 ± 7.0901
1.55	99.99630 ± 9.999630	110.7512 ± 11.07512	104.8263 ± 7.422
1.6	105.5954 ± 10.55954	114.5469 ± 11.45469	109.7078 ± 7.7639
1.65	108.2471 ± 10.82471	117.9528 ± 11.79528	112.6843 ± 7.9753
1.7	113.9749 ± 11.39749	123.4455 ± 12.34455	118.333 ± 8.3741
1.75	119.9108 ± 11.99108	127.4576 ± 12.74576	123.4542 ± 8.7336
1.8	123.7133 ± 12.37133	132.1228 ± 13.21228	127.6419 ± 9.0305
1.85	130.6263 ± 13.06263	134.1826 ± 13.41862	132.3567 ± 9.3599
1.9	136.1019 ± 13.61019	138.0203 ± 13.80203	137.0477 ± 9.691
1.95	141.3770 ± 14.13770	139.7445 ± 13.97445	140.5513 ± 9.9386
2	146.9303 ± 14.69303	140.9251 ± 14.09251	143.8025 ± 10.1706
2.05	152.3665 ± 15.23665	145.7371 ± 14.57371	148.9045 ± 10.5317
2.1	158.4026 ± 15.84026	153.9572 ± 15.39572	156.1167 ± 11.0402
2.15	164.8331 ± 16.48331	160.6854 ± 16.06854	162.7085 ± 11.5062
2.2	170.1389 ± 17.01389	168.1487 ± 16.81487	169.1321 ± 11.9597
2.25	175.1453 ± 17.51453	178.5950 ± 17.85950	176.8269 ± 12.5041
2.3	180.9686 ± 18.09686	187.3928 ± 18.73928	184.0687 ± 13.0176
2.35	185.7008 ± 18.57008	193.4562 ± 19.34562	189.4199 ± 13.3968
2.4	190.5797 ± 19.05797	197.8783 ± 19.78783	194.0919 ± 13.7268
2.45	198.0207 ± 19.80207	204.5616 ± 20.45616	201.1849 ± 14.2278
2.5	203.4998 ± 20.34998	212.4128 ± 21.24128	207.7654 ± 14.6946
2.55	208.2430 ± 20.82430	218.8230 ± 21.88230	213.271 ± 15.0852
2.6	213.9576 ± 21.39576	223.8816 ± 22.38816	218.6948 ± 15.468
2.65	219.1259 ± 21.91259	229.9099 ± 22.99099	224.2591 ± 15.8621
2.7	223.7978 ± 22.37978	238.0469 ± 23.80469	230.4831 ± 16.3054
2.75	228.0188 ± 22.80188	243.7030 ± 24.37030	235.34 ± 16.6502
2.8	234.3834 ± 23.43834	245.9491 ± 24.59491	239.8879 ± 16.9676

Table (2) Continued

$E_{n□}$(MeV)	**(□$_{n,□}$ in mb)**		
	Nakasima et.al. 1992 [3] (10%)	**Shibata and Matsunobu 2003 [2] (10%)**	**Weighted Average ± Error**
4.85	332.6140 ± 33.26140	343.7158 ± 34.37158	337.9827 ± 23.9022
4.9	333.4375 ± 33.34375	344.1036 ± 34.41036	338.6027 ± 23.9458
4.95	334.1212 ± 33.41212	345.6168 ± 34.56168	339.6746 ± 24.0221
5	334.9638 ± 33.49638	347.2314 ± 34.72314	340.8771 ± 24.1075
5.05	335.6622 ± 33.56622	349.4705 ± 34.94705	342.2882 ± 24.2083
5.1	335.6879 ± 33.46879	351.3875 ± 35.13875	343.1791 ± 24.2728
5.15	336.1857 ± 33.61857	353.6666 ± 35.36666	344.4834 ± 24.3665
5.2	337.0611 ± 33.70611	355.6876 ± 35.56876	345.8739 ± 24.4658
5.25	337.3776 ± 33.73776	356.5656 ± 35.65656	346.4414 ± 24.5065
5.3	338.1412 ± 33.81412	359.3990 ± 35.93990	348.1229 ± 24.6274
5.35	338.7646 ± 33.87646	358.8387 ± 35.88387	348.2245 ± 24.6334
5.4	339.2732 ± 33.92732	360.9924 ± 36.09924	349.4598 ± 24.7224
5.45	339.5859 ± 33.95859	361.6681 ± 36.16681	349.9323 ± 24.7562
5.5	339.5193 ± 33.95193	361.9644 ± 36.19644	350.0244 ± 24.7631
5.55	339.3821 ± 33.93821	363.7999 ± 36.37999	350.7441 ± 24.8163
5.6	339.9098 ± 33.99098	366.6802 ± 36.66802	352.2822 ± 24.928
5.65	340.8289 ± 34.08289	373.9598 ± 37.39589	355.862 ± 25.1903
5.7	341.3892 ± 34.13892	377.5791 ± 37.75791	357.6671 ± 25.3229
5.75	342.2945 ± 34.22945	378.7547 ± 37.87547	358.6857 ± 25.3953
5.8	342.7775 ± 34.27775	380.5566 ± 38.05566	359.6992 ± 25.4692
5.85	341.9173 ± 34.19173	381.9815 ± 38.19815	359.7388 ± 25.4763
5.9	342.6084 ± 34.26084	382.4201 ± 38.24201	360.3347 ± 25.5179
5.95	343.8505 ± 34.38505	383.0640 ± 38.30640	361.348 ± 25.5883
6	342.5726 ± 34.25726	384.5978 ± 38.45978	361.1646 ± 25.5808
6.05	342.9837 ± 34.29837	384.1378 ± 38.41378	361.2389 ± 25.5843
6.1	345.4842 ± 34.54842	385.7220 ± 38.57220	363.3955 ± 25.7348
6.15	345.2757 ± 34.52757	386.2159 ± 38.62159	363.4616 ± 25.7408
6.2	346.3491 ± 34.63491	388.2819 ± 38.82819	364.9297 ± 25.8464
6.25	345.4777 ± 34.54777	389.9077 ± 38.99077	365.0182 ± 25.8577
6.3	343.6108 ± 34.36108	391.4934 ± 39.14934	364.4464 ± 25.8249
6.35	343.9536 ± 34.39536	393.7367 ± 39.37367	365.5007 ± 25.9036
6.4	343.1968 ± 34.31968	394.4645 ± 39.44645	365.2847 ± 25.8918
6.45	342.3385 ± 34.23385	395.6925 ± 39.56925	365.1785 ± 25.8894
6.5	341.5059 ± 34.15059	396.7368 ± 39.67368	365.0123 ± 25.8824
6.55	340.1263 ± 34.01263	398.6062 ± 39.86062	364.7657 ± 25.8735
6.6	339.3204 ± 33.93204	399.9859 ± 39.99859	364.7084 ± 25.8755
6.65	338.1565 ± 33.81565	401.3959 ± 40.13959	364.4078 ± 25.8616

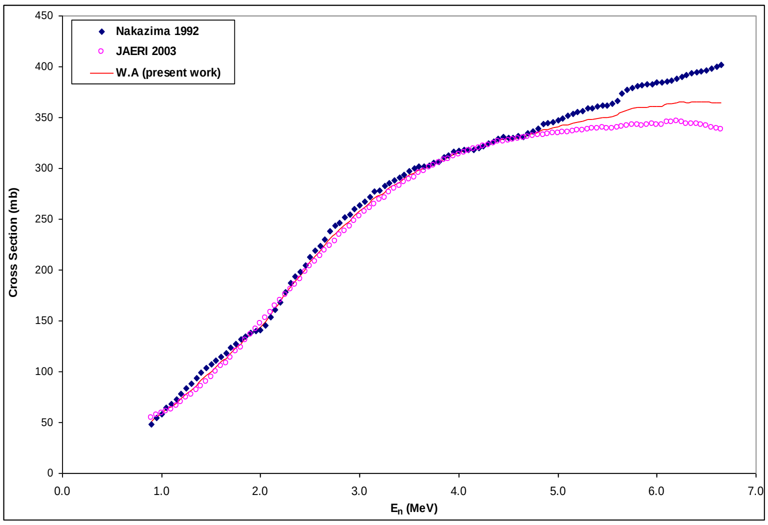

Figure (1): Cross Section as a function of neutron energy for the ^{22}Na(n,a)^{19}F Reaction.

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الملخص

تم حساب المقاطع العرضية للتفاعل النووي $^{22}Na(n,\alpha)^{19}F$ من المقاطع العرضية للتفاعل $^{19}F(\alpha,n)^{22}Na$ المنشورة في مراكز البيانات النووية NDC باستخدام مبدأ التوازن التفصيلي في مدى طاقة جسيمات ألفا MeV (0.9 – 6.65). في هذه الدراسة ولأول مرة تم تغطية مدى طاقي واسع للقذائف، حيث وجدت توافق جيد بين الرنائن المقاسة من التفاعلات المباشرة والعكسية.