

# Numerical Solution of Oscillatory Reaction – Diffusion System of $\lambda - \omega$ Type

<sup>1</sup>Saad A. Manaa & <sup>2</sup>Mohammed A. Moheemmed

<sup>1</sup>Department of Mathematics, College of Computers Sciences and Mathematics, Mosul University, Mosul, Iraq

<sup>2</sup>Department of Mathematics, College of Education for Women, Tikrit University, Tikrit, Iraq

## Abstract

Numerical solution of oscillatory reaction–diffusion system of  $\lambda - \omega$  type was done by using two finite difference schemes . The first one is the explicit scheme and the second one is the implicit rank–Nicholson scheme with averaging of  $f(u, v)$  and without averaging of  $f(u, v)$ . The comparison showed that Crank–Nicholson scheme is better than explicit scheme and Crank–Nicholson scheme with averaging of  $f(u, v)$  is more accurate but it needs more time and double storage and double time steps than Crank–Nicholson scheme without averaging of  $f(u, v)$ .

## 1. Introduction and Mathematical Model

$\lambda - \omega$  Systems are a class of simple examples of two coupled reaction – diffusion equations whose kinetics have a stable limit cycle:

$$\left. \begin{aligned} u_t &= u_{xx} + \lambda(r)u - \omega(r)v \\ v_t &= v_{xx} + \omega(r)u + \lambda(r)v \end{aligned} \right\} \quad (1a)$$

Here  $u$  and  $v$  are functions of space  $x$  and time  $t$ , with  $x \in R$  and  $t > 0$ , and  $r = \sqrt{u^2 + v^2}$  [1].  $\lambda - \omega$  Systems are an important prototype for the study of reaction – diffusion equations which have been used to model a number of biological and chemical systems [2]. Many systems in biology and chemistry are intrinsically oscillatory. In such cases, the stable state in the absence of spatial variation is not a stationary equilibrium, but rather consists of temporal oscillations in the interacting chemical or biological species. Examples include intracellular calcium signaling, the Belousov – Zhabotinskii reaction and some predator – prey interactions. These systems also exhibit spatial interactions, which are often modeled by diffusion. This combination of local oscillations and spatial diffusion produces a very wide range of spatiotemporal behaviors, including spiral waves, target patterns and spatiotemporal chaos. In one spatial dimension, the equivalent of both spiral waves and target patterns are periodic travelling waves, which are periodic functions of space and time, moving with constant shape and speed. Sherratt studied various features of the oscillatory reaction-diffusion systems of  $\lambda - \omega$  type in series of papers. He presented numerical evidence for a complex sequence of bifurcations in the unstable region of parameter space [6]. He also used numerical techniques to give suggestion that the spatiotemporal irregularities are genuinely chaotic [3]. He also made a comparison between the numerical solutions of the oscillatory reaction-diffusion systems of  $\lambda - \omega$  type [4]. He used numerical results suggesting that when this system is solved on semi-infinite domain subject to Dirichlet boundary conditions in which the variables are fixed at zero periodic travelling waves develop in the domain [5]. Borzi and Griesse presented the formulation, analysis, and numerical solution of distributed optimal control problems governed by lambda-omega systems [6]. Garvie and Blowey undertook the numerical analysis of a

reaction-diffusion system of  $\lambda - \omega$  type, their results are presented for a fully practical piecewise linear finite element method by mimicing results in the continuous case [7]. In this paper, the numerical solution of an oscillatory reaction-diffusion system of  $\lambda - \omega$  type by the use of two finite difference schemes is done to the following system [4].

$$\left. \begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + (1 - u^2 - v^2)(u - 3v) \\ \frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial x^2} + (1 - u^2 - v^2)(3u + v) \end{aligned} \right\} \quad (1b)$$

With initial and boundary conditions  $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$  at

$x = 0$  and  $u = v = 0.01$  at  $x = 0$ ,  $u = v = 0$ , for  $x > 0$  at  $t = 0$  and  $u = v = 0$  at  $x = L$  where  $L$  is sufficiently large number.

## 2. Discretization of the $\lambda - \omega$ System by Using Explicit Scheme

By using the finite difference approximations for the derivatives [8], we have

$$\begin{aligned} \frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \\ + (1 - (u_{i,j})^2 - (v_{i,j})^2)(u_{i,j} - 3v_{i,j}) \Rightarrow \\ u_{i,j+1} = r(u_{i-1,j} + u_{i+1,j}) + (1 - 2r + k)u_{i,j} - k(u_{i,j})^3 \\ + 3kv_{i,j}(u_{i,j})^2 - k(v_{i,j})^2 u_{i,j} - 3kv_{i,j} + 3k(v_{i,j})^2 \end{aligned} \quad (2)$$

Where  $r = k/h^2$

For the boundary condition  $u_x = 0$ , by using the central difference formula, we get

$$\frac{u_{1,j} - u_{-1,j}}{2h} = 0 \Rightarrow \quad (3)$$

$$u_{1,j} = u_{-1,j}$$

from (2), we have, for  $i = 0$

$$\begin{aligned} u_{0,j+1} = r(u_{-1,j} + u_{1,j}) + (1 - 2r + k)u_{0,j} - k(u_{0,j})^3 \\ + 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^2 \end{aligned} \quad (4)$$

Substitute (3) in (4), we obtain

$$\begin{aligned} u_{0,j+1} = 2ru_{1,j} + (1 - 2r + k)u_{0,j} - k(u_{0,j})^3 \\ + 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^2 \end{aligned} \quad (5)$$

Equation (5) represents the right boundary condition, with respect to equation (1b)

$$\begin{aligned} \frac{v_{i,j} + 1 - v_{i,j}}{k} &= \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{h^2} \\ &+ \left(1 - (u_{i,j})^2 - (v_{i,j})^2\right) (3u_{i,j} + v_{i,j}) \\ v_{i,j+1} &= r(v_{i-1,j} + v_{i+1,j}) + (1 - 2r + k)v_{i,j} - k(v_{i,j})^3 \quad (6) \\ &- kv_{i,j}(u_{i,j})^2 - 3k(v_{i,j})^2 u_{i,j} + 3ku_{i,j} - 3k(u_{i,j})^3 \end{aligned}$$

The boundary condition  $v_x = 0$ , by using the central difference formula, we have:

$$\frac{v_{1,j} - v_{-1,j}}{2h} = 0 \Rightarrow v_{1,j} = v_{-1,j} \quad (7)$$

From (6) we have, for  $i = 0$

$$\begin{aligned} v_{0,j+1} &= r(v_{-1,j} + v_{1,j}) + (1 - 2r + k)v_{0,j} - k(v_{0,j})^3 \quad (8) \\ &- kv_{0,j}(u_{0,j})^2 - 3k(v_{0,j})^2 u_{0,j} + 3ku_{0,j} - 3k(u_{0,j})^3 \end{aligned}$$

Substitute (7) in (8), we obtain

$$\begin{aligned} v_{0,j+1} &= 2rv_{1,j} + (1 - 2r + k)v_{0,j} - k(v_{0,j})^3 \quad (9) \\ &- kv_{0,j}(u_{0,j})^2 - 3k(v_{0,j})^2 u_{0,j} + 3ku_{0,j} - 3k(u_{0,j})^3 \end{aligned}$$

which represents the right boundary condition, with respect to equation (1b).

### 3. Discretization of the $\lambda - \omega$ System by Using of Crank - Nicholson Scheme without Averaging of $f(u, v)$

The diffusion term  $u_{xx}$  in this method is represented with central differences, with their values at the current and previous time steps averaged [8].

$$\begin{aligned} \frac{u_{i,j+1} - u_{i,j}}{k} &= \frac{u_{i+1,j+1} - 2u_{i,j} + u_{i-1,j+1}}{2h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2} \\ &+ \left(1 - (u_{i,j})^2 - (v_{i,j})^2\right) (u_{i,j} - 3v_{i,j}) \Rightarrow \\ &-r(u_{i-1,j+1} + u_{i+1,j+1}) + (2 + 2r)u_{i,j+1} = r(u_{i-1,j} + u_{i+1,j}) \\ &+ (2 - 2r + k)u_{i,j} - k(u_{i,j})^3 + 3kv_{i,j}(u_{i,j})^2 - k(v_{i,j})^2 u_{i,j} - 3kv_{i,j} + 3k(v_{i,j})^3 \quad (10) \end{aligned}$$

For the boundary condition  $u_x = 0$ , from central difference formula, we have

$$\frac{u_{1,j} - u_{-1,j}}{2h} = 0 \Rightarrow u_{1,j} = u_{-1,j} \quad (11)$$

from (10), we have, for  $i = 0$

$$\begin{aligned} &-r(u_{-1,j+1} + u_{1,j+1}) + (2 + 2r)u_{0,j+1} = r(u_{-1,j} + u_{1,j}) \\ &+ (2 - 2r + k)u_{0,j} - k(u_{0,j})^3 + 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^3 \end{aligned}$$

from (11), we get

$$u_{1,j+1} = u_{-1,j+1} \quad (12)$$

substitute (11) and (12) in (10), we obtain the equation for the left boundary condition which is

$$\begin{aligned} &-2ru_{1,j+1} + (2 + 2r)u_{0,j+1} = 2ru_{1,j} + (2 - 2r + k)u_{0,j} - k(u_{0,j})^3 \quad (13) \\ &+ 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^3 \end{aligned}$$

for (b), we have

$$\begin{aligned} \frac{v_{i,j+1} - v_{i,j}}{k} &= \frac{v_{i+1,j+1} - 2v_{i,j} + v_{i-1,j+1}}{2h^2} + \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{2h^2} \\ &+ \left(1 - (u_{i,j})^2 - (v_{i,j})^2\right) (3u_{i,j} + v_{i,j}) \Rightarrow \\ &-r(v_{i-1,j+1} + v_{i+1,j+1}) + (2 + 2r)v_{i,j+1} = r(v_{i-1,j} + v_{i+1,j}) \\ &+ (2 - 2r + k)v_{i,j} - k(v_{i,j})^3 - kv_{i,j}(u_{i,j})^2 - 3k(v_{i,j})^2 u_{i,j} + 3ku_{i,j} - 3k(u_{i,j})^3 \quad (14) \end{aligned}$$

For the boundary condition  $v_x = 0$ , from central difference formula, we have

$$\frac{v_{1,j} - v_{-1,j}}{2h} = 0 \Rightarrow v_{1,j} = v_{-1,j} \quad (15)$$

from (14), we have

$$\begin{aligned} &-r(v_{-1,j+1} + v_{1,j+1}) + (2 + 2r)v_{0,j+1} = r(v_{-1,j} + v_{1,j}) \\ &+ (2 - 2r + k)v_{0,j} - k(v_{0,j})^3 - kv_{0,j}(u_{0,j})^2 \\ &- 3k(v_{0,j})^2 u_{0,j} + 3ku_{0,j} - 3k(u_{0,j})^3 \end{aligned}$$

from (15), we have

$$v_{1,j+1} = v_{-1,j+1} \quad (16)$$

Substitute (15) and (16) in (14), we have the equation for the left boundary condition, with respect to equation (1b)-

$$\begin{aligned} &-2rv_{1,j+1} + (2 + 2r)v_{0,j+1} = 2rv_{1,j} + (2 - 2r + k)v_{0,j} - k(v_{0,j})^3 \quad (17) \\ &-k(v_{0,j})(u_{0,j})^2 - 3k(v_{0,j})^2 u_{0,j} + 3ku_{0,j} - 3k(u_{0,j})^3 \end{aligned}$$

The equations in above are especially pleasing to view in their tridiagonal matrix form  $A_1 X_1 = B_1$ , where  $A_1$  is the coefficient matrix, X is the unknown vector and B is the known vector as shown below:

$$\begin{bmatrix} 2+2r & -2r & & & & & \\ -r & 2+2r & -r & & & & \\ & & & & & & \\ & & -r & 2+2r & -r & & \\ & & & -r & 2+2r & -r & \\ & & & & -r & 2+2r & \\ & & & & & & 1 \end{bmatrix} \begin{bmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{p,j+1} \\ \vdots \\ u_{n-2,j+1} \\ u_{n-1,j+1} \\ u_{n,j+1} \end{bmatrix} =$$

$$\begin{bmatrix} (2-2r+k)u_{1,j} + 2ru_{2,j} - k(u_{1,j})^3 + 3kv_{1,j}(u_{1,j})^2 - k(v_{1,j})^2 u_{1,j} - 3kv_{1,j} + 3k(v_{1,j})^3 \\ ru_{1,j} + (2-2r+k)u_{2,j} + ru_{3,j} - k(u_{2,j})^3 + 3kv_{2,j}(u_{2,j})^2 - k(v_{2,j})^2 u_{2,j} - 3kv_{2,j} + 3k(v_{2,j})^3 \\ ru_{p-1,j} + (2-2r+k)u_{p,j} + ru_{p+1,j} - k(u_{p,j})^3 + 3kv_{p,j}(u_{p,j})^2 - k(v_{p,j})^2 u_{p,j} - 3kv_{p,j} + 3k(v_{p,j})^3 \\ ru_{n-3,j} + (2-2r+k)u_{n-2,j} + ru_{n-1,j} - k(u_{n-2,j})^3 + 3kv_{n-2,j}(u_{n-2,j})^2 - k(v_{n-2,j})^2 u_{n-2,j} - 3kv_{n-2,j} + 3k(v_{n-2,j})^3 \\ ru_{n-2,j} + (2-2r+k)u_{n-1,j} - k(u_{n-1,j})^3 + 3kv_{n-1,j}(u_{n-1,j})^2 - k(v_{n-1,j})^2 u_{n-1,j} - 3kv_{n-1,j} + 3k(v_{n-1,j})^3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+2r & -2r & & & & & \\ -r & 2+2r & -r & & & & \\ & & & & & & \\ & & -r & 2+2r & -r & & \\ & & & -r & 2+2r & -r & \\ & & & & -r & 2+2r & \\ & & & & & & 1 \end{bmatrix} \begin{bmatrix} v_{1,j+1} \\ v_{2,j+1} \\ \vdots \\ v_{p,j+1} \\ \vdots \\ v_{n-2,j+1} \\ v_{n-1,j+1} \\ v_{n,j+1} \end{bmatrix} =$$

$$\begin{bmatrix} (2-2r+k)v_{1,j} + 2rv_{2,j} - k(v_{1,j})^3 - 3ku_{1,j}(v_{1,j})^2 - 3k(u_{1,j})^2 v_{1,j} + 3ku_{1,j} - 3k(u_{1,j})^3 \\ rv_{1,j} + (2-2r+k)v_{2,j} + rv_{3,j} - k(v_{2,j})^3 - ku_{2,j}(v_{2,j})^2 - 3k(u_{2,j})^2 v_{2,j} + 3ku_{2,j} - 3k(u_{2,j})^3 \\ rv_{p-1,j} + (2-2r+k)v_{p,j} + rv_{p+1,j} - k(v_{p,j})^3 - ku_{p,j}(v_{p,j})^2 - 3k(u_{p,j})^2 v_{p,j} + 3ku_{p,j} - 3k(u_{p,j})^3 \\ rv_{n-3,j} + (2-2r+k)v_{n-2,j} + rv_{n-1,j} - k(v_{n-2,j})^3 - ku_{n-2,j}(v_{n-2,j})^2 - 3k(u_{n-2,j})^2 v_{n-2,j} + 3ku_{n-2,j} - 3k(u_{n-2,j})^3 \\ rv_{n-2,j} + (2-2r+k)v_{n-1,j} - k(v_{n-1,j})^3 - kv_{n-1,j}(u_{n-1,j})^2 - 3k(v_{n-1,j})^2 u_{n-1,j} + 3ku_{n-1,j} - 3k(u_{n-1,j})^3 \\ 0 \end{bmatrix}$$

#### 4. Discretization of the $\lambda - \omega$ System by Using of Crank – Nicholson Scheme with Averaging of $f(u, v)$

In this scheme the functions  $u$  and  $v$  in system (1b) will be replaced by their values at the current and previous steps averaged.

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2} \quad (18)$$

$$+ \left( 1 - \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right)^2 - \left( \frac{v_{i,j+1} + v_{i,j}}{2} \right)^2 \right)$$

$$\left( \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right) - 3 \left( \frac{v_{i,j+1} + v_{i,j}}{2} \right) \right) \Rightarrow$$

$$\frac{u_{i,j+1/2} - u_{i,j}}{k/2} = \frac{u_{i+1,j+1/2} - 2u_{i,j+1/2} + u_{i-1,j+1/2}}{2h^2} \quad (19a)$$

$$+ \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2} + (1 - (u_{i,j})^2) - (u_{i,j} - 3v_{i,j})$$

$$\frac{u_{i,j+1} - u_{i,j+1/2}}{k/2} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2h^2} \quad (19b)$$

$$+ \frac{u_{i+1,j+1/2} - 2u_{i,j+1/2} + u_{i-1,j+1/2}}{2h^2} + (1 - (u_{i,j+1/2})^2) - (v_{i,j+1/2})^2$$

$$(u_{i,j+1/2} - 3v_{i,j+1/2}) \Rightarrow$$

$$-r_1(u_{i-1,j+1} + u_{i+1,j+1}) + (2 + 2r_1)u_{i,j+1} = r_1(u_{i-1,j+1/2} + u_{i+1,j+1/2}) \quad (20a)$$

$$+ (2 - 2r_1 + k_1)u_{i,j+1/2} - k_1(u_{i,j+1/2})^3 + 3k_1v_{i,j+1/2}(u_{i,j+1/2})^2$$

$$- k_1(v_{i,j+1/2})^2 u_{i,j+1/2} - 3k_1v_{i,j+1/2} \quad (20b)$$

Where  $r_1 = k/2h^2$

By the same manner we have followed in (4), with repeat to right boundary condition  $u_x = 0$ , we get

$$-2r_1u_{1,j+1/2} + (2 + 2r_1)u_{0,j+1/2} = 2r_1u_{1,j} + (2 - 2r_1 + k_1)u_{0,j} \quad (21a)$$

$$-k_1(u_{0,j})^3 + 3k_1v_{0,j}(u_{0,j})^2 - k_1(v_{0,j})^2 u_{0,j} - 3k_1v_{0,j} + 3k_1(v_{0,j})^3$$

$$-2r_1u_{1,j+1} + (2 + 2r_1)u_{0,j+1} = 2r_1u_{1,j+1/2} + (2 - 2r_1 + k_1)u_{0,j+1/2} \quad (21b)$$

$$-k_1(u_{0,j+1/2})^3 + 3k_1v_{0,j+1/2}(u_{0,j+1/2})^2 - k_1(v_{0,j+1/2})^2 u_{0,j+1/2}$$

$$- 3k_1v_{0,j+1/2} + 3k_1(v_{0,j+1/2})^3$$

with repeat to (1b), we have

$$\frac{v_{i,j+1} - v_{i,j}}{k} = \frac{v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1}}{2h^2} + \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{2h^2} \quad (22)$$

$$+ \left( 1 - \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right)^2 - \left( \frac{v_{i,j+1} + v_{i,j}}{2} \right)^2 \right)$$

$$\left( 3 \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right) + \left( \frac{v_{i,j+1} + v_{i,j}}{2} \right) \right) \Rightarrow$$

$$\frac{v_{i,j+1/2} - v_{i,j}}{k/2} = \frac{v_{i+1,j+1/2} - 2v_{i,j+1/2} + v_{i-1,j+1/2}}{2h^2}$$

$$+ \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{2h^2} + (1 - (u_{i,j})^2) - (v_{i,j})^2 (3u_{i,j} + v_{i,j}) \quad (23a)$$

$$\frac{v_{i,j+1} - v_{i,j+1/2}}{k/2} = \frac{v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1}}{2h^2}$$

$$+ \frac{v_{i+1,j+1/2} - 2v_{i,j+1/2} + v_{i-1,j+1/2}}{2h^2} + (1 - (u_{i,j+1/2})^2) - (v_{i,j+1/2})^2$$

$$(3u_{i,j+1/2} + v_{i,j+1/2}) \Rightarrow$$

$$-r_1(v_{i-1,j+1/2} + v_{i+1,j+1/2}) + (2 + 2r_1)v_{i,j+1/2} = r_1(v_{i-1,j} + v_{i+1,j})$$

$$+ (2 - 2r_1 + k_1)v_{i,j} - k_1(u_{i,j})^3 - k_1v_{i,j}(u_{i,j})^2 \quad (24a)$$

$$- 3k_1(v_{i,j})^2 u_{i,j} + 3k_1u_{i,j} - 3k_1(u_{i,j})^3$$

$$-r_1(u_{i-1,j+1} + u_{i+1,j+1}) + (2 + 2r_1)u_{i,j+1} = r_1(u_{i-1,j+1/2} + u_{i+1,j+1/2})$$

$$+ (2 - 2r_1 + k_1)u_{i,j+1/2} - k_1(u_{i,j+1/2})^3 - k_1v_{i,j+1/2}(u_{i,j+1/2})^2 \quad (24b)$$

$$- k_1(v_{i,j+1/2})^2 u_{i,j+1/2} + 3k_1(u_{i,j+1/2})^3$$

By the same manner we have already followed in (4), with repeat to the right boundary condition, we get

$$-2r_1v_{1,j+1/2} + (2 + 2r_1)v_{0,j+1/2} = 2r_1v_{1,j} + (2 - 2r_1 + k_1)v_{0,j} \quad (25a)$$

$$-k_1(v_{0,j})^3 - k_1v_{0,j}(u_{0,j})^2 - 3k_1(v_{0,j})^2 u_{0,j} + 3k_1u_{0,j} - 3k_1(u_{0,j})^3$$

## Conclusions

In this study, we concluded that the Crank – Nicholson Schemes without averaging and with averaging of  $f(u,v)$  are more accurate than the explicit scheme which is simpler and needs less time and storage . Crank – Nicholson scheme with averaging of  $f(u,v)$  is more accurate than Crank – Nicholson scheme without averaging of  $f(u,v)$  but it doubles the time steps and needs more storage as shown in table (1) below. The figures (1), (2) and (3) show a comparison among

explicit, Crank-Nicholson without averaging and Crank-Nicholson with averaging schemes. The figures (4) and (5) show a comparison between explicit scheme and Crank-Nicholson without averaging scheme when  $a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4$  . From the figures we concluded that the solutions converge to the steady state solution  $u = 0, v = 0$  when gets large.

## Acknowledgement

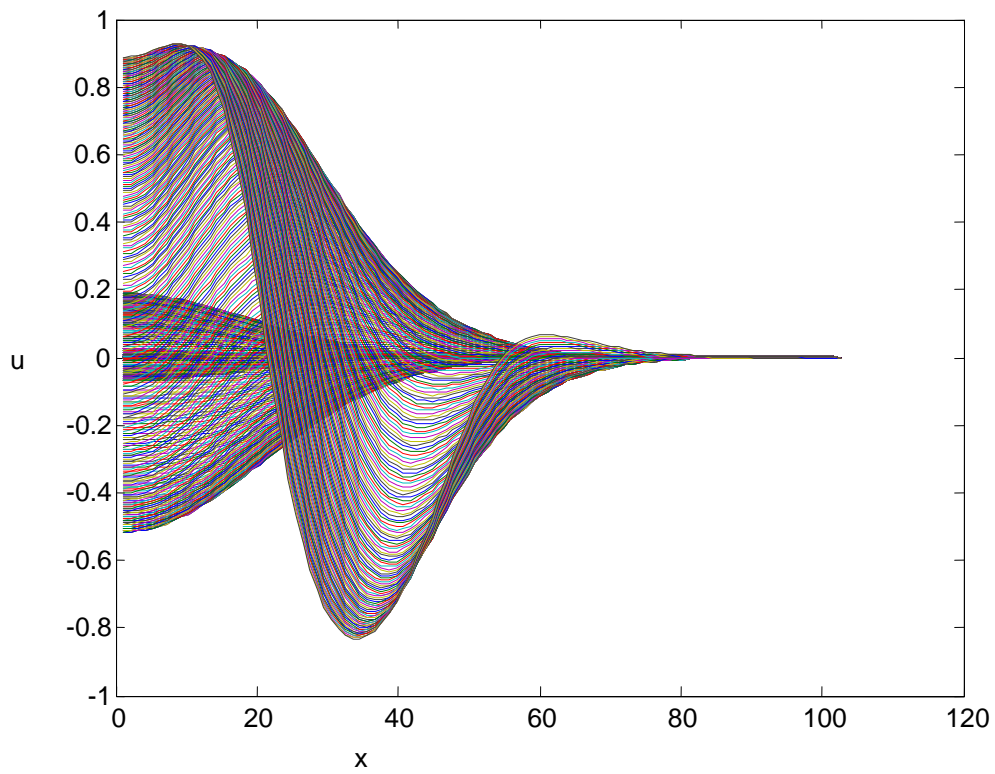
The authors would like to express their gratitude and indebtedness to their colleague A. F. Kassim (Mosul University, Iraq), for some references.

**Table (1)**

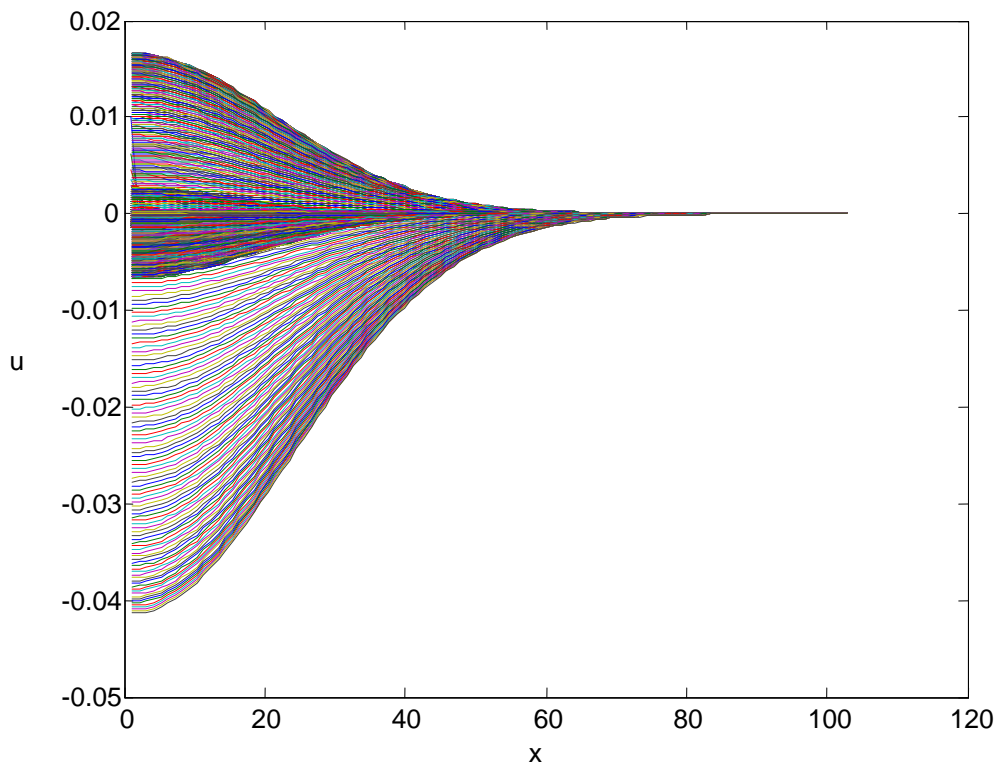
Explicit Scheme	Crank – Nicholson without averaging	Crank-Nicholson with averaging
0,01000	0,01000	0,1000
0,00192	0,00488	0,00689
0,00307	0,00334	0,00010
0,00191	0,00266	0,00412
0,00220	0,00227	0,00348
0,00161	0,00201	0,00304
0,00172	0,00182	0,00273
0,00149	0,00167	0,00241
0,00144	0,00100	0,00231
0,00131	0,00140	0,00217
0,00126	0,00121	0,00204
0,00117	0,00123	0,00194
0,00112	0,00118	0,00180
0,00106	0,00113	0,00177
0,00101	0,00106	0,00161
0,00960	0,00104	0,00164
0,00923	0,00101	0,00108
0,00883	0,00097	0,00103
0,00847	0,00094	0,00148
0,00812	0,00091	0,00144
0,00780	0,00089	0,00131
0,00749	0,00086	0,00136
0,00720	0,00084	0,00133
0,00693	0,00081	0,00121
0,00667	0,00079	0,00127

**Table (1) shows a comparison among explicit, Crank-Nicholson without averaging and Crank-Nicholson schemes with averaging of  $u$  when  $a = 1, b = 1, n = 11, m = 251, r = 0.4, k = 0.004, h = 0.1$ .**

**Figure (1)**  
explains the solution function  $u$  of the system by using explicit scheme when  
 $a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4$



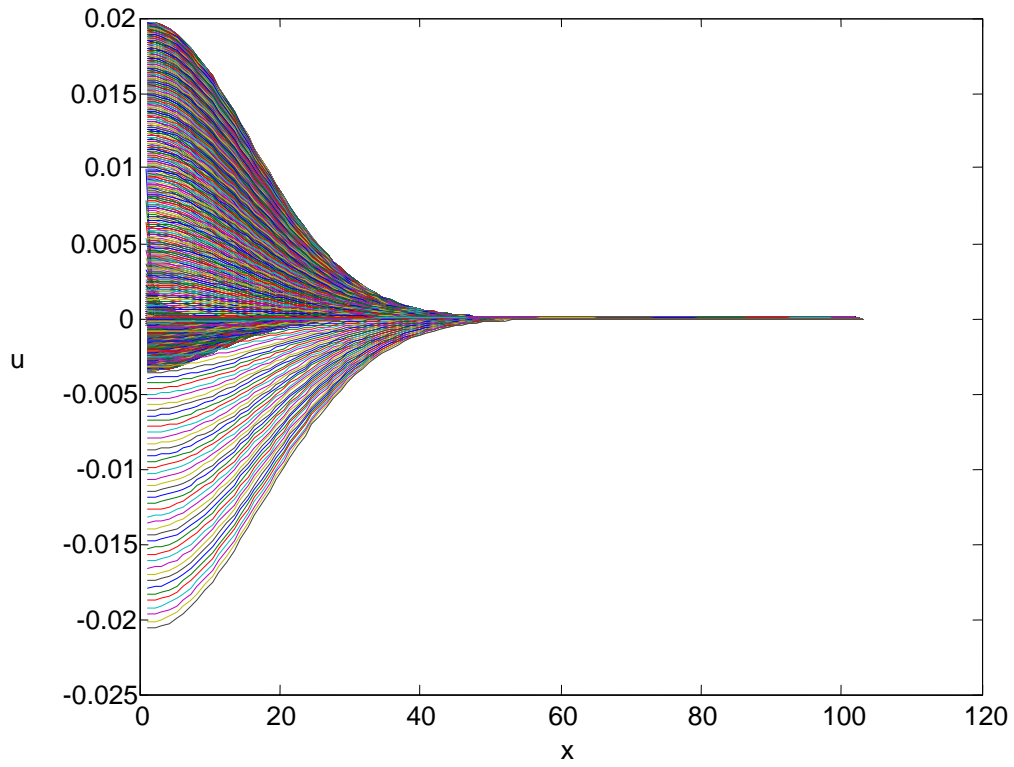
**Figure(2)**  
explains the solution function  $u$  of the system by using Crank- Nicholson scheme without averaging when  
 $a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4$



**Figure(3)**

explains the solution function  $u$  of the system by using Crank-Nicholson with averaging when

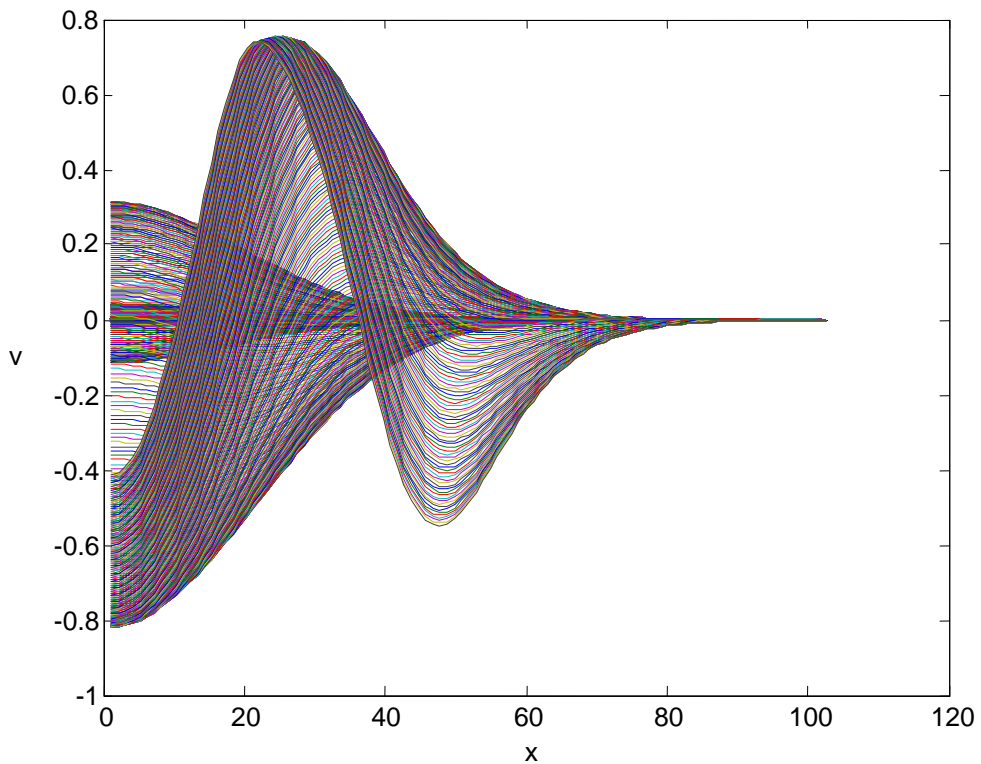
$$a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4.$$



**Figure (4)**

explains the solution function  $v$  of the system by using explicit scheme when

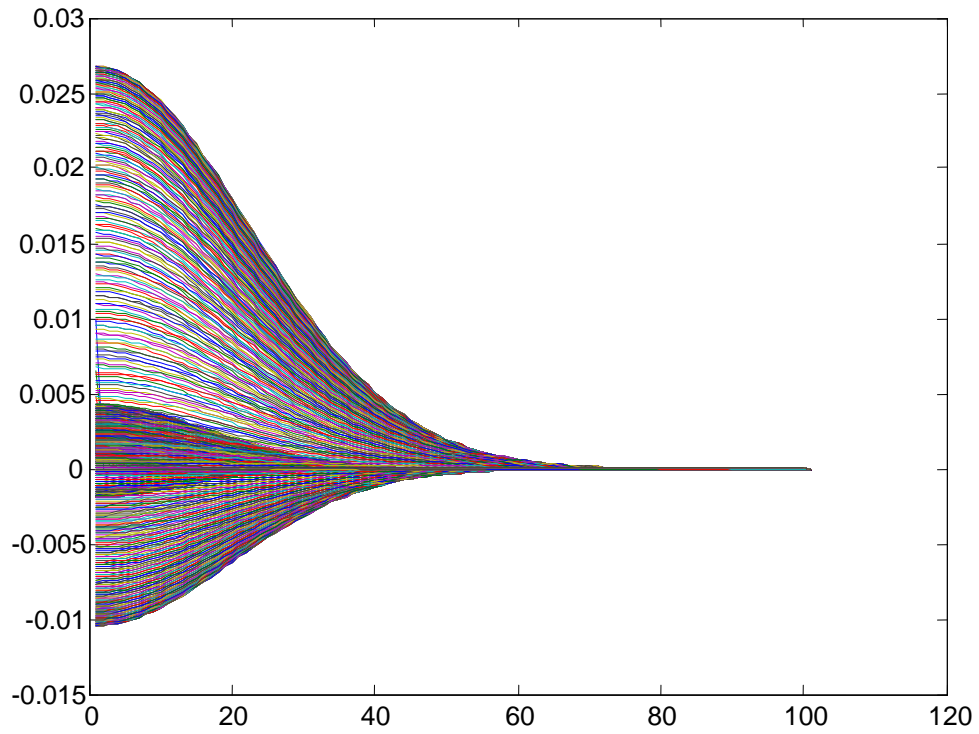
$$a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4.$$



**Figure (5)**

explains the solution function  $v$  of the system by using Crank-Nicholson scheme without averaging when

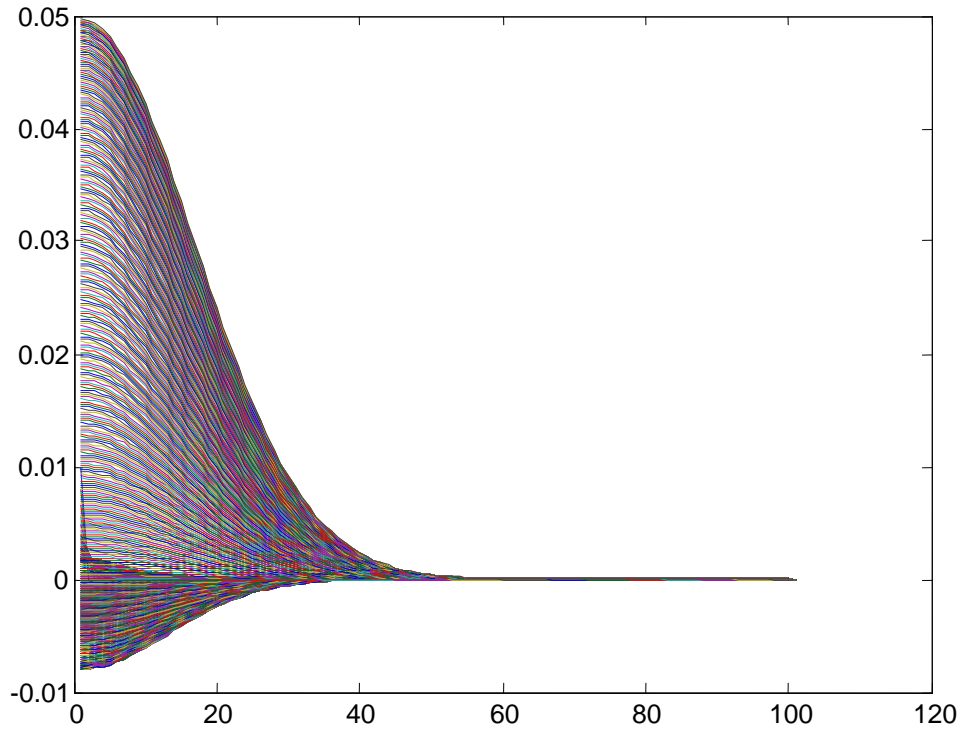
$$a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4.$$



**Figure (6)**

explains the solution function  $v$  of the system by using Crank-Nicholson scheme with averaging when

$$a = 10, b = 10, n = 101, m = 1001, h = 0.2, k = 0.01, r = 0.4.$$





## References

1. J. A. Sherratt, **On the Speed of Amplitude Transition Waves in Reaction-Diffusion Systems of  $\lambda-\omega$  Type**, IMA Journal of applied mathematics, 52, (1994), pp. 79-92.
2. J. A. Sherratt, **Unstable Wavetrains and Chaotic Wakes in Reaction-Diffusion Systems of  $\lambda-\omega$  Type**, Physica D 82, (1995), PP. 165-179.
3. J. A. Sherratt, **Periodic Waves in Reaction-Diffusion Models of Oscillatory Biological Systems**, Forma, 11, (1996), pp. 61-80.
4. J. A. Sherratt, **A Comparison of Two Numerical Methods for Oscillatory Reaction-Diffusion Systems**, Appl. Math. Lett. Vol. 10, (1997), No. 2, pp. 1-5.
5. J. A. Sherratt, **Periodic Travelling Wave Selection by Dirichlet Boundary Conditions in Oscillatory Reaction-Diffusion Systems**, SIAM J.APPL. MATH. Vol. 63, (2005), No. 5, pp. 1520-1583.
6. A. Borzi and R. Griesse, **Distributed Optimal Control of Lambda-Omega Systems**, [www.ricam.oeaw.ac.at/people/page/griesse/publications/oclosn.pdf](http://www.ricam.oeaw.ac.at/people/page/griesse/publications/oclosn.pdf).
7. Garvie, M. R. and Blowey, J. F., (2002), **A Reaction-Diffusion System of Type Part I I: Numerical Analysis**, Euro. Jnl of Applied Mathematics pp. 1-26.
8. J. H. Mathews and K. D. Fink, **Numerical Methods Using MATLAB**, Fourth Edition, (2004), Pearson Prentice Hall.

## الحل العددي لنظام الانتشار - التفاعل المتذبذب من النوع $\lambda - \omega$

<sup>1</sup> سعد عبد الله مناع و <sup>2</sup> محمد عبد محميد

<sup>1</sup> قسم الرياضيات ، كلية علوم الحاسبات والرياضيات ، جامعة الموصل ، الموصل ، العراق

<sup>2</sup> قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

### الملخص

تم حل نظام الانتشار- التفاعل المتذبذب من النوع  $\lambda - \omega$  باستخدام طريقتين من طرائق الفروقات المنتهية . الأولى هي الطريقة الصريحة والثانية هي طريقة Crank – Nicholson الضمنية في حالتين : الأولى بأخذ المعدل لـ  $f(u, v)$  ، والثانية بدون اخذ المعدل لـ  $f(u, v)$  . إذ بينت المقارنة إن طريقة Crank – Nicholson هي أفضل من الطريقة الصريحة وإن طريقة Crank – Nicholson بأخذ المعدل لـ  $f(u, v)$  هي أكثر دقة ولكنها تحتاج إلى زمن أكثر وإلى خطوات زمنية وخرن مضاعف من طريقة Crank – Nicholson بدون اخذ المعدل لـ  $f(u, v)$  .