Eng. & Tech. Journal, Vol.28, No.18, 2010

Derivation Mathematical Formulas For Tilt Angles of a Flat Plate Sun Trucking Collector with a Simulation In C++ Language

Fadhel Nooraldeen Abed Almousawi*

Received on: 24/1/ 2010
Accepted on: 3/ 6/ 2010

Abstract

In this paper a mathematical formula was driven to estimate the tilt angle of surface from horizontal Σ and the surface azimuth angle Ψ for a flat plate solar collector which gives maximum total incident solar radiation on a flat plate solar collector designed for heating water. According to the formulas of estimating Σ and Ψ which driven in this study a C++ program had been built and used to simulate the performance of a sun trucking solar collector, so the collector orientation varies with day time to give maximum incident solar radiation at each hour (in this case one hour time interval used) by estimating the surface azimuth angle and the tilt angle of surface from horizontal at the desired hour (time),and compare the results with the normal case of using stationary solar collector (oriented to the south with tilt angle equal to latitude). All calculations done for the city of Baghdad in January .Results show that the incident solar radiation and storage temperature in the case of sun trucking collector system is greater than it's in the case of stationary collector (normal case) which gives system more stability.

اشتقاق علاقات رياضية لقيم زوايا الميلان لمجمع مسطح متتبع للشمس مع برنامج محاكاة بلغة ++C.

الخلاصة

تم في هذا البحث أشتقاق علاقات رياضية لحساب قيم كل من زاوية ميلان السطح Σ وزاوية سمت السطح ψ لمجمع شمسي مسطح يستخدم لتسخين المياه ليعطي أكبر قيمة ممكنة للاشعاع الشمسي الكلي الساقط على المجمع وبناءا على قيم الزوايا المشتقة في هذا البحث تم أعداد برنامج حاسبة بلغة ++ C يمثل محاكاة لمجمع مسطح متتبع للشمس بحيث يتم إعادة توجيه المجمع الشمسي لكل ساعة (الفترة الزمنية ممكن أن تكون ساعة واحدة أو أي قيمة أخرى) من ساعات النهار بالشكل الذي يعطي أعلى قيمة ممكنة للإشعاع الشمسي الكلي الساقط عند تلك الساعة, ومقارنة النتائج مع الحالة الاعتيادية للمجمع الشمسي الثابت (باتجاه الجنوب ويميل بزاوية مع الأفق تمثل قيمة خط العرض للموقع الجغرافي). كل حسابات المجمع الشمسي كانت عند مدينة بغداد ولشهر كانون الثاني . أظهرت النتائج زيادة مقدار الإشعاع الشمسي الساقط بمقدار كبير عند استخدام المجمع الشمسي المتتبع للشمس خصوصا في الساعات الأولى والأخيرة من النهار. كذلك تبين الدراسة ارتفاع قيمة درجة حرارة ماء الخزان في حال المجمع الشمسي المتتبع للشمس بصورة ملحوظة مقارنة مع حالته في المجمع الشمسي الثابت مما يزيد من استقرارية المجمع عند نفس قيمة الحمل المسلط .

*Engineering College, University of Karbala/ Karbala

Introduction

The use of renewable energy sources has a positive influence on the environmental pollution and global warming processes. The dependence on expensive traditional energy sources and the necessity to economize them promoted the creation and development of new technologies of ecological energy systems. One of them is solar energy. Orientation of solar collector in space is the main factor influencing the quantity of absorbed solar radiation energy .One of the important parameters that affects the performance of a solar collector is its tilt angle with the horizontal. This is due to the fact that the variation of tilt angle changes the amount of solar radiation reaching the collector surface [1].

A review of literature sources were listed here, I.Luminosu [2] found that the thermo leading fluid temperature variation depends on the magnitudes α and β for a given value of the radiant power density in the collecting plane of the solar energy. The quantities α and β depend on: the collecting area A_c, the efficiency η (($\alpha = A_c\ \eta$)), the flow and the specific heat ($\beta = mc_p$). The coordinates ΔT, α and β and the magnitudes A_c, η, m, C_p can be established in the parametric graph of $\Delta T = f(\alpha, \beta)$. Omar Aliman and Ismail Muzamir [3] develop a new technique, by introducing a new rotational axis to the sun tracking frame, the slave mirrors of the same column or the same row can be arranged to share the same driving device. With this design, the focusing area can be as small as that of an element mirror or even smaller if the element mirrors are concave. in this technique, the alignment procedure can be simplified, thus, the engineering time can be greatly shortened.

Fabio Struckmann [4] suggested a way to describe the thermal performance of a Flat Plate Solar collector. He found that the most important measure is the collector efficiency and that the overall heat loss coefficient (U) and other factors as the heat removal factor (F_R) are not constant. Can Ertkin at all [5] revealed that the optimum tilt angles exhibit a strong seasonal trend with respect to the amount of maximum daily insolation incident on the collector surface. Monthly average optimum tilt angles were reasonably well estimated as a sinusoidal function of latitude and the day of the year over Turkey. The optimum tilt angle was low in the summer and high in the autumn and winter. Jurgita Grigonienė, and Mindaugas Karnauskas [1] stated the mathematical modeling of the optimal angles of a solar collector and a solar collector with a sunray concentrator. The quantity of solar energy depends on geographical position, the trajectory of the sun, on the intensity of solar radiation energy, of sunlight duration per day and per year, the reflection coefficient of SRR, etc. Therefore, in different seasons, we will have different optimal angles of a solar collector. In Lithuania, the optimal angle of tilt of a solar collector is determined experimentally is 15^o to 60^o for the whole year: $1-^o$...45^o in winter and 30°... 60° in summer. In this study the following objects were presented:

1. Derivation a formula for the tilt angles Ψ and Σ mathematically to (give optimal angles) by maximize the total solar irradiance ($dI_t/d\Psi=0$ gives optimal Ψ) and ($dI_t/d\Sigma=0$ gives optimal Σ).
2. Construction a C++ computer simulation program based on a sun trucking flat plate collector analysis and optimal tilt angles using climatic conditions for city of Baghdad.
3. Operation the simulation program (for 72 hours) to calculate the solar intensity, storage temperature, efficiency and other values for each simulation time.

Nomenclature

A_c Collector area m^2.
A_e Area of absorber element m^2.
C Diffuse radiation factor.
c, c_p Specific heat of fluid J/kg.C°.
F_R Collector heat removal factor
I_t Total short wave irradiance, W/m^2.
I_D Direct solar radiation, W/m^2.
I_d Diffuse sky radiation, W/m^2.
I_r Solar radiation reflected from surrounding surfaces, W/m^2.
L Local latitude.
$q_{s,ref}$ Reference value of collector energy.
$T_{f,in}$ Inlet fluid temperature, C°.
$T_{f,out}$ Outlet fluid temperature C°.
T_a Ambient temperature, C°.
$\bar{T}_a$ Average ambient temperature, C°
T_{st} Storage temperature C°.
T_{stag} Stagnation Temperature C°.
t Time interval.
U Collector overall heat transfer coefficient, W/m^2
β Solar altitude angle.
δ Control function value ,0 or 1.
Φ Solar azimuth angle.
ψ Surface azimuth angle.
Σ Tilt angle of surface from horizontal.
γ Surface solar azimuth angle.
$(\tau\alpha)_{eff}$ Effect value of transmissivity and absorptivity for solar energy.
$Q_u(t)$ Net enthalpy gain

Solar collector tilt angles formulas derivation

The irradiance on the surface aperture of the direct beam component I_b is the product of the direct normal irradiation I_{DN} and the cosine of the angle of incidence θ between the incoming solar rays and a line normal (perpendicular) to the surface The angle of incidence θ for any surface is defined as the angle between the incoming solar rays and a line normal to that surface. For the horizontal surface shown in Figure 1, the incident angle θ_H is QOV; for the vertical surface, the incident angle θ_V is QOP.For any surface, the incident angle θ is related to β, γ, and the tilt angle of the surface Σ as in equation (4)[6].

$$I_t = I_{DN}\cos q + I_d + I_r \qquad \text{....(1)}$$

$$I_d = CI_{DN}\frac{1+\cos\Sigma}{2} \qquad \text{.....(2)}$$

$$I_r = I_{DN}(C+\sin b)r_g\frac{1-\cos\Sigma}{2} \qquad \text{....(3)}$$

Where r_g is ground reflectivity, often taken to be 0.2 for typical mixture of ground surfaces [6].

$$\cos q = \cos b \cos g \cos \Sigma + \sin b \cos \Sigma \quad (4)$$

So we have

$$I_t = I_{DN} \cos b \cos g \sin \Sigma + I_{DN} \sin b \cos \Sigma + \frac{1}{2} C I_{DN} + \frac{1}{2} C I_{DN} \cos \Sigma + 0.1 I_{DN} (C + \sin b)(1 - \cos \Sigma)$$

$$I_t = I_{DN} \cos b \cos g \sin \Sigma + I_{DN} \sin b \cos \Sigma + \frac{1}{2} C I_{DN} + \frac{1}{2} C I_{DN} \cos \Sigma + 0.1 I_{DN} (C + \sin b) - 0.1 I_{DN} (C + \sin b) \cos \Sigma$$

1. ***Derive I_t with respect to tilt angle Σ and equal to zero.***

$$\frac{dI_t}{d\Sigma} = I_{DN} \cos b \cos g \cos \Sigma - I_{DN} \sin b \sin \Sigma - \frac{1}{2} C I_{DN} \sin \Sigma + 0.1 I_{DN} (C + \sin b) \sin \Sigma = 0$$

Divide by I_{DN} gives

$$\cos b \cos g \cos \Sigma - \sin b \sin \Sigma - 0.4C \sin \Sigma + 0.1 \sin b \sin \Sigma = 0$$

$$\cos b \cos g + 0.1 \sin b \tan \Sigma = 0.4C \tan \Sigma + \sin b \tan \Sigma$$

simplifed

$$\cos b \cos g = \tan \Sigma (0.4C + 0.9 \sin b)$$

Then we have a value for Σ gives maximum I_t.

$$\Sigma = \tan^{-1}\left(\frac{\cos b \cos g)}{0.4C + 0.9 \sin b}\right) \quad \text{….(5)}$$

2. **Derive I_t with respect to surface azimuth angle ψ and equal to zero.**

Start with equation (6) [6]

$$g = f - y \quad \text{….(6)}$$

From mathematics we have

$$\cos g = \cos f \cos y + \sin f \sin y$$

Substitute in equation (4) gives

$$\cos q = \cos b \cos \Sigma (\cos f \cos y + \sin f \sin y) + \sin b \cos \Sigma$$

$$\cos q = \cos b \cos \Sigma \cos f \cos y + \cos b \cos \Sigma \sin f \sin y + \sin b \cos \Sigma$$

Substitute in Equation (1), we have

$$I_t = I_{DN} (\cos b \cos \Sigma \cos f \cos y + \cos b \cos \Sigma \sin f \sin \Psi + \sin b \cos \Sigma) + I_d + I_r$$

Both IDN, I_d and I_r are not depend on ψ, thus

$$\frac{dI_t}{dy} = -\cos b \cos \Sigma \cos f \sin y + \cos b \cos \Sigma \sin f \cos y = 0$$

Thus

$$\sin f \cos y = \cos f \sin y$$

$$\frac{\sin f}{\cos f} = \frac{\sin y}{\cos y}$$

Then we have a value for ψ gives maximum I_t.

$$y = f \quad \text{…..(7)}$$

So we must use the two equations (5) and (7), (which derived in this research) to estimate the maximum total solar radiation I_t with respect to tilt angle Σ ,and to surface azimuth angle ψ.

Mathematical model of solar collector

The net enthalpy gain $Q_u(t)$ of the fluid following through the collector is given [7]

$$Q_u = d_c \overset{o}{m}_c\, c(T_{f,out} - T_{f,in}) \quad \text{....(8)}$$

The net enthalpy gain $Q_u(t)$ is related to the insolation I_t(t) through the definition of collector efficiency by equation (9).

$$Q_u(t) = hI_t(t)A_c \quad \text{....(9)}$$

Where A_c is the aperture area of the collector.

Where

$$, \quad q_a = \frac{T_a}{\bar{T}_a} \quad , \quad \text{and}$$

$$\Psi_1(t) = \frac{I_t(t)}{q_{s,ref}} \quad g_1 = \frac{d_c \overset{o}{m}_c\, c\bar{T}_a}{q_{s,ref}} \quad \text{....(10)}$$

$$T_{f,out} = T_{f,in}(1 - \frac{F_R b}{g_1})$$

$$+ \frac{F_R \bar{T}_a}{g_1}[(ta)_{eff}\Psi_1(t) + bq_a]$$

The parameter γ_1 is a measure of the ratio of the ability of the working fluid to remove energy from collector to the reference solar energy. The value of γ_1 can range from zero to very large values depending on the relative values of the mass flow rate through the collector ,the collector area , and the reference insolation. The non flow or stagnation temperature of the plate is given [7].

$$T_{stag}(t) = T_a(t) + \frac{(ta)_{eff}\Psi(t)}{b}\bar{T}_a$$

$$= T_a(t) + \frac{(ta)_{eff}A_c I_t(t)}{UA_e} \quad \text{...(11)}$$

The average plate temperature at stagnation is thus predicated to reach a value of $(ta)_{eff} I_t(t)/UA_e$ degrees above the ambient value of T_a. The term $(ta)_{eff} I_t(t)/UA_e$ is simply the absorbed solar energy divided by the convective loss per degree above ambient. For well-designed flat plate collector,$(\tau\alpha)_{eff}$ will have the values in range of 0.75 to0.95 .the value of ,$(\tau\alpha)_{eff}$ depends on the optical design of collector .The value above 0.9 exist for simpler unglazed collectors such as those designed for swimming pool heating ,while values of 0.75 to 0.85 apply to glazed collectors used in space heating and domestic hot water service. The value of bF_R depends on the thermal design of the collector ,and it determines the change of the collector efficiency with temperature .A swimming pool collector ,designed for operation near ambient temperature ,could have a value of bF_R as large as 15[7] .

A measure of a flat plate collector performance is the collector efficiency (η) defined as the ratio of the useful energy gain (Q_u) to the incident solar energy over a particular time period. The instantaneous thermal efficiency of the collector is [4].

$$h = F_R(ta)_{eff} - F_R U \frac{T_{f,in} - T_a}{I_t} \quad (12)$$

Discussion

The most important result in this study was the derivation of the formulas (5) and (7) that give the values of the angles Σ and ψ which gives the maximum value of the incident solar radiation received by the collector (optimum case). A C++ simulation program had built to study the performance of the sun trucking flat plate solar collector in city of Baghdad in January at which the angles Σ and ψ varies hourly. Calculations focus on the comparison between two cases one stationary solar collector (normal case) in this case solar collector oriented to the south, the angle with the altitude Σ have the value of latitude angle ,in Baghdad this angle have the value of L=33.3^o (so Σ= 33.3^o) . The other case was the sun trucking solar collector (optimum case due to optimal tilt angles which gives maximum solar radiation incident on collector), in this case the angles Ψ and Σ varies with time (with one hour time interval). Two dimensions of solar collector were used, one of 20m^2 of area with 2KW of maximum Load supplied and other of 30m^2 of area with 3KW of maximum load supplied

Fig (2) shows total solar radiation in two cases of stationary (normal) and sun trucking (optimum) solar collector installation, the large difference between two values can note clearly, at 8 A.M the difference approximate to 300 W/m^2, this value decrease to less than 100 W/m^2 at 12 P.M .The difference in two values of solar radiation result from the varying the values of the angles Σ and ψ. Morning the difference in values of the angle ψ in stationary and sun trucking cases was at maximum, so the difference between the incident solar radiations in two cases was at maximum also. At midday the difference in values of the angle ψ in two cases close to zero because of orientation of the stationary case which was to the south (ψ = 0) which is the same at the sun trucking case at midnight and the difference in solar radiations in two cases come just from the difference in values of the angle Σ in two cases.

Fig (3) shows the difference between storage temperature in two cases .Again sun trucking system case record the higher temperatures (from simulation program which build in this study) with collector area of 20m^2 and 2KW Load supplied and the reason was the solar radiation in sun trucking case was higher than form the other case for all simulation times. For example the difference in temperatures in two cases was 22 C^o at the hour 66 of the simulation time. In fig (4) the stability of system with load of 2KW in sun trucking case was clear with respect to normal case that because of the high capacity of sun trucking collector with respect to normal case. Fig (5) illustrates the same result with solar collector of 30m^2 and 3KW of load supplied. In this figure we can see the difference in temperatures of two cases for example the difference equal to 20 C° at hour 44 of simulation time. Finally Fig (6) illustrates the value of instantaneous collector efficiency in cases of stationary and sun trucking solar collector. The efficiency in sun trucking case was higher than the efficiency in normal case for intervals (8-9) A.M and (2-4) P.M , this because of net enthalpy gain $Q_u(t)$ which greater than its in normal case because the high storage temperature .In interval 9 A.M to 2 P.M the efficiency in sun trucking case was lower than the efficiency in normal case because of the value of I_t in sun trucking case would greater than its in normal case where $\eta = Q_u(t)/(I_t A_c)$ and the relation between η and I_t is inverse relation. On other hand the load was limited so the value of $Q_l(t)$ still at constant value. Table (1) listed the inputs data for the simulation program used .Table (2) listed the average ambient temperature for the city of Baghdad in January that used in this simulation.

Conclusions

The use of sun trucking flat plate solar collector technique for domestic water heating in the city of Baghdad By varying each angles Σ and ψ hourly according to formulas (5) and (7) which derived in this study enable the flat plate solar collector to receive more energy from sun and so we have higher water temperature in storage tank so its more useful than other types of flat plate collectors .This mean more depending on green energy , saving for electrical energy and reducing the environmental pollution and global warming processes.

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Table (1) Designed values used in simulation program.

Time of simulation, Hr	72
Calculation interval, Hr	1.0
$q_s(t)$, W/m^2	800
F_R value	1.0
Collector mass flow rate ,kg/s	0.1
Specific heat of fluid J/kg.C^o	4200
Storage capacity ,KJ/C^o	4,2000
U_c Collector heat transfer coefficient, W/m^{2}C^o	3.4
U_{st} Value of storage tank	0.2
A_{st} , Storage tank surface area , m^2	6.0
Initial storage temperature ,K^o	300
Minimum usable temperature ,F^o	300
Maximum safe temperature ,F^o	380
Load mass flow rate kg/min	0.1

Table (2) Ambient temperature for the city of Baghdad in January [8].

Time (Hr)	Temperature (C^o)	Time (Hr)	Temperature (C^o)
1	7.7	13	14.7
2	7.3	14	15.1
3	6.8	15	15.4
4	6.5	16	14.4
5	6.1	17	13.4
6	5.8	18	12.4
7	6.7	19	11.6
8	7.2	29	10.6
9	7.7	21	10.0
10	9.2	22	9.3
11	11.3	23	8.7
12	13.4	24	8.1

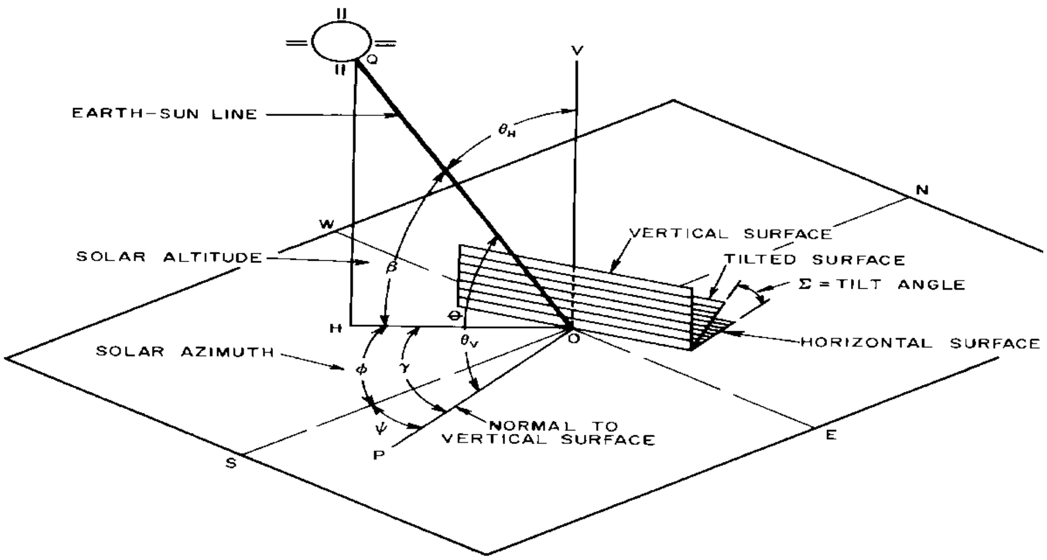
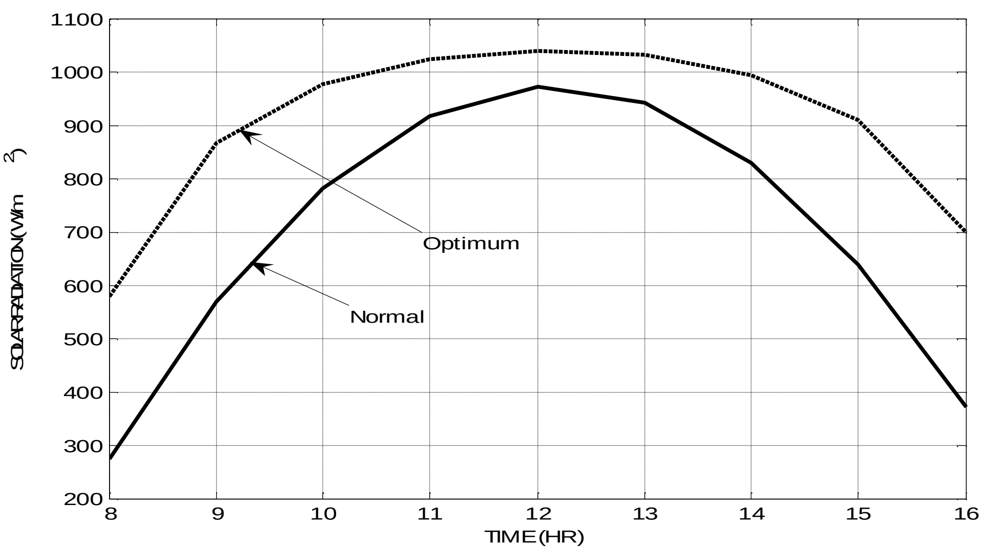
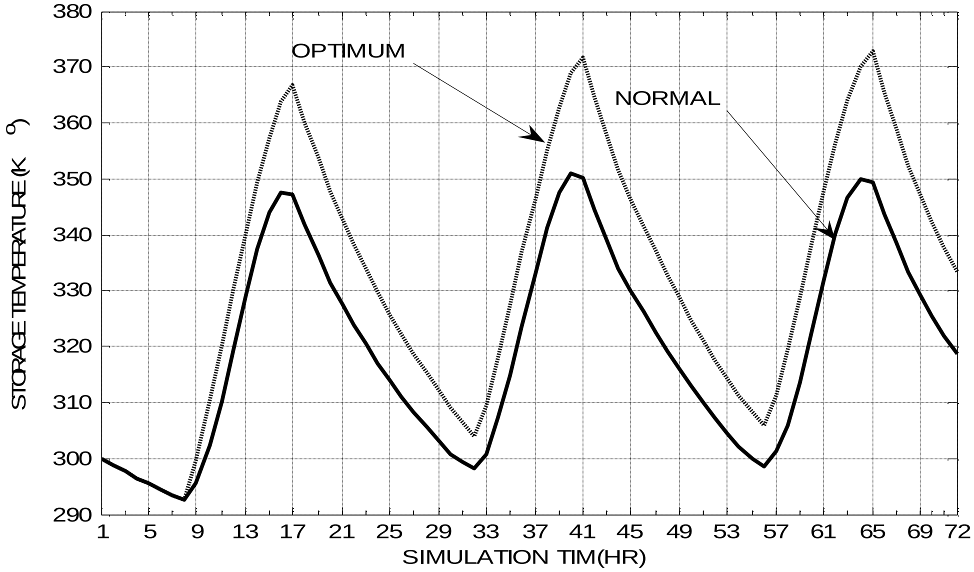


Figure (1) Solar Angles For Vertical And Horizontal Surfaces

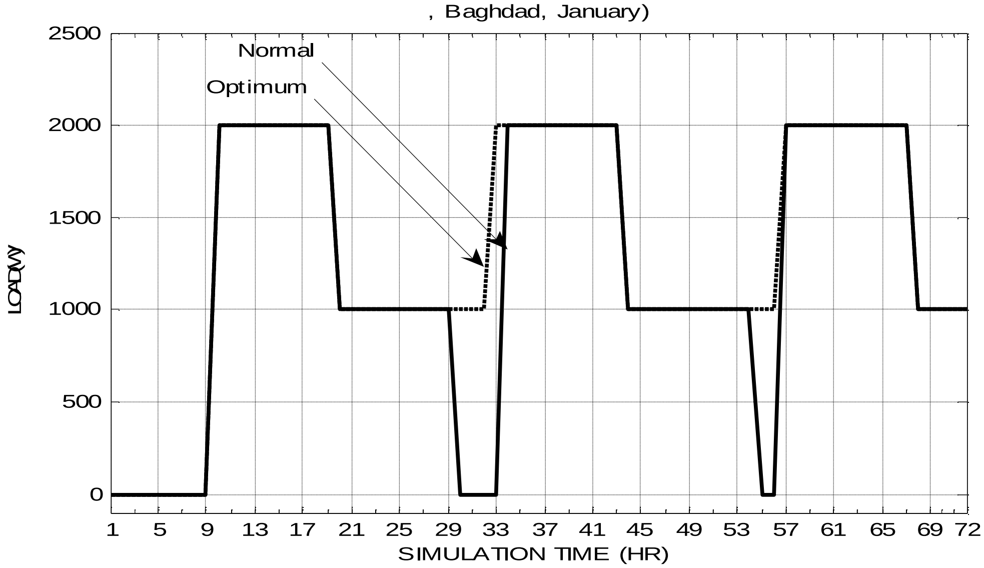
FIG(2) COMPARISON OF TOTAL SOLAR RADIATION ON SOLAR COLECTOR IN TWO CASES OF NORMAL AND OPTIMUM CASES IN CITY OF BAGHDAD ,JANUAURY 21.



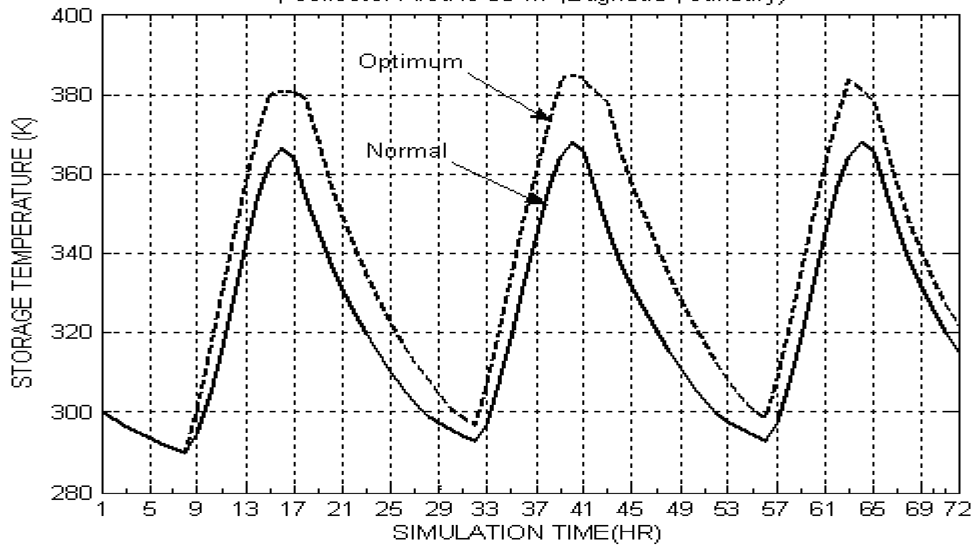
FIG(3) Storage temperature in normal case and optimum case (Max. Load is 2Kw ,Collector Area is 20 m^2,Baghdad, January)



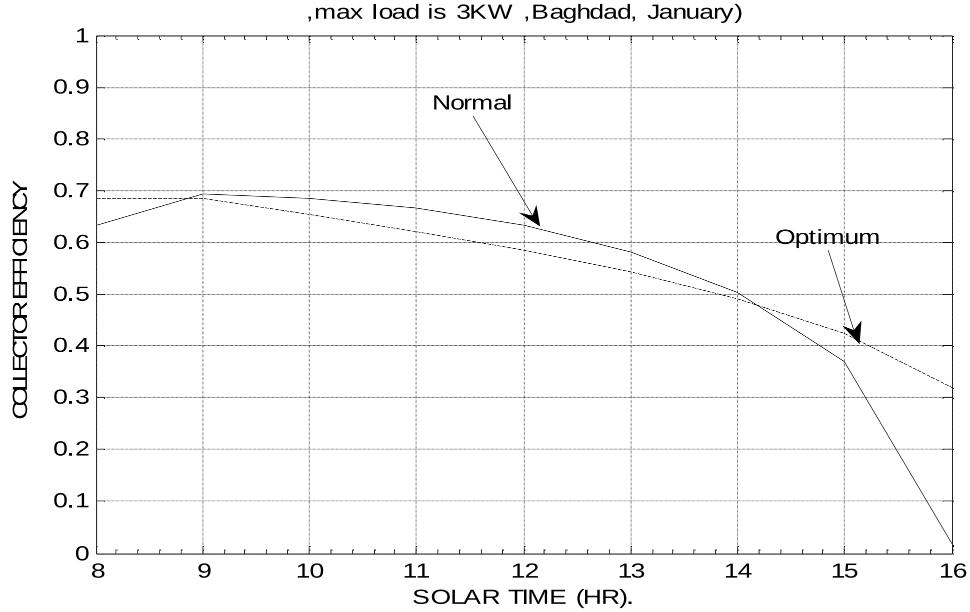
FIG(4) Loads throuth three days (Max. load 2KW ,Collector area is 20m^2 , Baghdad, January)



FIG(5) Storage temperature in normal and optimum cases (Max. Load is 3 KW , Collector Area is 30 m^2, Baghdad , January)



FIG(6) Efficiency of solar collector in normal and optimum cases (area is 30 m^2 ,max load is 3KW ,Baghdad, January)



Flow Chart of Computer Program