Mathematical model of optical amplifier using nonlinear stimulated Brillouin scattering (SBS) in optical fiber

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Abstract:
We demonstrate the results of a mathematical model for investigation the nonlinear Stimulated Brillouin Scattering (SBS), which can be employed to achieve high optical amplifier. The SBS is created by interaction between the incident light and the acoustic vibration fiber. The design criteria and the amplification characteristic of the Brillouin amplifier is demonstrated and discussed for fiber Brillouin amplifier using different pump power with different fiber length. The results show, high Brillouin gain can be achieved with high pump power and long effective fiber length.

Introduction:
SBS may be regarded as the modulation of light through thermal molecular vibrations within the fiber. The scattered light appears as upper and lower sideband, which are separated, from the incident light by the modulation frequency. The incident photon on this process produces a phonon of acoustic frequency as well as a scattered photon. This produces an optical frequency shift, which varies with the scattering angle because the frequency of the sound wave varies with acoustic wavelength [1]. SBS is a very efficient nonlinear to generates the amplification for the signal at longer wavelength or lower frequency. However unlike Stimulated Raman Scattering (SRS) depletes the transmitted signal by producing gain in the direction opposite to signal propagation, that is, back toward the source [2]. The schematic diagram of Brillouin amplifier is shown in figure 1.

The main different between the SRS and SBS is that optical phonons participate in Raman scattering, where as acoustic phonons participate in Brillouin scattering. Some of the nonlinear effect such as SRS and SBS can be used to advantage in the design of optical communication systems, since they can amplify an optical field by transferring energy to it from a pump field whose wavelength is suitably chosen [3] as shown in figure 2.

Figure 1: The schematic diagram of a Brillouin amplifier

Figure 2 - Received Spectrum after SBS is on a Long Fiber

Brillouin amplifiers can be useful for applications requiring selective amplification. On such application is based on a scheme in which receiver sensitivity is improved by selective amplification of the carrier while
leaving modulation sidebands unamplified. Another application of Brillouin amplifiers consists of using them as a tunable narrowband optical filter for channel selection in a densely packed multichannel communication system. In 1990, Boyed and Rzazewski describe a theoretical model that show how SBS is initiated by thermally acoustic waves distributed within a Brillouin active medium \[4\], and in 1991 Gatea and Boyd investigated the statistical properties of SBS in a single mode fiber \[5\]. The process of SBS in the presence of weak feedback in a single mode optical fiber is investigated theoretically and experimentally in 1992 by Gaeta and Boyed \[6\]. In 1998 \[7\] Boyed and Buckland reported a program aimed at clarifying that physical processes leading to the nonlinear optical response of silica fibers at studying the implications of optical nonlinearities on optical pulse propagation and optical switching devices. Ultrafast nonlinearities are used in 2004 to investigate all optical switching and wavelength conversion in semiconductor amplifier \[8\]. The first experimental demonstration of pulse advancement with high gain in optical fibers based on SBS by Song et.al in 2005 \[9\]. SBS process can be treated theoretically in a very straight forward manner for the case of an SBS amplifier, since the laser and stokes field are both applied externally. The mathmatical analysis is much more complicated for the cased of an SBS generator, because only the laser field is applied externally and the stokes wave is created within the medium.

**Mathematical Model:**

Since SBS can be viewed as scattering of a pump waveform an acoustic wave, the stokes shift \( \Omega_B = \omega_p - \omega_s \) where \( \omega_p \) is the angular frequency of the pump beam and \( \omega_s \) is the angular frequency of the signal beam. Corresponding to the acoustic frequency, must satisfy the dispersion relation\[10\]

\[
\Omega_B = |k_A| \nu_A = 2 \nu_A |k_p| \sin\left(\frac{\theta}{2}\right) --- 1
\]

where the acoustic wavevector \( k_A = k_p - k_s \) in order to satisfy the phase matching condition, \( k_p \) and \( k_s \) are the wavevectors of the pump and signal beams respectively, \( \theta \) is the scattering angle, and \( \nu_A \) represent the acoustic velocity. In optical fiber \( \theta \) can be only zero or \( \pi \) \[11\]. Equation 1 shows that vanishes in the forward direction \((\theta=0)\) and is maximum in the backward direction \((\theta=\pi)\). By using

\[
k_p = \frac{2 \pi n}{\lambda_p} --- 2
\]

where \( n \) is effective refractive index of the guided modes, and \( \lambda_p \) is the pump wavelength, the Brillouin shift is given by \[10\]

\[
\nu_B = \frac{\Omega_B}{2 \pi} = \frac{2 n \nu_A}{\lambda_p} --- 3
\]

When a weak signal is launched with a pump signal, it will be amplified due to SBS. In the continuous wave (CW) pump, the signal amplification is described by the following equations \[12\] in term of power transfer from the pump to the signal due to Brillouin interaction between the pump and the signal \[3\]

\[
\frac{dI_p}{dz} = -g_B I_p I_s - \alpha I_p --- 4
\]

\[
\frac{dI_s}{dz} = -g_B I_p I_s + \alpha I_s --- 5
\]
Where $g_B$ is the power Brillouin gain coefficient, $I_p$ and $I_s$ are the intensities of the waves at frequencies $\omega_p$ and $\omega_s$ respectively, and $\alpha$ is the fiber loss.

By neglected the pump depletion in equation 4 and using

$$I_p(z) = I_p(0) e^{-\alpha z}$$

in equation 5 and integration over the fiber length $L$, the stokes intensity is found to grow exponentially in the backward direction as [11]

$$I_s(0) = I_s(L) = \frac{g_B P_o L_{eff}}{A_{eff}} - \alpha L$$  ---6

where $P_o = I_p(0) A_{eff}$ is the input pump power, $A_{eff}$ is the effective core area, and the effective interaction length is given by [12]

$$L_{eff} = \frac{1}{\alpha} \exp\left(-\alpha L\right)$$  ---7

The Brillouin amplification gain can be expressed as

$$G_A = \exp\left(\frac{g_B P_o L_{eff}}{A_{eff}}\right)$$  ---8

Equation 6 shows how a stokes signal incident grows in the backward direction because of Brillouin in amplification occurring as a result of SBS. The exponential factor in equation 8 represent the optical Brillouin gain. The expression for the Brillion gain, assume CW pumping by a laser whose linewidth $\Delta \nu_p << \Delta \nu_B$ where $\nu_p$ and $\nu_B$ represent the linewidth of the pump laser and Brillouin respectively. For this reasons fiber Brillion amplifiers needed to be pumped by narrow linewidth semiconductor laser.

**Results and Discussion:**

In the present work, using acoustic velocity $V_A = 5.96$ km/sec and a mode index $n = 1.447$ as a typical values for silica fibers, then the Brillouin shift from equation 3 becomes $\nu_B = 11.1$ GHz at a pump wavelength 1550 nm. By using the parameter values typical of standard single mode optical fiber used in optical communication, $A_{eff} = 50 \mu m^2$, Brillouin gain coefficient $g_B$ is estimated to be about $5x10^{-11} m/w$.

Substituting these values in equation 8, the Brillouin gain as a function of the effective fiber length for different values of the pump power is shown in figure 3.

It may be observed from above figure, the Brillouin gain becomes larger when the fiber length increases, high pump power, and small effective area. Nevertheless, when the fiber is pumped with a CW laser at a power in the range 3-5 mw, gains in excess of 15 dB can be obtained as shown in figure 3.

![Figure 3: The Brillouin gain as a function of fiber length for different pump power](image-url)
Conclusion:

The SBS produced gain in an optical fiber can be used to amplify a weak signal whose frequency is shifted from the pump frequency by an amount equal to the Brillouin shift. SBS should occur only in the backward direction in single mode fibers because in forward direction $\theta=0$ and in backward direction $\theta=\pi$. The calculated amplifier factor of about 15 dB was achieved at a pump power (3-5) mw for effective fiber length about 2.8 km comparing with 15 dB was achieved when the fiber is pumped with a CW laser at a power in the range (5-10) mw in reference 1 and (30-35)dB with 5 mw pump power in reference 10. However, Brillouin amplifiers are capable of providing larger than 30 dB gain at a pump power under 10 mw. The gain of the fiber Brillouin amplifiers saturated when the amplified signal begin to deplete the pump due to increase of the fiber length. This calculation model is accurate to get the Brillouin gains in a standard single mode optical fiber, it can give us a projection of how much signal gain can be achieved in a specific fiber given its length and pump power.

References:
نموذج رياضي لمكبر ضوئي باستخدام استطارة بريلون المحتثة اللاخطية في الألياف البصرية

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الخلاصة:

تم استعراض نموذج رياضي لاستطارة بريلون المحتثة اللاخطية في الألياف البصرية والتي يمكن أن تطبق للحصول على مكبر ضوئي عالي. استطارة بريلون المحتثة تتولد من تفاعل المجال الضوئي الساقط مع الاهتزازات الصوتية المتولدة في اللف البصري. مبادئ التصميم وخواص التكبير لمكبر بريلون استعرضت وتوقشت لمكبر ليف بريلون باستخدام عدة طاقات ضخ مع عدة أطوال للف البصري. أوضحت النتائج أن ربح بريلون العالي يمكن الحصول عليه بطاقات ضخ عالية مع زيادة في طول اللف البصري الفعال.