Modeling the Effect of Operation Temperature on Characteristics of Rosen Type Piezoelectric Transformer

Dr. Nabil N. Rammo*, Dr. Rajaa R. Abbas**, Dr. Talib R. Abbas* & Inmar N. Ghazi***

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Abstract
A simplified lumped model that makes direct correlation between Rosen's piezoelectric transformer (PT) dimensions and material properties via equivalent circuit parameters was studied. The model calculation of the resonance characteristics and voltage gain was associated with changes in operating temperatures from 50°C to 70°C. It is found that for different load resistors, the resonance frequencies are shifted by less than 1.6% their normal values upon raising the operation temperature to 70°C, whereas the sensitivity of voltage gain is better at higher load resistors.

Each of the elastic compliances $S_{11}^E$, $S_{33}^E$ and $S_{33}^D$ of the PT material due to their temperature dependency cause the resonance frequency to shift below and above the normal value by less than 1% for very slight change in voltage gain in each case. The overall balance between mechanical parameters may play role in the optimum stability of the voltage gain.

Keywords: Piezoelectric transformer; Piezoelectric ceramic; Resonance characteristics; Equivalent circuit; Temperature.

Introduction:
Recently, the piezoelectric transformers (PT) have been widely supplied to the inverter power sources for backlight systems of liquid crystal displays (LCD) because their electrical characteristics are in accordance with required ones of...
LCD backlight that needs high ignition voltage at burst state and low operating voltage in static one, others have been developed as converters for alternating current (AC) adapters for commercial applications. PT offers several advantages compared to the electromagnetic ones such as higher electro-mechanical power density, no electro-mechanical noise, higher efficiency at resonance, low profile, small size, no winding coil, easier miniaturization, non-flammable and simple fabrication process (Hemsel, 2006; Joo, 2006 and Yang, 2006).

(Rosen,1961) has proposed a PT operating in the length vibration mode, but progress in this type of PT was not commercially pursued to a large extent due to poor materials reliability. Recently, researchers have manipulated new efforts at producing compact, efficient, and bulk-ceramic piezoelectric trans-formers. These efforts have yielded reported efficiencies from 82% to as high as 92% (Flynn, 1998). The rectangular shape Rosen-type PT is composed of driving part and generating part, the former works with piezoelectric transversal effect and the latter works with piezoelectric longitudinal effect (Kanayama, 1998) Such PT has the characteristic voltage gain and efficiency to be dependent on the ratio of length to width, as well as the ratio of length to thickness of the PT (Fikunaga, 1998).

Common materials used for PTs include lead zirconate titanate PZT-based ceramics (Yang,2006; Burianova, 2003& 2005 and Junhui, 2003). Temperature plays an important role on the electrical and mechanical behavior of the PT as an internal energy loss. Especially important is the dependency of piezoelectric material parameters on temperature. Actual data for different kinds of PZT ceramics showing the dependencies of such properties with temperature have been demonstrated by many investigators (Joo, 2006 and Burianova, 2003).

Many previous studies deals with mathematical analyses of PT (Limei, 2009; Joung, 2009 and Loyau,2009) Changes in resonance characteristics of the PT as a result of load variation associated with temperature rise utilizing finite element method have been reported ( Joo, 2006; Joo, 2001 and Chang, 2005). However, the effect of temperature dependency of piezoelectric material parameters on the resonance characteristics is lacking, so it is of interest to consider the electromechanical transformation in PT via equivalent circuit parameters.

**Equivalent Circuit:**

Fig.1 shows a typical Rosen-type piezoelectric transformer comprises of a driver section connected to an AC voltage $v_s$ and a generator section connected to a load resistor $R_L$. The poling direction is oriented normal to the thickness in the driver section and longitudinal in the generator section.

For operating principles of the PT, the driver section is driven by unstiffened mode (i.e. 31-mode) vibration due to converse piezoelectric effect employed for converting electrical energy to mechanical energy. The generator section is driven by stiffened mode (i.e. 33-mode) vibration due to direct piezoelectric effect employed for converting mechanical energy to electrical energy (Mason, 1964).
The analysis of piezoelectric actuator (driver section) or transducer (generator section) and their application to ultrasonic system can be carried forward on a continuum basis using the wave equation. However, it is frequently more convenient to use the equivalent circuit approach where both the electrical and mechanical portions of the system are represented by electrical equivalents. In this approach a single mode of vibration is considered, neglecting coupling phenomena along the other axis (Fikunaga, 1998; Mason, 1964; Dallago, 2001 and Syed, 2001).

The equivalent circuit for driver section or generator section of Rosen-type PT can be derived using its equation of motion (specific solution to the wave equation) and the appropriate piezoelectric equations together with appropriate boundary conditions at the two loaded faces of the resonator.

Assuming the PT operates at uniform temperature and both cross-sectional dimensions are small compared with its length, the driver section can then be considered as length expander bar with electric field perpendicular to length and the generator section with electric field parallel to length. Using six terminals (2 for electrical and 4 for mechanical) equivalent circuits are developed for each section separately following Mason (Mason, 1964). These two sections are placed on a cascade in order to conserve continuity of displacements and stress at the junction. A complete equivalent circuit is shown in Fig. 2 with the circuit parameters shown in Table 1.

\[
Z_{A1} = jZ_{o1} \tan \frac{wl}{2v_b} \cdot Z_{A1} = \frac{Z_{o1}}{j \sin \frac{wl}{v_b}}
\]

\[
v_p^E = \left( \frac{1}{\rho s_{31}^E} \right)^{\frac{1}{2}}, \quad Z_{o1} = \rho WH v_p^E
\]

\[
Z_{A2} = jZ_{o2} \tan \frac{wl}{2v_b^D} \cdot Z_{A2} = \frac{Z_{o2}}{j \sin \frac{wl}{v_b^D}}
\]

\[
v_p^D = \left( \frac{1}{\rho s_{33}^D} \right)^{\frac{1}{2}}, \quad Z_{o2} = \rho WH v_p^D
\]

\[
C_{o1} = \frac{Wle_{33}^T (1 - k_{31}^E)}{H}, \quad C_{o2} = \frac{WHe_{33}^T (1 - k_{33}^E)}{l}
\]

\[
\phi = \frac{Wd_{31}^E}{s_{31}^E}, \quad \psi = \frac{WHd_{33}^E}{s_{33}^E}
\]

**Table 1. Equivalent Circuit Parameters**

Where \( w \) is the angular frequency of driving voltage, \( l \) is driver or generator section and equal to \( L/2 \), \( W \) and \( H \) is the width and thickness respectively, \( \rho \) is the piezoelectric material density. \( C_{o1} \) and \( C_{o2} \) are shunt capacitors associated with the input and output electrodes respectively.

\( s_{31}^E \), \( s_{33}^E \) and \( s_{33}^D \) are the elastic compliance constants. \( e_{33}^T \) is the dielectric constant.

The superscripts \( E, D \), and \( T \) refer to a constant electric field, electric displacement and stress respectively. \( k_{31}^E \) and \( k_{33}^E \) are the electromechanical coupling coefficients. \( \phi \) and \( \psi \) model the electromechanical transformation. \( d_{31}^E \) and \( d_{33}^E \) are the piezoelectric strain constants.

Further simplification can be done to the equivalent circuit when...
operating near mechanical resonance frequency that attain highest output/input ratio (Dallago, 2001). The resultant simplified circuit is shown in Fig. 3 with the circuit parameters as shown in Table 2. (Mason, 1964).

\[L_1 = L_2 = \frac{\rho \cdot V}{2}, \quad V = lW. H\]

\[C_1 = \frac{2 \cdot k_{31}}{\pi^2 WH}, \quad C_2 = \frac{2 \cdot k_{32}}{\pi^2 WH}\]

\[R_1 = \sqrt{\frac{L_1}{C_1 Q_m}}, \quad R_2 = \sqrt{\frac{L_2}{C_2 Q_m}}\]

V refers to volume of each PT section.

\(Q_m\) refers to mechanical quality factor. \(C_{31}, C_{32}, \phi \) and \(\psi\) refer to same expressions given in Table 1.

Fig. 3 represents the well-accepted configuration of PT equivalent circuit for theoretical and experimental studies (Hemsel, 2006; Fikunaga, 1998 and Joo, 2003). \(L_1, L_2, C_1, C_2\) and \(R_1, R_2\) model inertial, potential mechanical energy and mechanical loss respectively. A more accurate quantification of equivalent circuit parameters can be obtained numerically using finite element method (Limei, 2009; Joo, 2003 and Tsuchiya, 2003). However the equivalent circuit parameters expressions shown in Table 2, are acceptable for conceptual investigation done in this study.

**Characteristics of PT due to operating temperature variation:**

The equivalent circuit developed in the previous section associated with the calculation procedure presented by (Theraja, 1976), has been used as an investigation tool. Fig. 4 shows change of resonance characteristics of a hypothetical PT due to change of operating temperature of the transformer from 50°C to 70°C under different load resistors. The length \(L\), the width \(W\) and thickness \(H\) are assumed to be 44 mm, 6.5 mm and 2.2 mm respectively (Chang, 2005). Ceramic material’s constants of the transformer extracted from graphs in (Joo, 2006) are listed in Table 3. Note that \(k_{ij}\) is not included in Table 3, since voltage gain is independent of \(C_{ij}\). Parametric values at any temperature higher than 50°C was calculated by the following equation

\[C(T) = C (50°C) + \delta (T - 50°C)\]

Where \(C\) is a particular parametric value, \(T\) is the temperature in °C and \(\delta\) is the temperature coefficient.

It is clear that for different load resistors, the resonance frequencies are shifted by less than 1.6% of their values upon raising the operation temperature to 70°C. Assuming that the PT is driven with frequency equal to resonance frequency at 50°C, the voltage gain \(V_L/V_S\) would decrease as the temperature increase. The sensitivity of voltage gain to temperature is higher as load \(R_L\) decrease. Since the quality factor of the equivalent circuit decrease as the load resistance increase reaching a minimum value and then increase again with load resistance. Similar observation was presented by Chang (Chang, 2005). Fig. 5 shows the way in which the percentage of voltage gain varies as a function of load. It is clear that the sensitivity of voltage gain is better at higher load resistor.

A slight change in voltage gain at resonance frequencies as temperature increase is caused by the collective effect of the change of equivalent
circuit parameters values. This can be deduced from inspection of values of these parameters (Table 2). It is clear that the inductances $L_1$ and $L_2$ are insensitive to temperature. $C_1$ and $C_2$ are decrease with temperature, while $R_1$, $R_2$ and $\psi$ are increase with temperature.

**Elastic compliance temperature dependency:**

The effect of change of each elastic compliances value of ceramic material, due to temperature variation, on the resonance characteristics of the PT was examined. Fig. 6 shows the PT characteristics due to the change of each parameter separately for a load of 150 kΩ.

The calculation declared that the change in dielectric and piezoelectric constants $\varepsilon_{33}, d_{31}, d_{33}, k_{31}$, $k_{33}$ and $Q_m$ have no significant effect on resonance characteristics. While the two elastic compliances $S_{E11}$ and $S_{D33}$ cause the resonance frequency to shift to a higher value and the third elastic compliance $S_{E33}$ cause the resonance frequency to shift to a lower value. These shifts are below and above the normal value by less than 1% for very slight change in voltage gain in each case.

The change in the values of the mechanical constants is governed by their temperature coefficients. So, the temperature coefficient of these parameters is the most significant factor that determines the voltage gain stability as the operating temperature change. Although the temperature coefficient of the three parameters $S_{E11}$, $S_{E33}$ and $S_{D33}$ (see Table 3) are negative, the effect of the first two parameters on resonance characteristics apposes the effect of the third parameter.

**Conclusions**

A model to predict the effect of operating temperature variation on Rosen's PT resonance characteristics using equivalent circuit is considered.

The calculation revealed that material's elastic compliances $S_{E11}$, $S_{E33}$ and $S_{D33}$ and their temperature coefficients have more impact on resonance characteristics and voltage gain than other material's constants, namely $\varepsilon_{33}, d_{31}, d_{33}, k_{31}, k_{33}$ and $Q_m$. Each acts as a counterpart with the other such that their net effect may provide a balance in the stability of voltage gain i.e. lower gain variation. This result may serve as a guide line toward PT ceramic material improvement.

**References:**


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Conference, PESC IEEE 32nd Annual, 4, 1761-1766.


List of symbols:

- $C_o1$ and $C_o2$: Shunt capacitors associated with the input and output electrodes respectively (Farad)
- $d_{31}$ and $d_{33}$: Piezoelectric strain constants (C/N)
- $H$: Thickness of the PT (m)
- $k_{31}$ and $k_{33}$: Electromechanical coupling coefficients (-)
- $L$: Length of the PT (m)
- $L$: Length of the generator section (m)
- $O_m$: Mechanical quality factor (-)
- $R_L$: Load resistor (Ohm)
- $S_{E11}$, $S_{E33}$: Elastic compliance
- $and S_{33}^D$: Constants (m²/N)
- $V$: Volume of each PT section (m³)
- $V_S$: Driving voltage (Volt)
- $V_L/V_S$: Voltage gain (-)
- $W$: Width of the PT (m)
- $w$: Angular frequency (Hz)
- $\varepsilon_{33}$: Dielectric constant (-)
- $\rho$: Density (Kg/m³)
Table (1) Piezoelectric Ceramic Material's Constants.

<table>
<thead>
<tr>
<th>Constants</th>
<th>Values (T=50 °C)</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{11}$ (m$^2$/N)</td>
<td>$0.732 \times 10^{-11}$</td>
<td>$-9 \times 10^{-15}$</td>
</tr>
<tr>
<td>$S_{33}^{E}$ (m$^2$/N°C)</td>
<td>$0.909 \times 10^{-11}$</td>
<td>$-6.5 \times 10^{-15}$</td>
</tr>
<tr>
<td>$S_{33}^{D}$ (m$^2$/N)</td>
<td>$0.646 \times 10^{-11}$</td>
<td>$-10.5 \times 10^{-15}$</td>
</tr>
<tr>
<td>$d_{31}$ (C/N)</td>
<td>$224.8 \times 10^{-12}$</td>
<td>$-8 \times 10^{-14}$</td>
</tr>
<tr>
<td>$d_{33}$ (C/N)</td>
<td>$138.4 \times 10^{-12}$</td>
<td>$9 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\varepsilon_{33}$ (C/Vm)</td>
<td>$7.28 \times 10^{-12}$</td>
<td>$14 \times 10^{-12}$</td>
</tr>
<tr>
<td>$k_{33}$</td>
<td>0.538</td>
<td>2 $\times 10^{-4}$ (°C$^{-1}$)</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>720</td>
<td>-4.25</td>
</tr>
</tbody>
</table>

Figure (1) Structure of a Typical Rosen Type PT (Joung, 2009)
Figure (2) Equivalent Circuit Model of the PT.

Figure (3) Simplified Equivalent Circuit Parameters
Modeling the Effect of Operation Temperature on Characteristics of Rosen Type Piezoelectric Transformer

Figure (4) Voltage Gain Versus Driving Frequency at 50 and 70°C.

Figure (5) Change in Voltage Gain Versus Load Caused by Operating Temperature Change from 50°C to 70°C.

Figure (6) Voltage Gain Versus Driving Frequency Due to One Parameter Variation while Others Held Constant, $R_L=150 \, \Omega$.