A simple nonlinear mathematical model for wind turbine power maximization with cost constraints

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Abstract—**In this paper we have proposed a non-linear mathematical model for a wind turbine. The objective function maximizes the power of the wind turbine and the constraints are related to the rotor and tower costs. Rotor diameter and hub height are the variables which affect on power of the wind turbine, so we have considered them as decision variable in our mathematical model. By increasing rotor diameter and hub height the power of the turbine will increase but the costs don't let the infinitive increase in rotor diameter and height. The model applied for a typical case study and the results of solving the model for it have shown in the paper.**

Keywords—wind turbine, non linear mathematical model, rotor diameter, hub height, optimization.

I. Introduction

Increasing fuel prices and environmental concerns create a great attention in the governments in developed and even developing countries on the renewable energy. Among the various renewable energy sources, wind energy could be, in the short term, one of the most promising renewable energy sources .The wind energy market is rapidly expanding worldwide .This rapid growth of the wind energy industry has led to cost reduction challenges and cost is the first concern for the wind farm owner.

Indeed, wind farm should produce the maximum energy by considering the minimum of the costs and in the most of the studies which have done on the wind energy these two factors have been considered simultaneously. Kusiak et al. proposed a multi objective model which maximizes the expected energy output, as well as minimizes the constraint violations to find the best wind turbine location in a wind farm [1]. Donovan formulates an integer program to determine the optimal positions of wind turbines within a wind farm. This integer programming model seeks to maximize power generated in accordance with constraints based on the number of turbines, turbine proximity, and turbine interference [4]. Elkinton et al. developed and combined Models of the principle costs of a wind farm and the energy production and losses to estimate of the levelized production cost (LPC). They employed Optimization algorithms to search for the wind farm layout with the lowest LPC [2].

Several studies have been done to analyze the cost components of the wind farm and wind turbines. Quraeshi et al. reviewed the costs and economics of wind turbine generator power plants and provides a brief discussion on the development status, costs, cost of energy, value of energy, type of credits and breakeven costs [5]. Ozer igra et al. proposed Cost estimates for the conventional, horizontal-axis, wind turbine and for the vortex-augmented wind turbine [6]. In the next section we will describe the main problem and the propose of the model and the technological formulas about the wind which are necessary for this research. In section 2 we will propose our nonlinear model which consists of one objective function and one constraint. In section 3 we will introduce a typical case study and we are solving the proposed model for it. In the last section the results and the efficiency of the model will be discussed.

II. Problem definition

The purpose of this paper is proposing a mathematical model to find the optimal power of the turbine by considering the cost of the rotor and tower. The objective function of this mathematical model is maximization of the power generated by the wind turbine, so at first we should know that which variables are effective in wind power and how can affect on it .to understand it we should notice these two points:

1) The power output of a wind generator is proportional to the area swept by the rotor - i.e. double the swept area and the power output will also double.

2) The power output of a wind generator is proportional to the cube of the wind speed - i.e. double the wind speed and the power output will increase by a factor of eight (2 * 2 *2)

We should remind that wind has power, and this power is created from the wind energy which it is a kinetic energy. Wind is made up of moving air molecules which have mass - though not a lot. Any moving object with mass carries kinetic energy in an amount which is given by:

$$\text{Kinetic Energy} = 0.5 * \text{Mass} * \text{Velocity}^2 \qquad (1)$$

Air has a known density (about 1.23 kg/m3 at sea level), so the mass of air hitting our wind turbine (which sweeps a known area) each second is given by the following equation:

$$\text{Mass/sec} = \text{Velocity} * \text{Area} * \text{Density} \qquad (2)$$

Therefore, the power in the wind hitting a wind turbine with a certain swept area is given by simply inserting the equation (2) into (1), resulting in the following equation:

Power = 0.5 * Swept Area * Air Density * Velocity^3 (3)

where Power is given in Watts, the Swept area in square meters, the Air density in kilograms per cubic meter, and the Velocity in meters per second. The swept area refers to the area covered by the turbine rotor, i.e. the area swept by the blades. It is also called the 'captured area'. Swept area can be easily calculated by:

Area Swept by the Blades = π × Radius² (4)

Now we can understand the verity of tow mentioned points on the top.
As it is mentioned before, the objective function of our mathematical model is maximization of the power generated by of the wind turbine given by (3). In other words, the objective function is:

$$\text{Max (P)} = 1/8\ \pi\ \rho\ D^2\ V^3 \qquad (5)$$

where D is rotor diameter, V is wind speed, and ρ is density of the air. As we can see in objective function by increasing rotor diameter and wind speed the wind turbine power will increase, but increase in these two variable will also increases the costs of rotor and tower. So the costs prevent from unlimited increase in these two variable. this is the logic of our model and it helps us to design the mathematical model which by solving it we can obtain the optimal power of the turbine.

III. A Simple Model

a) The objective function

We need one or more decision variables to make a mathematical model. These decision variables are often considered according to an objective function. In order to maximize the power, our decision variables are been defined according to equation (3). We can understand from (3) that the wind power depends on the swept area and the wind velocity. Equation (4) shows that swept area is a function of the rotor diameter. On the other hand, wind speed is related to the height, such that by increasing the height wind speed increases in the same region. So it shows that the second variable in the equation (3), i.e. the wind speed, is a function of height. There is an empirical mathematical relationship between wind speed and height which is used herein to formulate the turbine power as follows. For wind turbine engineering purposes, an exponential variation in wind speed with height may be defined relative to wind speed measured at a reference height of h_r. The reference height is usually chosen to be 10 meters. Thus, the relationship will be:

$$v_w(h) = V_{10}\,(h / h_{10})^a \qquad (6)$$

where:

$v_w(h)$ = velocity of the wind in [m/s] at an arbitrary height of h [m];
V_{10} = velocity of the wind at height, h_{10} = 10 meters
a = Hellman exponent

The exponent, a, is an empirically derived coefficient that varies dependent upon the stability of the atmosphere. For neutral stability conditions, a is approximately 1/7, or 0.143
Hence, if the wind speed in the reference height (10 [m]) is known, according to (6), we can obtain wind speed in each arbitrary height. Therefore, we substitute (6) instead of wind speed in the objective function, and the revised objective function is as following:

$$\text{Max } P(D, h) = \beta \times D^2 \times (V_{10}\,(h / h_{10})^{0.143})^3 \qquad (8)$$

with β = 1/8 π ρ.

b) Constraints

According to the objective function (8), by increasing D and h, the Power P increases infinitely, so, we should limit this increase with one or more unequal constraints. These boundaries are related to rotor and tower cost. It means that we have a specific amount of budget to invest on rotor and tower, and if the costs of rotor and tower violate from their budget we should decrease the rotor diameter and tower height. Also the rotor and tower cost boundaries should be a function of decision variables, rotor diameter and tower height. On the other hand it is not easy work to determine the costs of rotor and tower, and it needs to break the total cost into the component costs and after calculation of component costs we should aggregate them to total cost again. To do this process, we use a paper which calculates these component costs. Fingersh et al. divided the costs of the wind turbine into the components of the turbine, means rotor and tower, and then disaggregate the costs of the rotor and tower to their components which are related to rotor diameter and tower height [3].

Rotor cost:
According to mentioned paper, rotor is consisting of four main components as following:

- Blades
- Hub
- Pitch mechanisms and bearings
- Spinner, nose cone

The blade cost consists of blade material costs and labor costs, which both are related to rotor radius estimated as the following relationship:

Blade cost = blade material cost + labor cost =
$$[(0.4019 R^3 - 21051) + (2.7445 R^{2.5025})]/(1-0.28) \qquad (9)$$

The estimated relation assumes 28% of overhead.

The hub cost is related to the hub mass, which is calculated from the following approximate relation:

Hub mass = 0.945*(blade mass/2.61) + 5680.3 (10)

where the blade mass is estimated as:

mass = 0.4948 * $R^{2.53}$ per blade (11)

Then, the hub cost is calculated by:

Hub cost = hub mass*4.25 (12)

The total cost of pitch bearing for all three blades is calculated as a function of rotor diameter leading to:

Total pitch system cost (three blades) =
2.28 × (0.2106 × rotor diameter $^{2.6578}$) (13)

Nose cone cost depends on nose cone mass given below:

Nose cone mass = 18.5*rotor diameter-520.5 (14)
Nose cone cost=nose cone mass*5.57 (15)

After calculation of all costs related to the components, we should sum these costs up to obtain the total cost of the rotor. Therefore the total cost of the rotor is as follows:

Total cost of rotor =
blade cost + hub cost + total pitch bearing system cost + nose cone cost =
[(0.4019*$R^{2.96}$ – 21051) + (2.7445*$R^{2.35}$)]/0.72 + 18.06*hub_mass + (0.4802*rotor_diameter$^{2.66}$) + 5.57*nose_ cone_mass (16)

If we substitute the component cost relationship in equation (16) and simplify it, we will have an equation, which depends just on rotor diameter:

Total rotor cost = 0.06 $D^{2.96}$ + 1.45 $D^{2.35}$ + 0.48 $D^{2.65}$ + 103.05 D + 21242 (17)

As (17) shows, the total rotor cost is statistically estimated as a nonlinear function of the rotor diameter.

Tower cost:
Tower cost calculation is not as complex as rotor costs, and it is not necessary to calculate disaggregated costs to find the total cost. Indeed, the total tower cost is directly calculated according to Fingersh et al. paper. The tower cost depends on the tower mass which can be calculated as the following:

Tower mass= 0.2694*swept area*hub height + 1779 (18)
Tower cost = tower mass*1.5 (19)

If we substitute for π=3.14 in (4), we can convert equation (19) to find (20) which is a function of both the rotor diameter and the hub height:

Tower cost=12.68*R^2*h + 2668.5 (20)

Thus, the tower cost is a nonlinear function, as well as the total rotor cost.

Other costs:
There are additional costs due to some other parts in a wind turbine, among which we have just listed those related to the rotor diameter. It seems necessary to consider the following costs in our formulation. These costs are:

Low speed shaft cost = 0.01 $D^{2.88}$
Yaw cost = 0.06 $D^{2.96}$
Mainframe cost = 303.96 $D^{1.06}$

All of the above are obtained experimentally and are given in [3].

IV. CASE STUDY & COMPUTATIONAL RESULTS

To complete our model, we need some parameters, which are related to the studied region. These parameters are air density and wind speed in reference height (10m), which are indicated before by ρ and V_{10} respectively. Moreover, we assume that we have a certain predetermined quantity of budget, to invest on building a wind turbine. We have determined these values, rational as it is possible and they are not related to a specific region but they are obtained from several sites currently working as wind energy farms.
Suppose that ρ = 1.2 kg/m^3 (β = 0.471) and V_{10} = 4 m/s in the region which our wind turbine will be built. Furthermore, the budget that we have to invest on building the rotor and the tower is $384000. By such a budget one may be able to purchase a turbine in range of a medium size.
Now we can construct our mathematical model by applying the values assumed for parameters and the budget, in company with the objective function and constraints are obtained in the previous section. The final mathematical model for this case study is as follows:

Max $P(D, h) = 11.22\ D^2\ h^{0.429}$

Subject to:

$0.06\ D^{2.96} + 2.65\ D^{2.35} + 0.48\ D^{2.65} + 103.05\ D + 303.96\ D^{1.06} + 0.01\ D^{2.88} + 3.17\ D^2\ h < 360089.4$

Note that the constraint is a little approximated when simplifying. Figures (1) and (2) show the surfaces of the objective and the constraint respectively.
Obviously, to prevent blades strike to the ground we should consider another constraint simply shown by:

$h > D/2 + \varepsilon$

where ε is a usually small design parameter. The described model is coded by LINGO 8.0 and the following results are obtained executing the program for ε = 3 m:

D = 55.88 meters
h = 30.94 meters

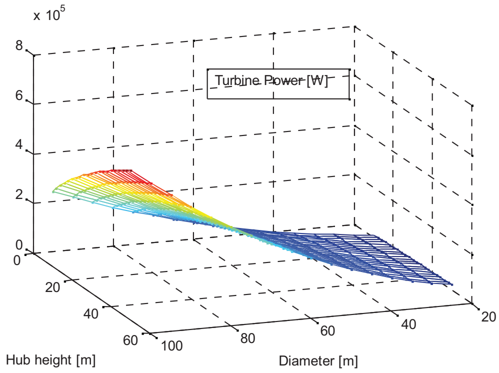

Figure (1): Surface of the objective

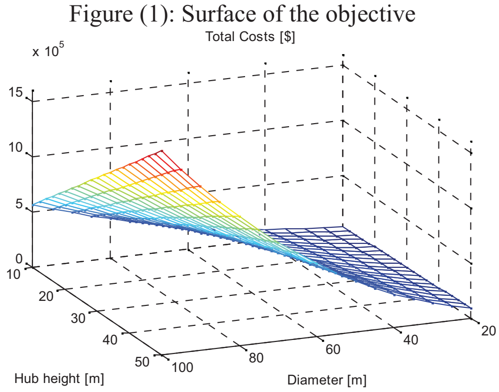

Figure (2): Surface of the constraint

Now we can calculate the power of the turbine by substituting the values of rotor diameter and tower height in the objective function:

$P = 11.22\ D^2\ h^{0.429} = 152.87$ kW.

This means a cost of 2.51 $ per one kW of wind power generation capacity, as its capital expenditures, which is an acceptable norm.
According to the values of D and h, we can understand that when the rotor diameter and hub height is equal to the maximum power of the turbine will be obtained. according to the technical information of the turbine which have built by the wind turbine manufactures we can understand that this result is true. for example for the 2500kW,1500kW and 300kW turbines, (D, h), are (100,100), (82.5,80), (30,33), respectively.
As we can see the hub height and rotor diameter are close together for these large turbines but this is not true for the small wind turbine which have been in the range of 500 W to 10000 W. In these turbines usually the hub height is more than rotor diameter because of the small size of these turbines. For example for a typical 5000W turbine the (D,h) is (5.4,9) which the equality of the rotor diameter and hub height is not true. In our case study, the blade distance from the ground is approximately 21.5 m which is an appropriate distance and the power of the turbine is approximately 106 kWs which is a rational power for this turbine with these features of rotor diameter and hub height.

V. CONCLUSION

In this paper we proposed a nonlinear mathematical model for a wind turbine to maximize the power of the turbine by considering the constraint of the rotor and tower costs. Decision variables are rotor diameter and hub height which the optimum values for them obtained in the last section. The results show that to reach the maximum power, the rotor diameter and hub height should be equal. This paper and the obtained results are useful for one turbine but we should notice that the results can be extended for a wind farm with more than one turbine. In fact we can apply this model for each turbine of a wind farm and affect the results from the problems which exist in a wind farm like wake effect, suitable distance between turbines and the optimum location of the turbines in the wind farm.
It is worthy to mention that based on the proposed model having a single turbine to generate a specified amount of power is more economical. However, in a wind farm there are so many other constraints and decision variables that make the problem quite different and much more complicated models should be applied for design.

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