Study of some topological concepts in bitopological spaces

Abstract:
A new definition of bitopological space is introduce in this paper with its separation axiom and continuity. Connected and compact set are defined with some theorems and also $T_{1/2}$, semi $T_{1/2}$, semi $T_{1/3}$ spaces are defined in this paper.

1- Introduction:
A bitopological space $(X,p_1,p_2)$[ J.C.Kelly "bitopological spaces", 1963] is an non-empty set $X$ with two topologies $p_1$ and $p_2$ on $X$.
 [Caldas,M, "semi-$T_{1/2}$-space" 1994] defined the concept of anew class of topological spaces called semi –$T_{1/2}$ spaces.
The purpose of this paper that is give a new definition of these concepts in Bitopological spaces.

2- Basic definitions and theorems
We would like to point out that all the definitions provided in this research has been formulated by researchers by adoption of their counterparts in topological spaces.

Definition (2-1)
Let $(X,p_1,p_2)$ be a bitopological space then a subset $A$ of $X$ is said to be open iff there exists $T_i$- open set $U$ such that $U \subseteq A$ and $\cap_{p_i}(U) \subseteq A_i$, $i=1,2$, this open set denoted by $\partial$-open set

Example(2-2):
Let $X\{a,b,c,d\}$, $T_1=X,\emptyset,\{a\},\{b\}\}$. $T_2=\{X,\emptyset,\{a\},\{c\}\}$. then $\partial$-open set=$\{X,\emptyset,\{a\},\{c\}\}$

Remark(2-3)
The intersection of two $\partial$-open sets is not necessary $\partial$-open while the union is $\partial$-open set

Proof: let $\{A_\lambda : \lambda \in \Lambda\}$ be any arbitrary collection of $\partial$-open set , then there exist $T_i$-open set $U_\lambda$ such that $U_\lambda \subseteq A_\lambda$ and $\cap_{p_i}(U_\lambda) \subseteq A_\lambda_i$, $i=1,2$ for each $\lambda$

Since
$\cup \lambda \in \Lambda (\cap_{i=1,2}cl_{p_i}(U_\lambda))$
Remark (2-4)
The set of all \(\partial\)-open sets is not a topological space.
- If \(A\) is pi-closed set for \(i=1,2\) then \(A\) is \(\partial\)-open set

Example (2-5):
Let \(X = a, b, c, d\) and \(T = \{X, \phi, \{a\}, \{b, c\}\}\), \(T = \{X, \phi, \{a\}, \{c, d\}\}\)
\(\partial\)-open set \(\{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}\)
Since \(\{a, b, d\} \cap \{c, b, d\} = \{b, d\}\) which is not \(\partial\)-open set then the set of all \(\partial\)-open sets is not topological space.

Notation (2-6)
Let \((X, p_1, p_2)\) be a bitopological space and let \(Y\) be a subset of \(X\) then a subset \(B\) of \(Y\) is said to be \(\partial_Y\)-open (the open set in \((Y, p_1, p_2, Y)\)) iff there exist \(p_Y\) open set \(W\) such that \(W \subseteq B\) and \(\cap \text{cl}_{p_Y}(W) \subseteq B\)

Theorem (2-7)
If \(A\) is \(\partial\)-open set and \(B\) is \(\partial_Y\)-open set where \(Y\) is a subset of \(X\) which is not \(\partial\)-open set then \(A \cap B\) is \(\partial_Y\)-open set and \(A \cup B\) is \(\partial\)-pen set

proof:- since \(A\) is \(\partial\)-open set then there exist \(T_1\)-open set \(U\) such that \(U \subseteq A\) and \(\cap \text{cl}_{p_1}(U) \subseteq A\) and since \(B\) is \(\partial_Y\)-open set then there exist \(T_Y\)-open set \(W\) such that \(W \subseteq B\) and \(\cap \text{cl}_{p_Y}(W) \subseteq B\), then
\[\cap \text{cl}_{p_1}(U) \cup \cap \text{cl}_{p_Y}(W) \subseteq A \cap B, \text{ and then}\]
\[\cap \text{cl}_{p_1}(U) \cap \text{cl}_{p_Y}(W) = (\text{cl}_{p_1}(U) \cap \text{cl}_{p_Y}(W)) \cap (\text{cl}_{p_1}(U) \cup \text{cl}_{p_Y}(W)) \cap (\text{cl}_{p_1}(U) \cup \text{cl}_{p_Y}(W)) \cap (\text{cl}_{p_1}(U) \cup p_{2Y}(W)) = [\text{cl}_{p_1}(U) \cap W] \cap \text{cl}_{p_2}(U \cup W) \cap [\text{cl}_{p_1}(U) \cup Y] \cap [\text{cl}_{p_2}(U) \cup Y] \cap [\text{cl}_{p_1}(U) \cup \text{cl}_{p_Y}(W)] \subseteq A \cup B, \text{ and then}\]
\[\cap \text{cl}_{p_1}(U) \cup \text{cl}_{p_Y}(W) \subseteq A \cup B.\]

Remark (2-8)
By theorem (1-5) we can define the subspace \(Y\) of a topological space \(X\) as follow:
- If \(Y\) is a subset of \(X\) such that \(Y\) is \(\partial\)-open set then \(\partial_Y\)-open set \(= \{G \cap Y: G \text{ is } \partial\text{-open set in } X\}\)

Notation (2-9)
Let \((X, p_1, p_2)\), \((Y, w_1, w_2)\) are two bitopological spaces, a function
\(f: (X, p_1, p_2) \rightarrow (Y, w_1, w_2)\) is said to be \(\partial\)-continuous iff \(f^{-1}(V)\) is \(\partial\)-open(\(\partial\)-closed) set in \(X\) for each \(V\) in \(\partial\)-open(\(\partial\)-closed) set in \(Y\)

Theorem (2-10)
A function \(f: (X, p_1, p_2) \rightarrow (Y, w_1, w_2)\) is said to be
1. \(\partial\)-open mapping iff \(f(U)\) is \(\partial\)-open whenever \(U\) is \(\partial\)-open set in \(X\)
2. \(\partial\)-closed iff \(f(U)\) is \(\partial\)-closed in \(Y\) whenever \(U\) is \(\partial\)-closed set in \(X\)
3. \(\partial\)-bi continuous iff \(f\) is \(\partial\)-open and \(\partial\)-continuous
4. \(\partial\)-homeomorphism iff \(f\) is bijection mapping, \(\partial\)-continuous and \(f^{-1}\) is \(\partial\)-continuous

Proof: We using the same proof in [1] with replacing the open set(closed set) by \(\partial\)-open(\(\partial\)-closed) set respectively.

Notation (2-11)
Let \((X, p_1, p_2)\) be a bitopological and \(A, C\) are two subsets of \(X\) we say that \(A\) and \(C\) are separated iff \(A \cap \text{cl}_{p_i}(C) = \emptyset\) and \(C \cap \text{cl}_{p_i}(A) = \emptyset\) for \(i = 1, 2\)

Remark (2-12)
If \(A, C\) are pi-separated, for \(i = 1, 2\) then it is not necessary that \(A, C\) are separated in \((X, p_1, p_2)\)
Example (2-13)
In example (2-5) \{a\} and \{b,c\} which are T_1-separated but are not separated in \((X,p_1,p_2)\)

Theorem (2-14)
Let Y be a subspace of a bitopological space X such that Y is \(\partial\)-open set and let A,C be two subset of Y . then A , C are pi-separated if and only if they are \(p_{1Y}\)-separated.
Proof: by using the relation \(\text{clip}(A) \cap Y = \text{clpi}_Y(A)\) and \(\text{clip}(C) \cap Y = \text{clpi}_Y(C)\) the result exist.

Notation (2-15)
Let \((X,p_1,p_2)\) be a bitopological space . a subset A of X is said to be \(\partial\)-disconnected if and only if it is the union of two non-empty pi-separated subsets for \(I=1,2\). A is said to be \(\partial\)-connected iff it is not \(\partial\)-disconnected.

Example (2-16)
\(X=\{a,b,c\}\) \(T_1=\{X,\phi,\{a\},\{b,c\}\}\) \(T_2=\{X,\phi,\{a,c\}\}\) then X is \(\partial\)-disconnected and \(\{a,c\}\) is \(\partial\)-connected

Theorem (2-17)
Let Y be a subspace of a bitopological space X and A subset of Y such that Y is \(\partial\)-open set then A is \(\partial\)-disconnected iff it is \(\partial_Y\)-disconnect and A is said \(\partial\)-connected iff it is \(\partial_Y\)-connected
Proof: by theorem (2-14) A is union of pi-separated sets iff it is \(p_Y\)-separated sets and hence the result.

Notation (2-18)
Let \((X,p_1,p_2)\) be a bitopological space and \(A\) be a subset of \(X\) is said to be compact iff every pi-open cover of A has pi-finite sub cover for \(I=1\) or 2

Example (2-19)
Let \(X=\{a,b,c\}\), \(T_1=\{X,\phi,\{a\},\{c\},\{a,c\}\}\), \(T_2=\{X,\phi,\{a\},\{b,c\}\}\) then \((X,p_1,p_2)\) is compact with respect to \(T_1\) or \(T_2\).

Remark (2-20)
If \((X,p_1)\) or \((X,p_2)\) is compact then \((X,p_1,p_2)\) is compact
Proof: by the definition of compactness in this paper clearly that if we take any open cover of X then this cover we take from \(p_1\) or \(p_2\) and hence the open cover containing a finite sub cover.

Theorem (2-21)
Let Y be subspace of a bitopological space X such that Y is \(\partial\)-open set and let \(D \subseteq Y\). then D is compact relative to X if and only if D is compact relative to Y.
Let D is compact relative to X and \(\{G_{\lambda} : \lambda \in \Lambda\}\) is pi-open cover of A in Y then \(A \subseteq \bigcup \{G_{\lambda} : \lambda \in \Lambda\}\), then there exist \(V_{\lambda}\) which is pi-open sets in X such that \(G_{\lambda} \cap Y\) for every \(\lambda \in \Lambda\) and then \(A \subseteq \bigcup \{V_{\lambda} : \lambda \in \Lambda\}\) then \(\{V_{\lambda} : \lambda \in \Lambda\}\) is open cover of D in X and since D is compact relative to X then there exist \(\lambda_1, \lambda_2, \ldots, \lambda_n\) such that \(D \subseteq V_{\lambda_1} \cup V_{\lambda_2} \cup \ldots \cup V_{\lambda_n}\) and then \(D \subseteq \bigcup \{V_{\lambda_1} \cup V_{\lambda_2} \cup \ldots \cup V_{\lambda_n}\}\cap Y = G_{\lambda_1}\) and there for D is compact relative to Y. and in the same way we proof the converse.

Theorem (2-22)
Let \((X,p_1,p_2)\) be a compact bitopological space then a pi-closed F of X is compact for \(i=1\)or 2 respectively. subset
Proof:
Let C=\(\{G_{\lambda} : \lambda \in \Lambda\}\) is pi-open cover of F where F is pi-closed set in X let D=\(\{G_{\lambda} : \lambda \in \Lambda\}\) \(\cup \{X-F\}\) form pi-open cover of X since X is compact the D has a finitely sub collection of D covers X and then covers F, and then F is compact.

Notation (2-23)
a bitopological space \((X,p_1,p_2)\) is say to be
1- \(\partial\)-\(T_0\) iff for each \(x, y\) in X thee exist \(\partial\)-open set U such that \(x \in U\), \(y \notin U\)
2- \(\partial\)-\(T_1\) iff for each two distinct point \(x, y\) there exist two \(\partial\)-open sets \(U, W\) such that \(x \in U\), \(y \notin W\)
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3- \( \partial \)-T\(_2\) space iff for each two distinct point x, y there exist two \( \partial \)-open sets U,W such that \( x \in U, y \notin W \) and \( U \cap W = \emptyset \)
4- \( \partial \)-T\(_{2\frac{1}{2}}\) space iff any two points of X can be separated by \( \partial \)-closed neighborhood
5- \( \partial \)-regular iff for each \( \partial \)-closed set F and \( y \notin F \) there exist two \( \partial \)-open sets U,W such that \( F \subseteq U, y \in W \) and \( U \cap W = \emptyset \)
6- \( \partial \)-T\(_3\) iff it is \( \partial \)-T\(_1\) and regular
7- \( \partial \)-completely regular space if and only if for an \( \partial \)-closed set F in X and x any point in X not in F there exist \( \partial \)-continuous map \( f:X \to [0,1] \) such that \( f(F) = \{1\} \), \( f(x) = 0 \)
8- \( \partial \)-T\(_{3\frac{1}{2}}\) space iff this space is \( \partial \)-completely regular and \( \partial \)-closed
9- \( \partial \)-normal iff for each two disjoint pi-closed sets F,H for i=1,2 there exist two \( \partial \)- open set U,W such that \( F \subseteq U, H \subseteq W \) and \( U \cap W = \emptyset \)
10- T\(_4\) space if and only if X is \( \partial \)-T\(_1\) and \( \partial \)-normal space
11- completely normal space if and only if for a separated sets A,C there exist two \( \partial \)-open sets U,W such that \( A \subseteq U \) and \( C \subseteq W \) \( U \cap W = \emptyset \) and we denoted this space by [CN]
12- \( \partial \)-T\(_5\) space iff it is both \( \partial \)-completely normal and hausdorff

**Theorem (2-24)**
Let \((X,p_1,p_2)\) be a bitopological space then every \( \partial \)-complete normal space is \( \partial \)-normal
Proof:
Let A,B are two disjoint pi-closed sets then \( A \cap cl_{p_i}(B) = \emptyset \) and \( B \cap cl_{p_i}(A) = \emptyset \) and there for A,B are two disjoint separated sets in X and since X is \( \partial \)-complete normal space there exist two disjoint \( \partial \)-open set U,W such that \( A \subseteq U \) and \( B \subseteq W \), then \((X,p_1,p_2)\) is \( \partial \)-normal space.

**Theorem (2-25)**
Let \((X,p_1,p_2)\) be a bitopological space and Y be a subset of X such that \( Y \in \partial \)-open(X) then \( \partial \)-T\(_0\), \( \partial \)-T\(_1\), \( \partial \)-T\(_2\), \( \partial \)-regular, \( \partial \)-T\(_3\), \( \partial \)-T\(_5\) are hereditary property
Proof: see [1] with replacing every open set (closed set) by \( \partial \)-open set (\( \partial \)-closed set) respectively .

3- new main results

**Notation (3-1)**
Let \((X,p_1,p_2)\) be a bitopological space then a subset A of X is said to be
a- semi-open set if \( A \subseteq cl_{p_i}(int_{p_i}(A)) \) and semi-closed if \( int_{p_i}(cl_{p_i}(A)) \subseteq A \) for \( i=1,2 \).
b- generalized closed set (briefly g-closed set) iff \( cl_{p_i}(A) \subseteq U \) where \( A \subseteq U \) and U is \( \partial \)-open set in \((X,p_i)\) for \( i=1,2 \).
c- semi-generalized closed (briefly sg-closed set) if \( scl_{p_i}(A) \subseteq U \) whenever \( A \subseteq U \) and U is semi-open set in \((X,p_i)\) for \( i=1,2 \).
d- generalized semi-closed (briefly gs-closed) iff \( scl_{p_i}(A) \subseteq U \), where \( A \subseteq U \) and U is \( \partial \)-open set in \((X,p_i)\) for \( i=1,2 \).
e- \( \psi \)-closed set if \( scl_{p_i}(A) \subseteq U \) whenever \( A \subseteq U \) and U is sg-open set in \((X,p_i)\) for \( i=1,2 \).

**Example (3-2)**
Let \( X = \{a,b,c\} \) and \( T_1 = \{X, \emptyset, \{a,b\}\} \) \( T_2 = \{X, \emptyset, \{a\}, \{b,c\}\} \) then \{a,c\} is \( \psi \)-closed
And \{b\} is sg-open set and sg-closed set but \{b,c\} is not \( \psi \)-closed set.

**Proposition (3-3)**
A subset A of X is semi-open set (respectively, semi-closed, sg-closed, gs-closed, g-closed, \( \psi \)-closed) in \((X,p_i)\) for \( i=1,2 \) then it is semi-open set (respectively, semi-closed, sg-closed, gs-closed, g-closed, \( \psi \)-closed) in \((X,p_1,p_2)\)
Proof:
Let A is pi-semi-open (respectively, pi-semi-closed, pi-sg-closed, pi-g-closed, pi-\( \psi \)-closed) set then \( A \subseteq cl_{p_i}(int_{p_i}(A)) \) (respectively, \( int_{p_i}(cl_{p_i}(A)) \subseteq A \) , \( scl_{p_i}(A) \subseteq U \) where \( A \subseteq U \) and U is pi-\( \partial \)-open set, \( scl_{p_i}(A) \subseteq U \) where \( A \subseteq U \) and U is pi-\( \partial \)-open set, \( cl_{p_i}(A) \subseteq U \) whenever \( A \subseteq U \) and U is pi-sg-open set ] and then A is semi-open set (respectively, semi-closed, sg-closed, gs-closed, g-closed, \( \psi \)-closed) in \((X,p_1,p_2)\).

**Notation (3-4)**
A bitopological space $(X,\pi_1,\pi_2)$ is said to be
a- $T_{1/2}$ space if every $g$-closed set is $\partial$-closed set
b- Semi-$T_{1/2}$ space [Caldas, M., 1994] if every sg-closed set is semi- closed set
c- semi-$T_{1/3}$ space [M.K.R.S. Veera Kumar, 1991] if every $\psi$-closed set in it is semi-closed

**Example (3-5)**
Let $X=\{a, b, c\}$ and $T_1=\{X, \emptyset, \{a\}, \{b, c\}\}$, $T_2=\{X, \emptyset, \{b\}, \{b, c\}\}$ then $(X,\pi_1,\pi_2)$ is semi-$T_{1/3}$ space and semi $T_{1/2}$ space

**Proposition (3-6)**
If $(X,\pi_i)$ is semi-$T_{1/3}$ space (respectively $T_{1/2}$ space, semi- $T_{1/2}$ space) for $i=1$ or $2$ then $(X,\pi_1,\pi_2)$ is semi-$T_{1/3}$ space (respectively $T_{1/2}$ space, semi- $T_{1/2}$ space)

**Proof:** Let $A$ be a $\psi$-closed set (respectively, $g$-closed set, sg-closed set) in $(X,\pi_1,\pi_2)$ by theorem (3-5) $A$ is $\psi$-closed set (respectively, $g$-closed set, sg-closed set) in $(X,\pi_i)$, $i=1$ or $2$ and since $(X,\pi_i)$ is semi-$T_{1/3}$ space (respectively $T_{1/2}$ space, semi- $T_{1/2}$ space) $A$ is semi-closed set (respectively, $\partial$-closed set, semi close set) then $(X,\pi_1,\pi_2)$ is semi-$T_{1/3}$ space (respectively, $T_{1/2}$ space, semi $T_{1/2}$ space).

**Remark:** (3-7) [7]
1- every semi closed set and thus every $\partial$-closed set is $\psi$-closed set
2- every $\psi$-closed [M.K.R.S. Veera Kumar, 1991] is sg-closed set and also gs-closed set
3- every semi-closed set is sg-closed set

**Proposition (3-8)**
every semi-$T_{1/2}$ space is semi $T_{1/3}$ space

**Proof:** let $A$ be a $\psi$-closed set in $X$, by remark (3-7) $A$ is sg-closed set and since $X$ is semi $T_{1/2}$ space $A$ is semi closed set and then $(X,\pi_1,\pi_2)$ is semi-$T_{1/3}$ space.

The converse is not true as we show in the following example

**Example (3-9)**
let $X=\{a, b, c\}$, $\pi_1=\{X, \emptyset, \{a\}, \{b, c\}\}$, $\pi_2=\{X, \emptyset, \{a, b\}\}$

Then $(X,\pi_1,\pi_2)$ is not semi-$T_{1/2}$ space since $\{a, c\}$ is sg-closed but not semi closed set, however $(X,\pi_1,\pi_2)$ is semi $T_{1/3}$ space.

**Definition (3-10) [6]**
For any subset $E$ of $(X,\pi_1,\pi_2)$, $\text{scl}_{\pi_i}(E)=\cap\{A: E \subseteq A \text{ such that } A \in \text{sd}(X,\pi_1,\pi_2)\}$ where $\text{sd}(X,\pi_1,\pi_2)=\{A: A \subseteq X \text{ and } A \text{ is sg-closed in } (X,\pi_1,\pi_2)\}$ and $\text{SO}(X,\pi_1,\pi_2)\ast=\{B: \text{scl}_{\pi_i}(B^c)=B^c\}$ for $i=1$ or $2$.

**Proposition (3-11)**
A bitopological space $(X,\pi_1,\pi_2)$ is a semi-$T_{1/2}$-space if and only if $\text{SO}(X,\pi_1,\pi_2)=\text{SO}(X,\pi_1,\pi_2)$.

**Proof**
Since the semi closed sets and the sg-closed sets are coincide by assumption $\text{scl}_{\pi_i}(E)=\text{scl}_{\pi_i}(E)$ holds for every subset $E$ of $(X,\pi_1,\pi_2)$ there for we have $\text{SO}(X,\pi_1,\pi_2)=\text{SO}(X,\pi_1,\pi_2)$

Conversely let $A$ be sg-closed set of $(X,\pi_1,\pi_2)$ then we have $A=\text{scl}_{\pi_i}(A)$ and hence $A^c \subseteq \text{SO}(X,\pi_1,\pi_2)$ thus $A$ is semi close set there for $(X,\pi_1,\pi_2)$ is semi $T_{1/2}$ space

**Proposition (3-12)**
A bitopological space $(X,\pi_1,\pi_2)$ is semi $T_{1/2}$ space if and only if for each $x \in X$, $\{x\}$ is semi open or semi closed

**Proof**
Suppose that for some $x \in X$, $\{x\}$ is not semi closed, since $X$ is the only semi open set containing $\{x\}$, the set $\{x\}$ is sg-closed set so it is semi closed set in the semi$T_{1/2}$ space $(X,\pi_1,\pi_2)$, therefore $\{x\}$ is semi open set

Conversely, since $\text{SO}(X,\pi_1,\pi_2) \subseteq \text{SO}(X,\pi_1,\pi_2)$ holds by theorem (3-11) it is enough to prove that $\text{SO}(X,\pi_1,\pi_2)\ast \subseteq \text{SO}(X,\pi_1,\pi_2)$. let $E \in \text{SO}(X,\pi_1,\pi_2)\ast$ . Suppose that $E \in \text{SO}(X,\pi_1,\pi_2)\ast$. Then $\text{scl}_{\pi_i}(E^c)=E^c$ and $\text{scl}_{\pi_i}(E^c) \neq E^c$ hold. There exist a point $x$ of $x$ such that $x \in \text{scl}_{\pi_i}(E^c)$ and $x \notin E^c (=\text{scl}_{\pi_i}(E^c))$ since
x ∉ SC_{pi}(E^c) there exist sg-closed set A such that x ∉ A and E^c ⊆ A. By the hypothesis the singleton {x} is semi-open set or semi-closed set.

Now if {x} is semi-open set, since {x}^c is semi-closed set with E^c ⊆ {x}^c, we have scl_{pi}(E^c) ⊆ {x}^c. i.e, x ∉ scl_{pi}(E^c). This contradicts the fact that x ∉ scl_{pi}(E^c). Therefore E ∈ SO(X,p_1,p_2).

If {x} is semi-closed set, since {x}^c is semi-open set containing the sg-closed set A (⇒ E^c) we have scl_{pi}(E^c) ⊆ scl_{pi}(A) ⊆ {x}^c. Therefore x ∉ scl_{pi}(E^c). This is contradiction. Therefore E ∈ SO(X,p_1,p_2).

Hence in both cases we have E ∈ SO(X,p_1,p_2), i.e, SO(X,p_1,p_2) *⊆ SO(X,p_1,p_2).

**Proposition (3-13)**
(X,p_1,p_2) is semi-T_{3/2} space if and only if every subset of X is the intersection of all semi-open sets and all semi-closed sets containing it.

**Proof**
Let (X,p_1,p_2) be a semi-T_{3/2} space with B ⊆ X arbitrary. Then B = \( \cap \{ x^c : x ∉ B \} \) is an intersection of semi-open sets and semi-closed sets by the above theorem the results follow.

Conversely, for each x ∈ X, {x}^c is the intersection of all semi-open sets and all semi-closed sets containing it. Thus {x}^c is either semi-open sets or semi-closed set and hence X is semi-T_{3/2} space.

**References**
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