INSTABILITIES AND CHAOS IN OPTICALLY INJECTED SEMICONDUCTOR LASERS

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Abstract

The stability of the output intensity of a semiconductor laser with external optical injection is studied by the numerical analysis based on the rate equations model. We investigate the effects of varying the laser operating control parameters on the dynamical behavior of the optically injected semiconductor. The time-series intensity and the corresponding phase – space portraits and the intensity spectra are plotted. We find that the variations of the control parameters lead to different dynamical behaviors through sequence of bifurcations, such as stable oscillations, periodic oscillations, quasi-periodic oscillations, and chaotic oscillations. These laser dynamical behaviors are studied for different values of the laser control parameters.

Keywords: Instabilities, Chaos, Bifurcations, Semiconductor lasers.
1. Introduction
Semiconductor lasers with optical injection have attracted a great considerable attention because they are excellent test systems for an investigation of the nonlinear dynamics and also because of their wide-ranging applications in many areas. Therefore we can consider the semiconductor laser as a representative of the other nonlinear dynamical systems. It is proved that the optically injected semiconductor laser can be used in the secure optical communications [1,2], chaos synchronization [3-5], optical chaos cryptography [6], messages encoding / decoding [7,8], photonic microwave generation[9], and other important applications.

Optically injected single-mode semiconductor laser system that is considered for our present work consists of two lasers, one of them is called the master laser (ML), whose output light signal is injected into the cavity of the second laser which is called the slave laser (SL). This system can exhibit a rich variety of nonlinear dynamical behaviors, such as stable, periodic, and chaotic output characteristics depending on the values of the laser operating parameters [1,10].

In this paper we present a numerical study of the effects of varying the main control parameters of the optically injected semiconductor laser on its dynamical behavior. Our numerical calculations are performed by using a simple rate equations describing the dynamics of the optically injected semiconductor laser. Different nonlinear dynamical behaviors are obtained from these calculations, ranging from stable laser output oscillation state to irregular chaotic oscillation state through a sequence of phase transitions.

2. Theoretical Model
The fundamental equations modeling an optically injected single-mode semiconductor laser is a set of differential rate equations describing the time evolutions of the complex optical electric field $E$ and the carrier population $N$. These equations can be written in dimensionless form [11-13] as follows:

$$\dot{E} = (1 - i \, b) \, G \, E + \eta \, e^{-i \, \Delta t} \tag{1}$$

$$T \, \dot{N} = P - N - P \, (1 + 2 \, G) \, |E|^2 \tag{2}$$

where

$$G = N - \varepsilon \, P \, (|E|^2 - 1) \tag{3}$$

Here $b$ is the line-width enhancement factor, $G$ is the laser material gain, $\eta$ is the injection field strength, $\Delta$ is the frequency detuning between the master laser and the slave laser, $T$ is the ratio of the carrier lifetime to the photon lifetime, $P$ is the pumping current, and $\varepsilon$ is the gain saturation coefficient.

3. Results and Discussion
We have numerically calculated Eqs. (1) – (3) by employing a fourth order Runge – Kutta method and verified various transitions among steady, periodic, quasi-periodic, and chaotic states. Here we show representative examples for the dynamical behaviors of the optically injected semiconductor laser that is obtained from our numerical investigation.

We choose the injection field strength ($\eta$) and the gain saturation coefficient ($\varepsilon$) as the main laser control parameters, and change their values. The other laser control parameters are kept fixed in the present work. Figs. (1) and (2) show the time –
series (the laser output intensity as a function of time), and the corresponding phase – space portrait (electric field imaginary part against electric field real part) and the intensity spectrum, which is calculated by using the fast Fourier transformation (FFT) method.

In Fig.1, we varied the value of the injection field strength ($\eta$) over the range $\eta = 0.010 – 0.018$, for the case of the absence of the gain saturation effect ($\varepsilon = 0$), when $b = 5$, $P = 0.7$, $T = 165$, and $\Delta = 0$ (no frequency detuning between the master laser and the slave laser). The dynamical behavior of the semiconductor laser starts with the period – one (P1) oscillation state, when $\eta = 0.01$, as shown in Fig.1(a1). The corresponding phase – space portrait is shown in Fig.1 (a2) and it is a single limit cycle, while the corresponding intensity spectrum is shown in Fig.1 (a3), and contains only single frequency that represents the fundamental laser frequency. When the value of $\eta$ is slightly increased to 0.011, the laser dynamical behavior changes to period-two (P2) oscillations (Fig.1(b1)), and the corresponding phase – space portrait shows two asymmetric orbits, as shown in Fig.1(b2). Another frequency appears in the corresponding intensity spectrum as shown in Fig.1(b3). As the value of $\eta$ increases continuously, the dynamical behavior of the laser system changes from one type to another type: Period-four (P4) oscillations, when $\eta = 0.0113$ (Figs.1(c1) – (c3)), quasi-periodic oscillations, when $\eta = 0.012$ (Figs.1(d1) – (d3)), period – six (P6) oscillations, when $\eta = 0.015$ (Figs.1 (e1) –(e3)), period – eight (P8) oscillations, when $\eta = 0.018$ (Figs.(f1) – (f3)), period – four (P4) oscillations, when $\eta = 0.0185$ (Figs2.(g1) – (g3)), and finally the laser reaches to the chaotic state at $\eta = 0.019$, where the laser starts to display output with irregular oscillations, and the corresponding phase – space portrait and intensity spectrum become more complicated (Figs.2(h1) – (h3)). In this case (i.e. when the gain saturation effect is taken into account), we have not observed the period – three (P3) and the period – six (P6) oscillation states. The difference between the laser dynamical behavior in the two cases, without and with gain saturation effect, is also evident from the comparison, for example, between Fig.1(f) and Fig.2(f), when $\eta = 0.018$, we see that when $\varepsilon = 0$ (Fig.1(f)) the dynamical behavior is chaotic while it is period–two (P2) oscillations when $\varepsilon = 0.0085$ (Fig.2(f)). Furthermore, we note that when the gain saturation effect is taken into account ($\varepsilon \neq 0$), the laser system takes a slightly longer range of $\eta$ value to reach the chaotic state.
compared with the case of the absence of the gain saturation effect ($\varepsilon = 0$).

4. Conclusions

We have numerically studied the dynamical behavior of the optically injected semiconductor laser using a simple theoretical model consists of two rate equations. We have presented results from the numerical calculations of these equations which show different types of laser dynamical behaviors.

By varying the value of the injection field strength parameter ($\eta$), period-doubling and quasi-periodic route to chaos are observed in this laser system. We find that when the gain saturation effect is taken into account ($\varepsilon \neq 0$), the feature of the laser dynamical behavior is significantly changed.

The results we have obtained in the present investigation reveal the effects of the laser control parameters on the dynamical behavior of the semiconductor laser system. These laser parameters can play important roles on the behavior of the laser output and therefore can be used as suitable parameters for controlling the laser system for the practical applications.
Fig. 1 Time series of the laser output intensity (first column) and the corresponding phase–space portraits (electric field imaginary part versus electric field real part) (second column) and intensity spectra (third column), as the value of $\eta$ increases, for $b = 5.0$, $P = 0.7$, $T = 165$, $\Delta = 0$, and $\varepsilon = 0$. 
Fig. 1 (Continued)
Fig. 2 Time series of the laser output intensity (first column) and the corresponding phase-space portraits (electric field imaginary part versus electric field real part) (second column) and intensity spectra (third column), as the value of $\eta$ increases, for $b=5.0$, $P=0.7$, $T=165$, $\Lambda=0$, and $\varepsilon=0.0085$. 
Fig. 2 (Continued)
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References