Adaptive Sliding Mode Control Design for a Class of Nonlinear Systems with Unknown Dead Zone of Unknown Bounds

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***Abstract*—The control problem for a class of a nonlinear systems that contain the coupling of unmeasured states and unknown parameters is addressed. The system actuation is assumed to suffer from unknown dead zone nonlinearity. The parameters bounds of the unknown dead zone to be considered are unknown. Adaptive sliding mode controller, unmeasured states observer, and unknown parameters estimators are suggested such that global stability is achieved. Simulation for a single link mechanical system with unknown dead zone and friction torque is implemented for proving the efficacy of the suggested scheme.**

I. INTRODUCTION

Dead zone is classified as one of the most important hard nonlinearities encountered frequently in practice and usually arises from continuous usage of actuators. This phenomenon has a significant effect on tracking performance of the plant and may lead to system instability. Dead zone was firstly pioneered by Tao and Kokotovic when they developed a dead zone inverse for compensating the dead zone effect [1, 2]. Furthermore, if the dead zone output is measurable, then perfect asymptotic adaptive cancellation for the unknown dead zone can be achieved [3].

Moreover, fuzzy and neural control systems were successfully used to construct dead zone compensators that would eliminate the effects of dead zone [4, 5]. Other researchers were capable of using robust adaptive control algorithms to achieve better performance [6, 7].

Depending on the dead zone properties, other approaches were developed for dealing with the unknown dead zone existence without requiring the construction of a dead zone inverse [8, 10]. Furthermore, in [9, 10] the unknown dead zone existence was handled without constructing a dead zone inverse.

On the other hand, a new controller and observer design was introduced for challenging nonlinear systems that contain the coupling of unmeasured states and unknown parameters in the measured states dynamics [11]. The results were promising when the authors implemented their theory on a single-link mechanical system with a friction torque described by LuGre model [11]. However, it is important to mention that a single-link mechanical system is likely to suffer from dead zone behavior at its actuation.

In [12], the author derived stable robust adaptive control law, unmeasured states observer, and unknown parameters update law considering the problem described in [11] with unknown dead zone in the system actuation. Moreover, in [12] a robust approach was employed for the unknown dead zone. Based on this approach, the uncertainty of the unknown dead zone is required to be reasonably small and the unknown dead zone bounds to be known.

In this paper we design an adaptive sliding mode controller, unmeasured states observer, and unknown parameters estimators for a class of SISO nonlinear systems that contains i. the coupling of the unmeasured states and unknown parameters ii. unknown dead zone with unknown bounds. So, in the scheme suggested in this paper, the bounds of the dead zone are not necessarily to be known.

The rest of the paper is organized as follows. The problem statement, that includes the class of nonlinear systems to be considered, is presented in Sec. 2. In Sec. 3, the plant and dead zone assumptions are mentioned and the main result is developed. Simulations are given in Sec. 4 and concluding remarks are stated in Sec. 5.

II. PROBLEM STATEMENT

Consider the following system [11]:

$$\dot{x} = Mx + h\left[u + f^T(x)\Theta + Z^T G(x)\Theta\right] \tag{1.a}$$

$$\dot{Z} = a(x) + B(x)Z \tag{1.b}$$

Where $x \in R^n$ is the vector of measured states and $Z \in R^P$ is the vector of unmeasured states, $u \in R$ is the system input, $\Theta \in R^P$ is the unknown constant parameter vector, $M \in R^{n\times n}$ and $h \in R^n$ are known constant matrix and vector, respectively, of the forms given in (2). $f(x) \in R^n$, $G(x) \in R^{p\times p}$, $a(x) \in R^p$, and $B(x) \in R^{p\times p}$ are known smooth functions of *x*.

$$M = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix} \text{ and } h = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ w \end{bmatrix} \tag{2}$$

Where I_{n-1} is (n-1)×(n-1) identity matrix and w>0.

Consider that we have the dead zone shown in Fig.1 at the system actuation. Such a dead zone can be described by:

$$u(t) = mv(t) + d(v(t)) \tag{3}$$

where $u(t)$ and $v(t)$ are the dead zone output and input respectively, m is the slope of the dead zone, and $d(v(t))$ can be modeled as:

$$d(v(t)) = \begin{cases} -m\,b_r & for \quad v(t) \ge b_r \\ -m\,v(t) & for \quad b_l < v(t) < b_r \\ -m\,b_l & for \quad v(t) \le b_l \end{cases} \tag{4}$$

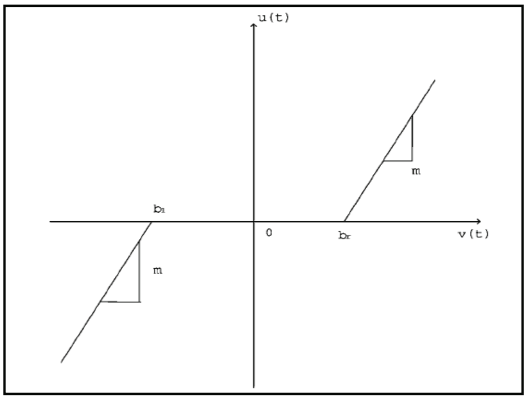

Fig. 1. Dead zone model

On the other hand, if we focus on (1) we can see that it contains a coupling of the unmeasured state and unknown parameter. Such a class of nonlinear systems was addressed in [12] considering unknown dead zone in the system's actuator. The parameters b_l and b_r were supposed in [12] to have known upper and lower bounds.

The main objective of this paper is to design a control law, states observer and parameters estimator such that a desirable tracking performance is achieved for system (1) considering that the upper and lower bounds of b_l and b_r are unknown.

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III. Main Results

The following assumptions are considered for the dead zone and plant considered in this paper:

(A1) The dead zone parameters b_r, b_l, and *m* are unknown but their signs are known, i.e. $b_r>0$, $b_l<0$, $m>0$, and the dead zone output *u(t)* is not available for measurement.

(A2) The dead zone parameters b_r, b_l, and *m* are bounded, but their bounds are unknown.

(A3) The pair *(M, h)* is controllable.

(A4) There exist positive definite matrices P_Z and Q_Z such that $B^T(x)P_Z + P_Z B(x) \le -Q_Z$. Also, for every bounded *x(t)*, the solution of *Z(t)* is bounded for any initial condition $Z(t_0)$.

(A5) Each parameter, θ_i, $i=1...p$, has a bounded unknown magnitude and a known sign.

(A6) The functions *f(x)*, *G(x)*, *a(x)*, and *B(x)* are bounded functions of *x*.

Remark 1: Assumption (A1) is common in many practical systems such as servomotors and servovalves. If *u(t)* is available for measurement then the dead zone problem of this paper becomes trivial and easy to be solved. For more details on assumptions (A3)-(A6) justifications, interested readers are recommended to review [11].

For the desired states trajectory, the following assumption should be achieved.

(A7) The desired trajectory, $x_d = \left[x_d, \dot{x}_d, \ldots, \overset{(n-1)}{x_d}\right]$ is continuous, bounded and available for measurement.

Let us define $\phi = \frac{1}{m}$, $\Psi = \phi.\Theta$ and $b = \max(|b_l|, |b_r|)$.

Moreover, a filtered tracking error is defined as:

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}(t) \tag{5}$$

where $\lambda > 0$. We can rewrite (5) as:

$s(t) = \Lambda^T \tilde{\mathbf{x}}(t)$

with $\Lambda^T = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, 1]$, and $\tilde{x} = x(t) - x_d(t)$.

Defining $\Lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, (n-1)\lambda]$, from (3) and (5) we obtain:

$$\dot{s}(t) = \Lambda_v^T \tilde{x}(t) + \tilde{x}^n(t) = \Lambda_v^T \tilde{\mathbf{x}}(t) + w[mv(t) + d(v(t)) + f^T(x)\Theta + Z^T G(x)\Theta] \tag{6}$$

Note: It has been shown that the filtered error described by

(5) has the following properties: (i) the equation $s(t)=0$ defines the time-varying hyperplane in $\boldsymbol{R}^n$, on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero, (ii) if $\tilde{\mathbf{x}}(0)=0$ and $|s(t)|\le\varepsilon$ with constant ε, then $\tilde{\mathbf{x}}(t)\in \boldsymbol{\Omega}_\varepsilon = \left\{ \frac{\tilde{\mathbf{x}}(t)}{\tilde{\mathbf{x}}_i} \le 2^{i-1}\lambda^{i-n}\varepsilon, i=1,\ldots,n \right\}$ for $\forall t\ge 0$ and (iii) if $\tilde{\mathbf{x}}(0)\neq 0$ and $|s(t)|\le\varepsilon$ then $\tilde{\mathbf{x}}(t)$ will converge to $\boldsymbol{\Omega}_\varepsilon$ within a time constant (n-1)/ λ [13, 14].

THEOREM 1: If the plant described by (1) satisfies assumptions A1-A7, then all the closed loop signals are bounded and the equilibrium state $\tilde{\mathbf{x}}(t_0)=0$ is asymptotically stable under the following control law, parameter estimators and state observer:

$$v(t) = -k_d s(t) + \frac{\hat{\phi}}{w}\left\{x_d^{(n)} - \Lambda_v^T\tilde{x}\right\} - f^T(x)\hat{\Psi} - \hat{Z}G^T(x)\hat{\Psi} - \hat{b}|s| \quad (7)$$

$$\dot{\hat{\Psi}} = s.(\Gamma^T)^{-1}.\left(\hat{Z}^T G(x) - f^T(x)\right) \quad (8)$$

$$\dot{\hat{\phi}} = \eta\, s\frac{\left\{x_d^{(n)} - \Lambda_v^T\tilde{x}\right\}}{w} \quad (9)$$

$$\dot{\hat{b}} = |s| \quad (10)$$

$$\dot{\hat{Z}} = a(x) + B(x)Z + s.P_Z^{-1}.\operatorname{sgn}(\Psi)G(x) \quad (11)$$

Where k_d, η are positive constants, $\Gamma\in R^{p\times p}$ and $P_Z\in R^{p\times p}$ are positive definite matrices with $P_Z = P_Z^T$.

Proof: Substituting the control law (7) into (6) gives:

$$\dot{s}(t) = \Lambda_v^T\tilde{x}(t) + w\left[\frac{1}{\phi}\left(-k_d s(t) + \frac{\hat{\phi}}{w}\left\{x_d^{(n)} - \Lambda_v^T\tilde{x}(t)\right\} - f^T(x)\hat{\Psi} - \hat{Z}^T G(x)\hat{\Psi} - b|s|\right) + d(v(t)) + f^T(x)\Theta + Z^T G(x)\Theta\right] - x_d^{(n)} \quad (12)$$

Consider a Lyapunov candidate:

$$V = \frac{1}{2}\left[\frac{\phi}{w}s^2 + \tilde{\Psi}^T\Gamma\tilde{\Psi} + \frac{1}{\eta}\tilde{\phi}^2 + \tilde{Z}^T\Lambda_{|\Psi|}P_Z\tilde{Z} + \tilde{b}^2\right]$$
(13)

Where $(\tilde{\cdot}) = (\hat{\cdot}) - (.)$ and $\Lambda_{|\Psi|}$ is a diagonal matrix whose ith diagonal element is the absolute value of the ith element in the parameter vector Ψ, that is $\Lambda_{|\Psi|} = diag(|\theta_1|,|\theta_2|,\ldots,|\theta_p|)$ where $diag(.)$ denotes a diagonal matrix. Differentiating (13) and substituting for $\dot{s}$ from (12) we obtain:

$$\dot{V} \le -k_d s^2 + \tilde{\phi}\left(\frac{\dot{\hat{\phi}}}{\eta} - s\frac{\left(x_d^{(n)} - \Lambda_v^T\tilde{x}\right)}{w}\right) + \left(s.f^T(x) + \dot{\hat{\Psi}}^T\Gamma\right)\tilde{\Psi} - s\hat{Z}^T G(x)\hat{\Psi} + sZ^T G(x)\Psi - \hat{b}|s| + |s|b + \tilde{Z}^T\Lambda_{|\Psi|}P_Z\dot{\tilde{Z}} + \tilde{b}\dot{\hat{b}} \quad (14)$$

Where we employed $b=\phi d(v(t))$. Since $\dot{\tilde{Z}} = \dot{\hat{Z}} - \dot{Z}$, and using (11) and (1b), we can rewrite $\dot{\tilde{Z}}$ as:

$$\dot{\tilde{Z}} = B(x)\tilde{Z} + sP_Z^{-1}\operatorname{sgn}(\Psi)G(x) \quad (15)$$

Substituting (8), (9), (10) and (15) into (14), and using the fact that $\Lambda_{|\Psi|}\operatorname{sgn}(\Psi)=\Psi$, we obtain:

$$\dot{V} \le -k_d s^2 + \tilde{Z}^T\Lambda_{|\Psi|}P_Z B(x)\tilde{Z}$$

$$\dot{V} \le -k_d s^2 + \frac{1}{2}\tilde{Z}^T\Lambda_{|\Psi|}(P_Z B(x) + B^T(x)P_Z)\tilde{Z}$$

$$\dot{V} \le -k_d s^2 - \frac{1}{2}\tilde{Z}\Lambda_{|\Psi|}Q_Z\tilde{Z}$$

$$\dot{V} \le -k_d s^2 - \frac{1}{2}\lambda_{\min}\left(\Lambda_{|\Psi|}Q_Z\right)\tilde{Z}^T\tilde{Z} \quad (16)$$

From (16) it is clear that $s\in L_2\cap L_\infty$, and $\tilde{Z},\tilde{\Theta},\tilde{\phi},\tilde{b}\in L_\infty$. Since Z,Θ,b and ϕ are bounded and $\tilde{Z},\tilde{\Theta},\tilde{\phi},\tilde{b}\in L_\infty$, then $\hat{Z},\hat{\Theta},\hat{b}$ and $\hat{\phi}$ are also bounded. Then using (12), we can easily conclude that $\dot{s}\in L_\infty$. We have $s\in L_2\cap L_\infty$ and $\dot{s}\in L_\infty$, then $s\to 0$ as $t\to\infty$ according to Barbalat's lemma. This would make the equilibrium state $\tilde{\mathbf{x}}(t_0)=0$ asymptotically stable, where the problem of tracking the state vector $\mathbf{x}_d(t)$ can be replaced by first-order stabilization in s [13].

Since Q_Z is positive definite matrix, then from (16) we can easily conclude that $\tilde{Z}\in L_2\cap L_\infty$, and from (15) we get $\dot{\tilde{Z}}\in L_\infty$. Therefore, based on Barbalat's lemma $\tilde{Z}\to 0$ as $t\to\infty$. ■

IV. SIMULATION RESULTS

Consider a single-link mechanical system with an unknown dead zone exits at the actuator. The system dynamics can be described by:

$$J\ddot{x} = D(v(t)) - f_f \quad (17)$$

Where J is the inertia of the link, x is the angular position of the link, $D(v(t))$ is the actuator dead zone output, $v(t)$

is the control input, and f_f is the friction torque described by the following LuGre dynamic model:

$$f_f = \theta_1 z + \theta_2 \dot{z} + \theta_3 \dot{x} \tag{18}$$

$$\dot{z} = \dot{x} - \sigma \frac{|\dot{x}|}{g(\dot{x})} z \tag{19}$$

$$g(\dot{x}) = F_c + (F_s - F_c) e^{-\left(\dot{x}/\omega_s\right)^2} \tag{20}$$

Where σ, $\theta_1, \theta_2, \theta_3, F_s, F_c$, and ω_s are friction coefficients. The three coefficients θ_1, θ_2 and θ_3 are unknown. The objective is to control the link and make the position and velocity of the link track a predefined trajectory x_d and $\dot{x}_d$ respectively.

Our system, described by (17)-(20), can be cast into the form given by (1), where:

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, Z = \begin{bmatrix} z \\ z \\ z \end{bmatrix}, f(x) = \begin{bmatrix} 0 \\ x_2 \\ x_2 \end{bmatrix},$$

$$G(x) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sigma \frac{|x_2|}{g(x_2)} & 0 \\ 0 & 0 & 0 \end{bmatrix}, a(x) = \begin{bmatrix} x_2 \\ x_2 \\ x_2 \end{bmatrix}$$

and

$$B(x) = \begin{bmatrix} -\sigma \frac{|x_2|}{g(x_2)} & 0 & 0 \\ 0 & -\sigma \frac{|x_2|}{g(x_2)} & 0 \\ 0 & 0 & -\sigma \frac{|x_2|}{g(x_2)} \end{bmatrix}$$

(See [11] for more details on casting the system under study into this form).

In our paper, the following numerical values are considered:

$J = 3.4$, $\sigma = 340$, $F_s = 11$, $F_c = 1.557$, $\omega_s = 0.14$, $\eta = 1$ and $k_d = 10$ (See [11,12] for more details on the justifications in choosing those values).

In our simulation we selected the values 0.5, 1.8 and 1 as initial values for x_1, ϕ and b respectively. For x_2, $\hat{\mathbf{Z}}(t)$ and $\hat{\Psi}$ we used zero initial values. The base link is controlled to follow a sinusoidal trajectory, $x_d(t) = \sin(0.4\pi t)$.

Using the control law, parameters estimators and unmeasured states observer given in (7-11), we obtained the results shown in Fig .2 and Fig. 3. It is clear that from graphs A, B, C, and D of Fig. 2, excellent tracking was obtained, and this would emphasizes that both errors $e_{1,2} \to 0$ as $t \to \infty$ and this is a natural consequence since it was proved that $s \to 0$ as $t \to \infty$. Graphs E, F of Fig. 2, A, B, C, D, E, and F of Fig. 3 show the boundedness of all signals and parameters involved, which reinforce the efficacy of the control scheme suggested in the theorem. It is worth noting that the parameter b is also bounded which is the estimated bound measure for the dead zone parameters.

V. Conclusion

In this paper, an adaptive sliding-mode controller, unmeasured states observer and unknown parameters estimator were designed for a class of nonlinear systems that contain a coupling of unmeasured states and unknown parameters with the presence of unknown dead zone of unknown bounds at the system actuation. The validity of the main results was shown through a case study of a single link mechanical system with a dynamic friction LuGre model and all results were promising. Excellent tracking performance was obtained with all closed loop signals are bounded. The parameter *b*, which is a measure of the unknown dead zone bound, was successfully estimated and the dead zone bounds are no more needed.

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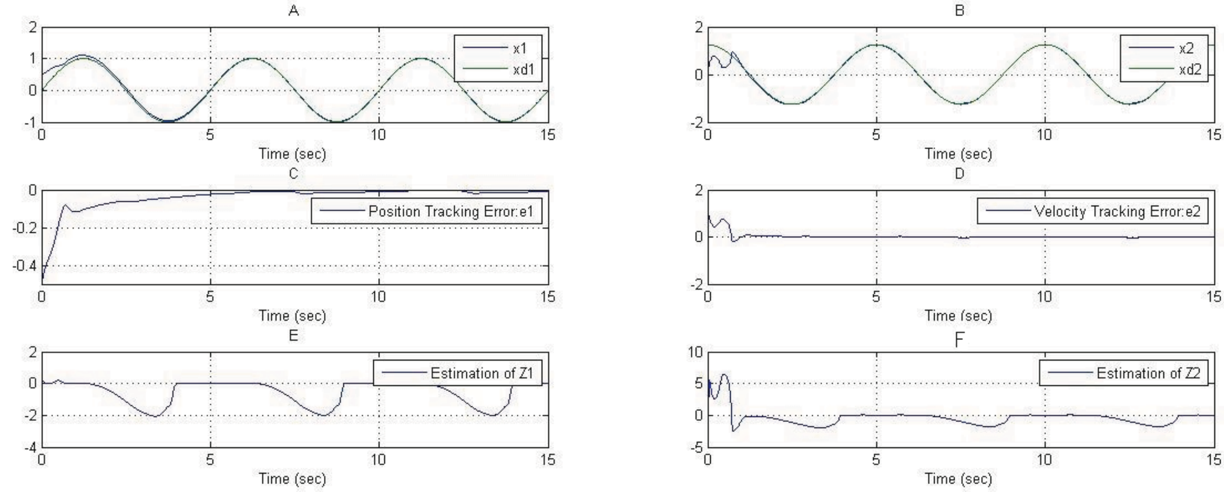

Fig. 2 A. Position tracking performance (in rad). B. Velocity tracking performance (in rad/sec). C. Position tracking error (in rad). D. Velocity tracking error (in rad/sec). E. Estimation of z1. F. Estimation of z2.

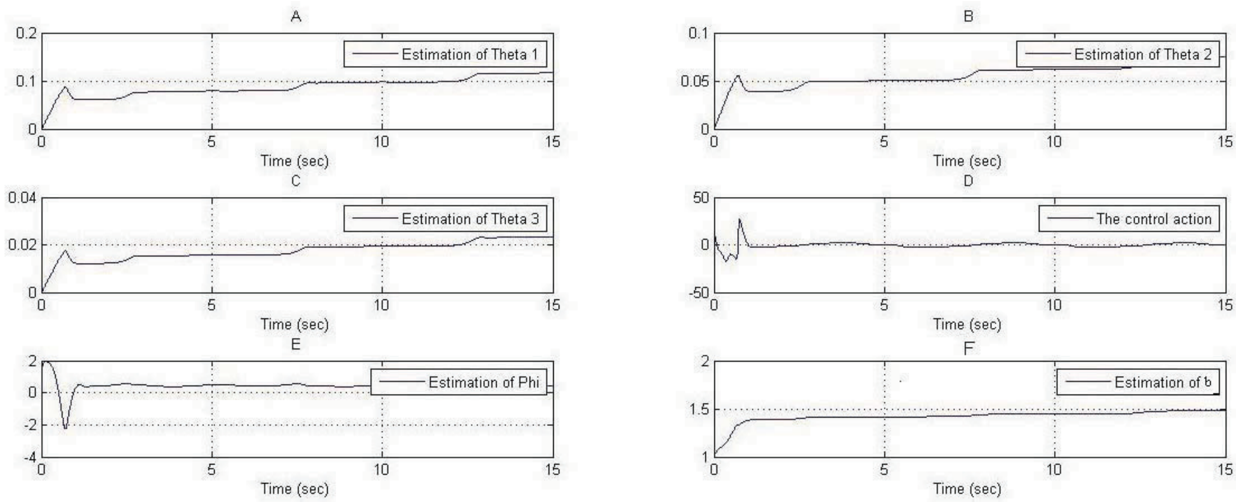

Fig. 3 A. Estimation of θ_1. B. Estimation of θ_2. C. Estimation of θ_3. D.

Control signal v(t). E. Estimation of φ. F. Estimation of b.