Numerical study of nonlinear Fabry- Perot cavity filled with a self-focusing medium

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Abstract:
In this paper a numerical study of a nonlinear Fabry- Perot cavity filled with a self- elf focusing are solved numerically using finite differences method with iterative approach. The results are in agreement with experimental data available by other workers.

Key words: numerical analysis, Fabry- Perot, self- focusing

1. Introduction:

The effect of transverse profile of laser beam on a fabry- perot cavity was included by McCall [1], Marburger and Felber [2] and Ballagh etal [3]. Then this subject was studied more generally by Rosanov and Semenov [4] and by Monoly and Gibbs [5]. Recently there are many studies dealing with this problem, for example Weaire etal [6] and Reinisch and Vitrant [7], but their studies are dealing with self- focusing in two dimensions which may not hold for three dimensional problem. In this work a numerical study of Fabry-Perot cavity contains a self-focusing three dimensional medium is presented. A computer program was written for the numerical solution algorithm.

2. Theory:

When a Gaussian laser beam of radius is incident on a Fabry- Perot cavity filled with a nonlinear medium (see figure 1), i.e.

$$n = n_o + n_2|E|^2 \qquad \ldots(1)$$

where n_o is the linear refractive index, n_2 is the nonlinear one, $|E|^2$ is the beam intensity which assumed to be a cylindrically symmetric Gaussian beam. For $n_2 > 0$ we have self-focusing and for $n_2 < 0$ we have self-defocusing.

Using Maxwell's equations, one can find:

$$\nabla^2 E(r,z) + k^2 n^2 E(r,z) = 0 \qquad \ldots(2)$$

where k is the propagation constant.

If we assume the solution of eq (2) is of the form:

$$E(r,z) = E_f(r,z)\exp(ikz) + E_b(r,z)\exp(-ikz) \qquad \ldots(3)$$

where E_f, E_b are the forward and backward waves in the cavity respectively (see figure 1).

Then for slowly varying fields, it can be shown that:

$$\nabla_T^2 E_f + 2ik\frac{E_f}{\partial z} + \frac{n_2}{n_0}k^2(|E_f|^2 + |E_b|^2)E_f = 0 \qquad \ldots(4)$$

$$\nabla_T^2 E_b - 2ik\frac{E_f}{\partial z} + \frac{n_2}{n_0}k^2(|E_f|^2 + |E_b|^2)E_b = 0 \qquad \ldots(5)$$

where $\nabla_T^2 = \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}$

with the boundary conditions:

$$E_f(r,0) = (1-R)E_{in}(r) + RE_b(r,0)$$

$$E_b(r,L) = Rexp(ikLE_f(r)$$

where R is the reflectivity of the mirrors.

The quantity kL is the linear phase shift in the cavity which assumed to be equal $2n\pi$, where n is an integer, and

$E_{in}(r) = E_0 \exp\left(-r^2/\rho^2\right)$ where

ρ is the incident beam radius.

The treatment becomes simple with dimensionless quantities , so it is convincement to introduce the following definitions:

$$F = \frac{1}{ka}\left(\frac{n_2}{n_o}\right)E_f \qquad \text{...(6a)}$$

$$B = \frac{1}{ka}\left(\frac{n_2}{n_o}\right)E_b \qquad \text{...(6b)}$$

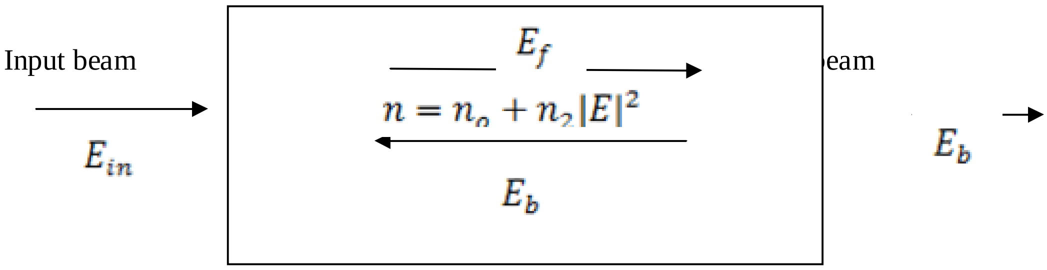
$$Z = \frac{1}{ka^2}z \qquad \text{...(6c)}$$

$$r^* = r/\rho \qquad \text{...(6d)}$$

Substituting these definitions into eqs. (4,5) gives:

$$\nabla_T^2 F + i\frac{\partial F}{\partial z} + (|F|^2 + |B|^2)F = 0 \quad \text{...(7)}$$

$$\nabla_T^2 B - i\frac{\partial B}{\partial z} + (|F|^2 + |B|^2)B = 0 \quad \text{...(8)}$$



Fig(1) A nonlinear Fabry- Perot cavity

3. Results and discussion:

The coupled equations (7) and (8) are solved numerically using finite differences method involve iterative approach. A computer program for this method has been written to serve the solution. The numerical solution is applied for the following experimental data, which are used by Firth etal [8], and Abraham etal [9]. They use a cw laser with wavelength 5.5 μm has a Gaussian beam of diameter 200 μm incident on Fabry- Perot cavity of length 200 μm contains a nonlinear medium (INSb) where its refractive indices are n_o=4 and $n_2 = 10^{-5} cm^2/W$. The surfaces of the cavity are used as mirrors of reflectivity 90%.

Figure (2) shows the transmitted power versus the input power. It is seen that there is an optical bistability region switching up to *25 mW* input power. The profile of the transmitted beam at switch up point and the input beam are shown in figure (3). It is seen that profile at the switch up point exhibits dramatic changes. These results are in agreement with experimental results of [8,9]. The solution is repeated for different input beam radius ρ , it is found that the optical bistability loop is expanded as ρ decrease, and the switch up and switch down point are enhance when is decrease, these results are in agreement with those of [7].

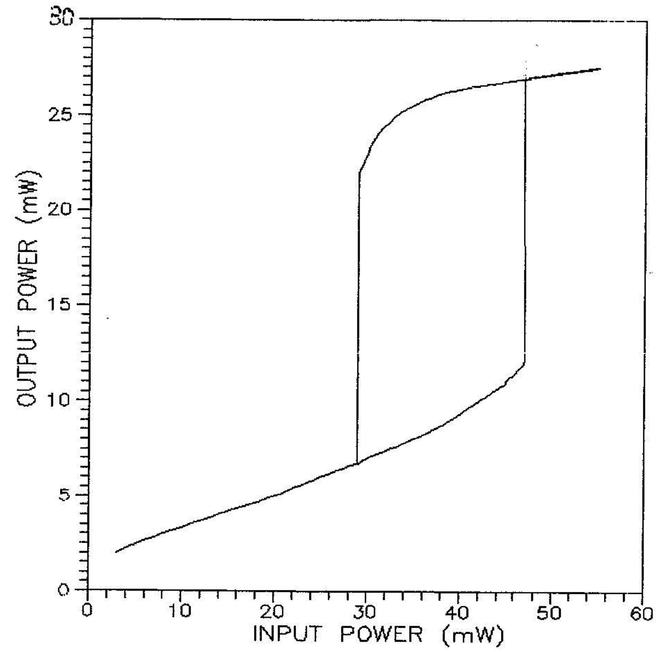


Fig (2) The transmitted power versus the input one $\lambda = 5.5\ \mu m$,

$\rho = 200\ \mu m, L = 200\ \mu m, n_2 = 10^{-5} cm^2/W$.

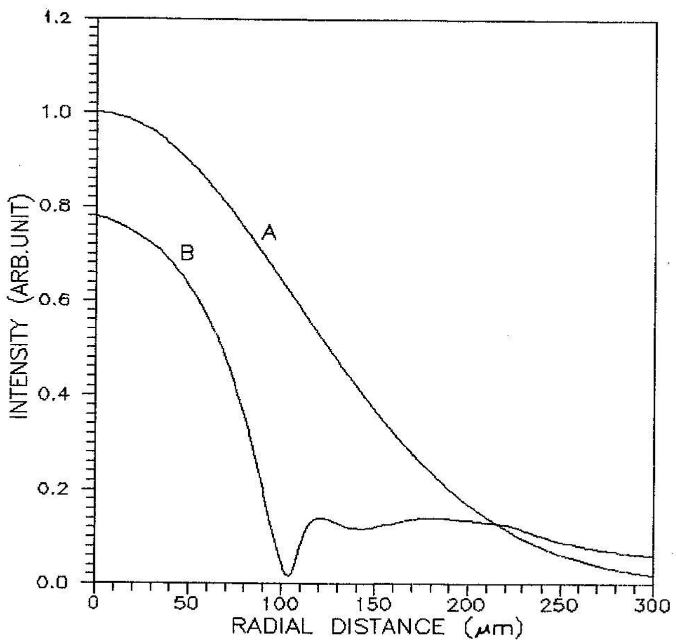


Fig (3) The normalized intensity profile : A)at the input, B) the transmitted at the switch up point.

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دراسة عددية لتجويف فابري - بيرو مملوء بوسط للتبؤر الذاتي

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الملخص:

هذا البحث دراسة عددية لتجويف فابري- بيرو لاخطي مملوء بوسط للتبؤر الذاتي. تم حل معادلات التبؤر الذاتي باستخدام طريقة الفروقات المنتهية ذات التقارب المتتالي. النتائج المحصلة من الحل متوافقة مع النتائج العملية المتوفرة لباحثين أخرين.