Solving Infinite Nonlinear Boundary Value Problems of Three Orders by Using Shooting Method

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ABSTRACT

The purpose of this research is to design an algorithm for solving nonlinear boundary value problems of third orders with an infinite number of boundary conditions. Newton-Raphson method was used for finding ($\infty$) value then we solved it by using shooting method. good results were obtained with a very small error value.

1. Introduction

Studied the boundary layer flow over a continuous moving surface in another quiescent fluid medium.[2,12] have paid attention to the solution’s existence and uniqueness, however, the approximate analytical solution remains unanswered. The goal to get it can be achieved by implementing the double-parameter transformation perturbation method which has been used in obtaining the approximate solutions of a wide class of differential equations.[13, 14, 15, 16] Engineers, mathematicians and other scientists systems who use mathematical models in order to describe and analyze physical often need to answer the question How
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does the system response change as the system parameters change?" For example, how does the air flow around an airplane wing change as the shape of the wing changes, and how does this affect drag? Sensitivity analysis seeks to answer such questions. Techniques used for this analysis vary from those which focus on the original mathematical model to those which consider such questions only after the model has been discretized and a numerical method has been introduced. Continuous Sensitivity Equation Methods (CSEMs) examine the mathematical model—which usually consists of one or more partial differential equations and appropriate boundary conditions—at the infinite dimensional level. These methods seek to derive a (PDE) sensitivity equation, with boundary conditions, which describes how the state variables change with respect to small changes in the design variables. Up to this point, CSEMs have used formal techniques in order to derive the sensitivity equation. In this method the solution is considered as the summation of an infinite series which usually converges rapidly to the exact solution. This simple method has been applied to solve linear and non-linear equations of heat transfer. Essentially, the PDE state equation and the boundary conditions are implicitly differentiated in order to derive the sensitivity PDE. For a specific class of state equations, one can construct an operator framework and use the Implicit Function Theorem in order to rigorously derive the sensitivity equation. [4, 6, 7, 8, 9].

2. Sensitivity Equations

The intent of this paper is to document the line of sight sensitivity equations for a Cassegrain telescope. These equations express the image motions at the focal plane in terms of the motions of primary, secondary, and focal plane. A numerical example is given for the Gemini F/16 IR configuration. The results are checked against those obtained from Code V Computer software for the same optical configuration.[5] Sensitivity analysis is used for a wide range of engineering problems, including characterization of complex flows, fast evaluation of nearby flows. Of methods for fast evaluation of nearby flows provides a rigorous framework to answer some difficult questions. For instance: what is the effect on the flow of changing the angle of attack, the thickness or the camber of an airfoil? First order Taylor series in parameter space yield quick and inexpensive estimates of the flow over the whole domain for nearby values of the shape parameter. Taylor series are very cost effective because the sensitivities can be solved for a fraction of the cost of the flow computation. Sensitivity analysis is also a scientifically rigorous tool for assessing the effects of input data uncertainty such as manufacturing tolerances on the performance of an airfoil. In essence, sensitivity information allow us to
cascade input data uncertainty through the CFD code to obtain uncertainty estimates of the flow response. Here, uncertainty analysis will be performed using first order sensitivity information combined to second order statistics.[11]

3. Shooting methods

Based on dividing the integration interval to subintervals at the beginning of each subinterval values estimated for the given dependent variables then the ODEs of the problem integrated in the subinterval.[2] Methods for numerically solving stochastic initial-value problems have been under much study [1] and the references therein. However, the theory and numerical solution of stochastic boundary-value problems have received less attention [17]. Generally, these stochastic boundary-value problems cannot be solved exactly and numerical methods must be used to obtain an approximate solution.

4. Algorithm

In this part of the research we develop a new algorithm to solve infinite nonlinear boundary value problem of third orders by using shooting method.

Let us consider boundary value problem

$$\frac{d^3y}{dx^3} = f(x, y, y', y'')$$

With the boundary conditions

$$y(a) = A, \quad y'(b) = B, \quad y'(\infty) = C$$

**Step 1:** we change the boundary condition at infinity when

$$y'(L) = C$$

**Step 2:** we must change the boundary value problem into an initial value problem. Define the variables

$$y_1 = y, \quad y_2 = y', \quad y_3 = y''$$

The equations are then

$$\frac{dy_1}{dx} = y_2, \quad \frac{dy_2}{dx} = y_3, \quad \frac{dy_3}{dx} = f(x, y, y', y'')$$

$$y_1(a) = A, \quad y_2(b) = B, \quad y_3(\infty) = C$$

We cannot impose the condition

$$y_2(\infty) = C$$

We must choose a value of

$$y_3(0) = k$$

and adjust k so that the condition is satisfied, the boundary condition are then

$$y_1(a) = A, \quad y_2(b) = B, \quad y_3(0) = k$$
Step 3: we use the Newton-Raphson method to adjust the value of $k$, and this requires we define some sensitivity equations. define
\[ y_4 = \frac{\partial y_1}{\partial k}, \quad y_5 = \frac{\partial y_2}{\partial k}, \quad y_6 = \frac{\partial y_3}{\partial k}. \]

Step 4:
\[ \frac{dy_4}{dx} = y_5, \quad \frac{dy_5}{dx} = y_6, \]
\[ \frac{dy_6}{dx} = \frac{d}{dx} \left( \frac{\partial y_3}{\partial k} \right) = \frac{\partial}{\partial k} \left( \frac{dy_3}{dx} \right) = \frac{\partial}{\partial k} f(x, y, y', y'') \]
The conditions are
\[ y_4(a) = A, \quad y_5(b) = B, \quad y_6(0) = C \]

Step 5:
\[ y_2(L, k) = C, \quad \phi(k) = y_2(L, k) - C = 0 \]
and
\[ \frac{d\phi}{dk} = y_5(L) \]

Step 6:
\[ k^{r+1} - k^r = -\frac{\phi(k^r)}{\phi'(k^r)} \]

5. Numerical applications
In this part we will solve two of nonlinear boundary value problems of third orders with an infinite number of boundary conditions using the algorithm that was mentioned in the third part of this paper and give the amount of error between the exact solution and numerical solution with the drawing by using MATLAB program.

Problem(1):
\[ y''' + y''y + y' + y = 0 \]
with boundary conditions
\[ y(0) = 1, \quad y'(0) = -1, \quad y'(\infty) = 0 \]

Solution:
Step 1:
\[ y'(L) = 0 \]
Step 2:
\[ y_1 = y, \quad y_2 = y', \quad y_3 = y'' \]
and the equations are then
\[ \frac{dy_1}{dx} = y_2, \quad \frac{dy_2}{dx} = y_3, \quad \frac{dy_3}{dx} = -y_1y_3 - y_1y_2 - y_1 \]
\[ y_1(0) = 1, \quad y_2(0) = -1, \quad y_2(\infty) = 0 \]
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change the boundary conditions at infinity when
\[ y_1(0) = 1, \quad y_2(0) = -1, \quad y_3(0) = k \]

Step 3 :
\[ y_4 = \frac{\partial y_1}{\partial k}, \quad y_5 = \frac{\partial y_2}{\partial k}, \quad y_6 = \frac{\partial y_3}{\partial k} \]

Step 4 :
\[ \frac{dy_4}{dx} = y_5, \quad \frac{dy_5}{dx} = y_6, \]
\[ \frac{dy_6}{dx} = \frac{d}{dx}(y_6) = \frac{d}{dx}\left(\frac{\partial y_1}{\partial k}\right) = \frac{\partial}{\partial k}\left(\frac{dy_3}{dx}\right) = \frac{\partial}{\partial k}\left(-y_1y_3 - y_1y_2 - y_1\right) \]
\[ = -y_1y_6 - y_3y_4 - y_1y_5 - y_2y_4 - y_4 \]

are and the conditions
\[ y_4(0) = 1, \quad y_5(0) = -1, \quad y_6(0) = 0 \]

Step 5 :
\[ y_2(L, k) = 0, \quad \phi(k) = y_2(L, k) - 0 = 0 \]

and
\[ \frac{d\phi}{dk} = y_5(L) \]

Step 6 :
\[ k^{r+1} - k' = -\frac{\phi(k')}{\phi'(k')} \]

We obtained on maximm error between numerical solution and exact solution of problem (1) \( 4.8697e-005 \)

Fig(1): comparison between numerical solution and exact solution of problem (1)
Problem (2):

\[ y'''' + yy'' + y' y - xy'' = 0 \]

with boundary conditions

\[ y(0) = 1, \quad y'(0) = 0, \quad y'(\infty) = 1 \]

Solution:

Step 1:

\[ y'(L) = 0 \]

Step 2:

\[ y_1 = y, \quad y_2 = y', \quad y_3 = y'' \]

and the equations are then

\[ \frac{dy_1}{dx} = y_2, \quad \frac{dy_2}{dx} = y_3, \quad \frac{dy_3}{dx} = -y_1y_3 - y_3y_2 + xy_3 \]

\[ y_1(0) = 1, \quad y_2(0) = 0, \quad y_2(\infty) = 1 \]

change the boundary conditions at infinity when

\[ y_1(0) = 1, \quad y_2(0) = 0, \quad y_3(0) = k \]

Step 3:

\[ y_4 = \frac{\partial y_1}{\partial k}, \quad y_5 = \frac{\partial y_2}{\partial k}, \quad y_6 = \frac{\partial y_3}{\partial k} \]

Step 4:

\[ \frac{dy_4}{dx} = y_5, \quad \frac{dy_5}{dx} = y_6, \quad \frac{dy_6}{dx} = \frac{d}{dx}(y_6) = \frac{d}{dx}\left(y_3\frac{\partial y_1}{\partial k}\right) = \frac{\partial}{\partial k}\left(y_3\right) = \frac{\partial}{\partial k}\left(-y_1y_3 - y_3y_2 + xy_3\right) \]

\[ = -y_1y_6 - y_3y_4 - y_3y_5 - y_2y_6 + xy_6 \]

are and the conditions

\[ y_4(0) = 1, \quad y_5(0) = 0, \quad y_6(0) = 1 \]

Step 5:

\[ y_2(L, k) = 1, \quad \phi(k) = y_2(L, k) - 1 = 0 \]

and

\[ \frac{d\phi}{dk} = y_6(L) \]

Step 6:

\[ k^{r+1} - k^r = -\frac{\phi(k^r)}{\phi'(k^r)} \]

We obtained on maximm error between numerical solution and exact solution of problem (2) \((1.4576e-005)\)
6. Conclusions

In this paper an algorithm is designed to solve of nonlinear boundary value problems of third orders with an infinite number of boundary conditions. where using Newton-Raphson method to find value ($\infty$) and then solve this problem using shooting method, and obtained good results where error ratio is small, finally drawing the exact solution and numerical solution of two problems by using MATLAB program.

References


3) Bashir M. Khalaf, New parallel algorithms for BVP in ODEs, college of education, Mosul university, (2006).

