One-Dimension Simulation of Plasma Flow in the Cylindrical Hall Thruster

One-dimension (1D) magnetohydrodynamic (MHD) model is employed to study physics of the acceleration process inside the cylindrical Hall thruster is independently designed. And some influencing factors which are the plasma ionization, neutralization, collision, Bohm diffusion as well as anode sheath, etc. are considered. Using the Runge-Kutta method obtains the distributions of the speed, ion number density, temperature. The ion number density increases, but near exit drops and the neutral number density gradually reduces in the channel. The ion speed reaches the maximum at exit and the electron velocity has the great gradient distribution in the downriver. These distributions are relevant with electromagnetic distribution, collision and electronic impedance. Finally make the analysis the connection between electron temperature distribution and velocity as well as magnetic field.

Keywords: Plasma, Cylindrical hall thruster, Magnetohydrodynamics, Runge-Kutta method

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1. Introduction

The principal advantage of electric propulsion over traditional chemical propulsion is the high specific impulse, which makes electric thruster systems capable of using significantly less propellant mass to produce the same impulse as chemical thrusters, and enhance spacecraft capabilities and reduce life cycle cost. The high specific impulse of electric thruster is not without penalties. Since electric thrusters require electrical energy, the engineering trade-off is that thrust levels are limited by the availability of electrical power. The inherent power limitations of a spacecraft relegate electric propulsion to the low thrust. Low thrust, high specific impulse applications include satellite station-keeping, reposition missions, attitude control, and orbit transfer in deep exploration missions. Among different kinds of electric thrusters, Hall thrusters are a nearly mature technology for satellite propulsion.

In Hall thrusters, the gas discharge mainly occurs in ExB region. When electrons move through magnetic field, majority of them are restricted by magnetism, and forms ring electric current (namely Hall electric current). The electrons are restricted which promote the gas ionization. At first in the 1960s, the view which ions are accelerated in the quasi-neutral plasma was proposed, and done massive experiments and the fundamental research. The main research results have stated in the recent summary literature [1-2].

Generally, the Hall thruster is divided into two kinds: one is stationary plasma thruster (SPT). Another is anode layer thruster (TAL). Two kinds of types have the same basic working principle. In SPT, because electrons and ceramic wall mutual action is bigger, so it has higher secondary electron emission (SEE) from the walls. This causes a lower electron temperature in SPT. In TAL, the ion acceleration takes place over a very short length near the anode.

Models of Hall thrusters have been developed using fluid [3-5] of hybrid [9-11] approaches to aid in the optimization of the performance of the thrusters. They are one-dimensional (1D), quasi-1D or 2D, and steady state or transient. Most of models are based on the quasi-neutrality assumption. These studies aim to predict high fidelity solution details inside the thruster while simulating real flight conditions, and towards better prediction of the performance and design issues.

This paper mainly utilizes the 1D magnetohydrodynamic model and simulates the discharge plasma peculiarities inside the cylindrical anode layer Hall thruster under the stable state.

2. Formulation of the Model

2.1 Physical processes in a thruster plasma

The Hall thruster plasma is partially ionized gas, consisting of electrons, ions, neutral particles. The plasma is formed primarily through the collision ionization via the electron impact with the incoming neutral propellant, in ExB region. The plasma in the thruster is assumed to be quasi-neutral, i.e., electron number density nₑ is locally equal to ion number density nᵢ. The assumption of quasi-neutral is valid except the sheath. The plasma in the thruster is sustained within the cylindrical discharge chamber by an axial electric field Eₓ established between down electrode pole and anode. The electrons move toward the anode across the radial magnetic field that is produced by magnet. The interaction of these electrons with the crossed axial electric field Eₓ and radial magnetic field Bᵧ redirects the electron in the...
angle’s direction, greatly reducing the electron conductivity in axial direction.

In order to simulate partially ionized plasma several important elastic and inelastic collision can take place simultaneously. The elastic collision involves only exchange momentum and energy between colliding particles, whereas inelastic processes like ionization recombination, charge-exchange collision, plasma-wall interaction, secondary emission, sputtering etc. can be responsible for redistributing the number density of the particles along with its momentum and energy. But is not all processes are equally important. For example, the momentum exchange due to Coulomb interaction is not as important as the plasma-neutral momentum exchange. But in this simulation we make further supposition, for example, neglecting the pressure gradient of ions, the plasma and the wall interaction and so on.

2.2 Governing equations

Based on the 1D magnetohydrodynamic model to simulate the plasma in the Hall thruster, supposed all the physical quantities are uniformity at radial direction. The length of acceleration channel \( L = 0.02 \text{m} \), radial direction area \( A = 0.0012 \text{m}^2 \). We take \( z = 0 \) at anode, and \( z = L \) at channel exit. In our simulated region, the magnetic mirror forms of magnetic field, it only considered the radial magnetic field, but at axial distribution is mutative (see Fig. 1). It come from the results of electromagnetic simulation in acceleration channel, and use curve fitting least square method get this magnetic field distribution:

\[
B(z) = -644z^2 + 8z + 0.1
\]  

Though the Debye length is generally very small (of the order of 10–100\( \mu \text{m} \)) compared to the channel length. The thickness of the sheath only has several Debye lengths. But the plasma in the sheath is nonlinear. And further because exist sheath potential, so ions must flow into sheath from quasi-neutral region near the anode. So the computation quasi-neutral plasma region must except for the anode sheath region. Therefore we take the simulation region at \( 6 \times 10^{-6} - 2 \times 10^{-2} \text{m} \).

The macroscopic model considers that the plasma is composed by three independent fluid: electrons (e), ions (i), and neutrals (n), and formulates equations for particle, momentum, and energy conservation of the three species. The set of differential equations for the stationary response of the quasi-neutral plasma is:

\[
\frac{\partial (\beta n V_i)}{\partial z} = -\beta n_i \frac{\partial n}{\partial z} 
\]  

\[
V_i \frac{\partial n_i}{\partial z} = -\beta n_i \frac{\partial n}{\partial z} 
\]

\[
\frac{\partial (nT)}{\partial z} = e n \frac{\partial V}{\partial z} - v_{nm} n V_e 
\]

\[
V \frac{\partial V}{\partial z} = -\frac{e}{m_i} \frac{\partial \phi}{\partial z} - \beta n_i \frac{\partial (V_i - V_e)}{\partial z} 
\]

\[
\frac{\partial (nT)}{\partial z} = e n \frac{\partial V}{\partial z} - v_{nm} n V_e - \beta n_i \left( \alpha_i E_i + \frac{3}{2} T \right) 
\]

where \( m_{\alpha}, n_{\alpha} ( \alpha = e, i, n ) \) are, respectively, mass, velocity, particle number density \( n \approx n_e \approx n_i \); \( \phi \) is the electric potential and \( e \) is the unit of electric charge (all ions are considered singly charged); \( T \) is electron temperature; \( E_i \) is ionization energy of the gas (for Argon \( E_i \approx 15.75 \text{eV} \)) and \( \alpha_i \) takes into account the effective energy loss per actual ionization, due to excitation collisions \( (\alpha_i \approx 2-3) \) [3]; There are two important parameters: the effective ionization rate per electron \( \beta \) and the effective frequency for the electron axial diffusion \( v_{dl} \).

![Fig. (1) The distribution of (a) magnetic lines and (b) magnetic field distribution along axis](image)

The effective ionization rate per electron \( \beta \) is the temperature function, it is expressed [6] as:

\[
\beta = \sigma_i \varepsilon_i \left( 1 + \frac{2 kT}{E_i} \right) e^{-\frac{E_i}{kT}} 
\]

and for Argon \( \sigma_i = 3.6 \times 10^{-20} \text{m}^2 \), \( E_i = 12.1 \text{eV} \), \( m_i = 2.2 \times 10^{-25} \text{kg} \), \( k = 1.38 \times 10^{-23} \text{J/K} \), \( \varepsilon_i = \frac{8 kT}{\pi m_i} \).
Equations (5) and (6) for the electron dynamics requires some discussion. Those equations are based on the quasi-neutral drift approximation, which request satisfies following two conditions [7]:

\[ |V_e| << \left( \frac{T_e}{m_e} \right)^{\frac{1}{2}} \]

and \( v_e << \omega_e \) (\( \omega_e = eB/m_e \)) is the electron gyrofrequency, and \( v_e \) is the total collision frequency for electron momentum. The conditions make negligible the contribution of the convective terms to the momentum equation; as a result the electron kinetic energy of mean motion is also negligible in the energy equation. Then neglect inertia terms in the azimuthal component of the momentum equation. From condition \( v_e << \omega_e \), we can know angular velocity \( \omega = -\omega_e V_e/v_e \) is very large. Substituting \( \omega \) from this equation into the axial component of the momentum equation, one obtains Eq. (5). And in this equation \( v_e \) is expressed as:

\[ v_e = \frac{\alpha_0^2}{\nu_e} = \Omega \omega \] (8)

where Hall parameter is given by

\[ \Omega = \frac{\nu_e}{\nu_e + \nu_{e+n} + \nu_{e+m}} \]

(typical Hall parameter value between 100-1000) [5], where \( \nu_{e+m} = \alpha_B \omega_e \) is Bohm diffusion, an \( \alpha_B \) is Bohm parameter (generally \( \alpha_B \approx (1/80) - (1/100) \)) [4].

### 2.3 Boundary conditions and numerical method

In computational method, using curve fitting least square method obtains the electromagnetic field distribution, then using Runge-Kutta method to solve the set equations. Finally obtains the main parameters distribution of plasma in thruster. This is different with numerical simulation of Michael Keider [4]. This set of differential equations (2)-(6) must satisfy some certain boundary conditions to be able to solve. Refer actual experiment datum, we give the boundary conditions:

\( V_e(x=0.02)=1 \times 10^4 \text{ m/s} \),
\( V_e(x=0)=1.6 \times 10^4 \text{ m/s} \),
\( T_e(x=0)=4.5 \times 10^4 \text{ K} \),
\( n_e(x=0)=3 \times 10^{17} \text{ m}^{-3} \),
\( K(3.8 \text{ eV}) \),
\( n(x=0)=6 \times 10^{15} \text{ m}_0^{-3} \)

Neutral particle velocity and discharge voltage are known, the ion at the inlet channel is supersonic. The mass rate is \( m = 0.3 \text{ mg/s} \) and discharge current is about 0.8A.

We use Runge-Kutta algorithm to solve the set of differential equations (2)-(6). We take the relative error is \( 10^{-2} \) and the absolute error is \( 10^{-6} \). So we can get the distributions of electron temperature, ion velocity, ion number density, electron velocity, neutral particle number density. In this simulation, we have not considered the electric potential of the exterior negative pole electron launcher. The down electrode pole was treated as zero potential. So hypothesis electric potential distribution is known. This is according to the electromagnetic simulation result and carries on curve fitting least square method to obtain the distribution. In quasi-neutral region, the electromagnetic is affected by the plasma is very small, so the calculation method is reasonable. In 1D simulation we suppose the potential is same at surface of anode and ceramic. Finally according to the actual situation, except for the anode sheath potential and modify this distribution, so we can get the potential (see Fig. 2):

\[ \varphi(x) = 1.75 \times 10^6 x^2 - 5.8 \times 10^4 x + 476 \] (9)

![Fig. (2) The electric potential distribution along axis](image)

### 3. Result and discussion

Figure (3) is ion number density and neutral particle number density distribution. Ion number density increases rapidly from a base value of \( 6 \times 10^{15} \text{ m}^{-3} \) that is the value at the inlet, and attains a maximum value \( 2.4 \times 10^{10} \text{ m}^{-3} \) downstream of the acceleration channel before it decreases near the exit. The change is slight between 0.012m and 0.02m. This is consistent with the magnetic field distribution, because radial magnetic field distribution is quite big nearby 0.01m. Thus the massive electrons are restricted by the magnetic field. The neutral gas ionization by impact finally increased, so produces massive ions.

On the other hand, from the neutral particle number density gradually decreases, also can explain the increase of ion number density. The neutral particle number density from the initial \( 3 \times 10^{17} \text{ m}^{-3} \) reduces to \( 1.06 \times 10^{15} \text{ m}^{-3} \). This also is because the propellant enter the channel occurs ionization. Further shows ion number density and neutral particle number density have close correlation with gas ionization in the acceleration channel.
Fig. (3) The ion and neutral particle number density distribution

The ion number density decreases slightly nearby the acceleration channel exit, but the neutral particle number density continuously drops. This kind of situation looks like does not ally with the above analysis. We know that ions are accelerated rapidly under the electric field to leave the channel. But the velocity of neutral particle enters the channel is much smaller than the velocity of ions which leave the channel. Although the ions occur backflow phenomena, the ions are produced by the ionization can’t supplement the reduction of ion density. This has concern with temperature distribution, because the ionization rate is the temperature function. Higher temperature and bigger ionization rate, otherwise is smaller. This is why the ion number density slightly decreases nearby the exit.

Figure (4) is ion velocity distribution, ion velocity arrives to the maximum at the channel exit, and we can see the change is big before 0.012m, afterwards the speed trend relaxes. There is main relate with the gradient distribution of electric potential. The potential’s affect of the exterior negative pole launcher is not considered this simulation. The wall only is treated as zero potential. Therefore the electric potential distribution like Fig. (4) shows.

Fig. (4) The ion velocity distribution along axis

The gradient of the electric potential is big before 0.012m, so the change of ion velocity is large. Afterward the electric potential tends to relax, so the ion velocity change is small and arrive to the maximum $2.48 \times 10^7$ m/s at nearby the exit. This result is basically consistent with the value that is obtained from the formula

$$V_i \approx \sqrt{\frac{ZeU}{m_i}}$$

(The mass of Argon $m_i=2.2 \times 10^{-25}$ kg, discharge voltage $U_d=500$ V, $V_i=2.6 \times 10^4$ m/s). So, we can use the specific impulse formula

$$I_{sp} = \frac{F}{g_0 m_i} \approx \frac{V_i}{g_0}$$

(See reference [12], F is the thrust and the $g_0$ is the earth gravitation constant) to obtain the specific impulse $I_{sp}=2500$ m/s.

From Fig. (5), we can know the electron velocity profile. The gradient of electron velocity is big between 0.01m and the exit. Afterwards the change tends to relax. This is consistent with the magnetic field and the electric potential profile. The electron should be accelerated towards the anode. But, on the one hand, the electron gyration radius is far smaller than ion gyration radius in the magnetic field, so electrons are magnetized and ions are unmagnetized. Thus the electrons are restricted and produce azimuthal electron drift velocity. Moreover, the charged particle magnetic moment is invariable in magnetic field, so the axial velocity reduces and the angular velocity increases. On the other hand, because of the electrons move towards anode occur collision ionization with the neutrals and neutralize with ions. This can cause electric axial velocity reduces. Therefore the electron velocity gradually reduces during move towards the anode process. Namely the electronic axial impedance is very big. The situation can be concluded from Fig. (5).

Fig. (5) The electron velocity distribution along axis

Because in the thruster the magnetic field structure is the magnetic mirror form (Fig. 1a), the magnetic flux density is big nearby the wall of the
down electrode pole. Therefore there are majority of electrons are restricted. This not only tallies with electronic velocity distribution, but also with the electron temperature distribution (Fig. 6) is consistent. The electron temperature gradually increases in the channel, at 0.014m arrive to maximum, afterwards tends to reduce. This kind of temperature distribution with the reference [7] also is consistent. As result of magnetic restraint at 0.012-0.014m, the electron density is bigger, then produces Hall electric current also bigger, therefore produces the Joule heat is very big. So the electron temperature has a maximum at 0.012-0.014.

Although the 1D numerical simulation can’t give the electron temperature profile in detail (because the temperature also has great gradient existence in the radial direction, Subrata Roy [8] have given the radial temperature gradient). It can approximately give the change tendency and the scope of electron temperature in the channel.

4. Conclusions
We use 1D fluid model to simulate the cylindrical HALL thruster plasma. In this simulation the electromagnetic distribution is change at axial. According to experiment we give the boundary conditions. Use Runge-Kutta method and get the results: the ion number density increases, but near exit drops; the neutral number density gradually reduces in the channel; the ion speed reaches the maximum at exit and the electron velocity has the great gradient distribution in the downriver; through calculation the specific impulse is \( I_{sp}=2500 \text{m/s} \). All results can really reflect the plasma physical process in the thruster from above analysis of the simulation result.

As a result of some limits of the supposition, it is unable to reflect the physical process in some special regions in the thruster channel, like the physical process in anode sheath. Therefore, our future works mainly research anode sheath and the wall sheath as well as develop the 2D numerical simulation of thruster plasma.

References: