COMPUTING NODES AND LINKS APPEARANCES ON GEODESICS IN NETWORKS TOPOLOGIES USING GRAPH THEORY

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Abstract

This paper proposes two important mathematical models related to network topology which helps in computing some of the efficiency or reliability factors of communication network as well as design purposes. Each of these models represents a topology property. The first (second) of these models is used to compute the number of appearances of any link (node) in the geodesics between nodes in a given network topology, and so can be used to help in uniformly distributing the data flow through links (nodes), as well as helping in measuring the degree of survivability of the network in case of failure of some of its links (nodes). The two models have been developed using "Graph Theory", and so, giving the advantage of using the very wide range of ideas, tools, and theorems of this field in case of developing other network topology formulas based on the two models proposed in this paper.

Keywords: Network topology, Graph theory, Topology modeling and properties, Node and link utilization.

حساب عدد مرات ظهور العقد و الوصلات في المسارات الاقصر ضمن طوبولوجيات الشبكات باعتماد نظرية حالة الاشكال

الخلاصة

يقترح هذا البحث نموذجين رياضيين مهمين يخصان طوبولوجية الشبكة. يساعد هذين النموذجين في حساب بعض عوامل الكفاءة و الاعتمادية لشبكات الاتصال، كما يخدمان بعض اهداف التصميم في الشبكات. ان كل من هذين النموذجين يمثل خاصية من خصائص الطوبولوجية. النموذج الاول (الثاني) من هذين النموذجين يستخدم لحساب عدد مرات ظهور أي وَصلة (عقدة) في المسارات الاقصر بين العقد ضمن طوبولوجية شبكة ما، و بالتالي يمكن ان يستخدم في المساعدة في التوزيع المنتظم لانسياب البيانات عبر الوَصلات (العقد) و كذلك المساعدة في قياس درجة قدرة الشبكة على البقاء في حالة فشل بعض وَصَلاتها (عقدها) . لقد تم تطوير النموذجين باعتماد نظرية حالة الاشكال، و هذا بدوره اعطى ميزة امكانية الاستفادة من المدى الواسع من الافكار و الادوات و النظريات التي يتيحها هذا الحقل المعرفي عند الحاجة لتطوير دوال اخرى ذات صلة بطوبولوجية الشبكة اعتمادا على النموذجين المقترحين في هذا البحث.

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1- Introduction

Network topology, and hence topology modeling, plays a very important role in many issues related to network design such as network reliability and network performance. In reality, many of the measures or factors of reliability and performance of a network are based on its topology model. Moreover, choosing a proper topology model can help in easing network design. Topology modeling could be done many ways and depends on the purpose behind modeling. The degree of complexity of the topology model also depends on the purpose behind modeling[1-8].

When it comes to communication network design, one of the important issues is the traffic distribution over links and nodes in the network. It is an important matter that the network utilizes its nodes and links as uniformly as possible. Unfortunately, determining the traffic distribution in a communication network is a very complex process and in many cases is unpredictable which makes some use statistical methods to help in predicting to some extent the nature of the traffic or load in the network. The traffic distribution and link and node utilization is a function of many parameters among which are: the topology, the relative positions and number of the source and destination nodes, and the presence of high, medium or low traffic carrying source-destination node pairs [9].

Considering the difficulties in estimating links and nodes utilization mentioned above, it will be helpful to find an estimation approach which is only topology dependent and so avoiding the uncertainty and unpredictability of other factors. This will, at least, help in designing a topology which provides a fair ground for uniform use of nodes and links . But any such approach will need computing the number of appearances of nodes and links on all possible routes.

Here, two important mathematical topology models (properties) are proposed which could be used for computing links and nodes appearances on routes in the network.

The two models or properties are developed using graph theory approach.

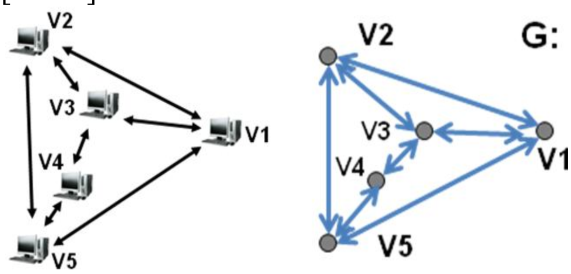
2- Definition of Nodes and Links Utilization

In general, the amount of traffic between a source-sink pair is the amount of information carried between them. The measure of traffic is dependent on the switching strategy followed by the system. If a packet switched network is considered then the minimum packet size may be defined as one unit of traffic. If a circuit switched network is considered, an arbitrary small amount of time can be considered as the amount of traffic. But for either of the cases it will be assumed that a direct or virtual connection has been established between the source-sink pair prior to initiation of the information transfer. So while routing the information through the pre-established path some specific links and nodes will be utilized. When a unit amount of traffic is routed through a link or a node, it is said that that particular link or node has been utilized once [10].

3- Modeling Network Topology with Graph Theory

Graph theory is a branch of mathematics concerned mainly with structures. This theory is concerned with patterns of relationships among pairs of abstract elements and has applications in many science and engineering fields like communication engineering, computer science and engineering, electrical engineering, and computational biology.

Graph theory is quite useful when the main interest is in the structural properties of any empirical system as it provides concepts, theorems, and methods appropriates to the analysis of structures Graph theory represents structures as graphs which in turn will be represented as matrices. These matrices of the way nodes are connected in the graph are called adjacency matrices. Graph theory will be used here for two main purposes the first of which is to model the network topology in question, keeping in mind that the topology of a network is in reality a structure, and the second will be in computing topology properties. Figure (1) shows a simple network with its corresponding graph and adjacency matrix [11-18].



A-Network topology B- Topology Graph

$$A(D) = \begin{array}{c} \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{array} \begin{array}{c} \begin{array}{ccccc} v1 & v2 & v3 & v4 & v5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C-Adjacency Matrix

Figure (1): A sample network topology with its representative graph and adjacency matrix.

4- Computing the "Number of Appearances of Links and Nodes on Geodesics" Properties

There are different strategies of data movement over the network through the links and nodes. One well known and used strategy is the shortest distance path (geodesic) strategy [19]. This strategy will be depended when developing the properties here.

A-Developing the "Number of Appearances of Nodes on Geodesics" Array

The first property to be developed is the number of appearances of nodes on geodesics (NAG(D)) array. This array shows the number of times a given node is on the geodesics between a given connected nodes pair in a given graph D. Here a formula (F_1) is derived to support finding the array ($NAG_{ij}(N_k)$), which is a three dimension array and stands for the number of times node k is on the geodesics from node i to node j. The NAG array is found as follows:

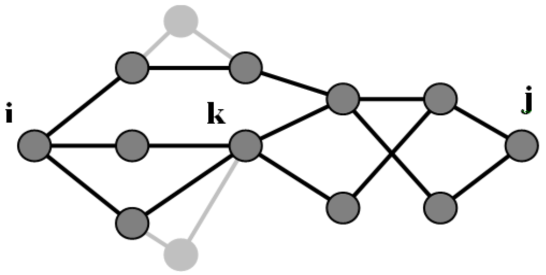
a- Find the distance matrix.(Refer to appendix A for how to find the distance matrix)

b- For a given i, j, k (where: $1 \leq i,j,k \leq n$ and $k \neq i \neq j$, n=number of nodes in the given network or graph), check if the node k is on the geodesics from node i to node j. This is done by checking if the distance from node i to node k plus the distance from node k to node j is equal to the distance from node i to node j. If this is true then apply formula (F_1) which states:

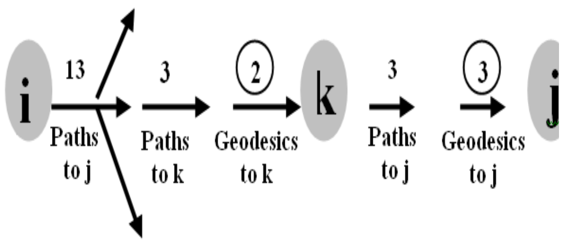
"If node k is on one or more of the geodesics from node i to node j, then the number of times node k is on the geodesics from node i to node j is equal to the number of geodesics from node i to node k multiplied by the number of geodesics from node k to node j".

Figure (2) illustrates this formula which is to be proved as follows:

i- Since it is assumed that k is on one (or more) of the geodesics from node i to node j, then all geodesics from node i to node k and all geodesics from node k to node j are part of these geodesics. This is obtained from the theorem which states that a point k is on a geodesic from point i to point j if and only if $d_{ik} + d_{kj} = d_{ij}$, and the converse is right [11].



A- Part of a sample topology showing all possible paths from node i to node j.

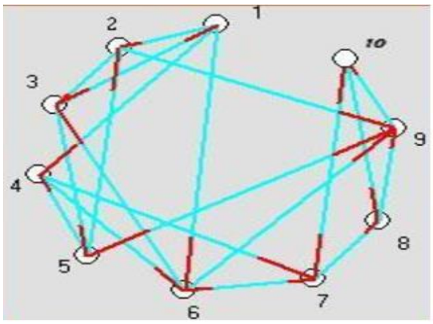


B- Details of the paths from node i to node j through node k. Number of times node k is on the geodesics from node i to node j = 2 × 3 = 6.

Figure (2): Illustration of the formula (F_1).

ii- Each geodesic from node i to node k can be completed by any of the geodesics from node k to node j to form a geodesic from node i to node j. This is obtained from the theorem which states that every subpath of a geodesic is a geodesic, then the number of times k will appear on the geodesics from node i to node j is equal to the number of geodesics from node i to node k multiplied by the number of geodesics from node k to node j [11].

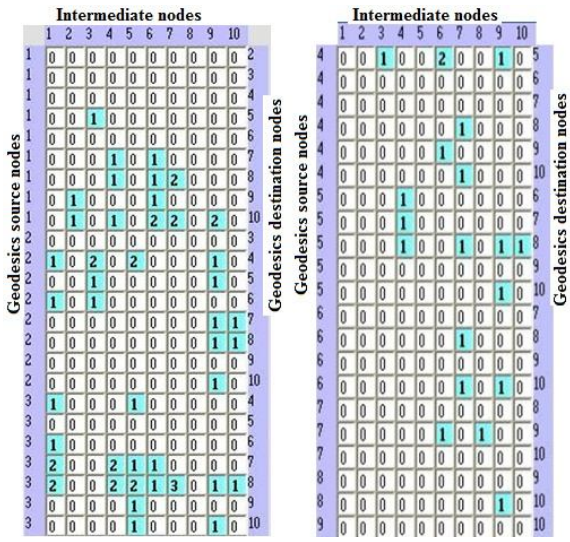
Figure (3) shows a 10-nodes sample topology with its adjacency Matrix and the number of appearances of nodes on geodesics.



A) A 10 nodes network

A	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	1	0	0	0	0
2	0	0	1	0	0	0	0	0	1	0
3	1	0	0	0	1	0	0	0	0	0
4	0	0	0	0	0	1	1	0	0	0
5	0	1	0	1	0	0	0	0	1	0
6	0	0	1	0	0	0	1	0	1	0
7	0	0	0	0	0	1	0	1	0	1
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	1	0	0	0	0	1
10	0	0	0	0	0	0	1	1	0	0

B) Adjacency matrix



C- Number of times a given node is on the geodesics between 2 connected nodes

Figure (3): Number of appearances of nodes on geodesics.

B-Developing the "Number of Appearances of Links on Geodesics" Matrix

The second property to be developed is the number of appearances of links on geodesics (LAG(D)) matrix. This matrix shows the number of times a given link is on the geodesics between a given linked nodes pair. To form LAG(D) means finding the number of times a given link (ab) is on the geodesics from node i to node j, for all i's and j's (where i≠j). A formula is derived here for this purpose and is given below:

$$LAG_{ab} = \sum_{i=1}^{n} \sum_{j=1_{(j \neq i)}}^{n} LAG_{ab}(i,j) \ldots . \ (1)$$

where;

$$LAG_{ab}(i,j)_{i \neq j} = \begin{cases} NAG(i,j,a) & ; b=j, a \neq i \\ NAG(i,j,b) & ; a=i, b \neq j \\ (NAG(i,b,a)) \times (NAG(a,j,b)) & ; a \neq i, b \neq j \\ 1 & ; a=i, b=j \end{cases}$$

where;

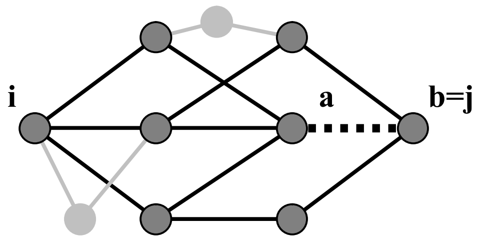
- LAG_{ab}: number of times link (ab) appears on network's geodesics.
- $LAG_{ab}(i,j)$: number of times link (ab) is on the geodesics from node i to node j.
- n: number of nodes in the network.
- NAG(i,j,b): number of times node b is on the geodesics from node i to node j, and similarly for the other NAGs.

Examples for the four cases or possibilities covered by this formula are shown in Figure (4).

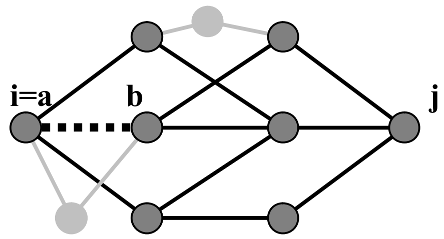
The LAG formula (equation 1) could be proved as follows:

i- If b=j, this means that node a is directly linked to node j (i.e. distance from node a to node j is 1), and so the link ab will appear on the geodesics from i to j as much as node a appears on these geodesics because all the geodesics passing from node i to node j through node a have to pass through the unique geodesic of length one, namely link ab. This is given by: NAG(i, j, a).

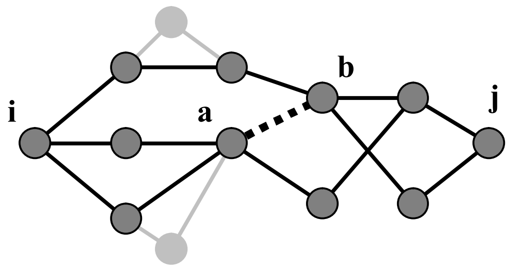
ii- If a=i and b=j, then ab is a unique geodesic of length one between node i and node j. Hence, the link ab appears once whenever node i approaches node j.



Case 1
(There are 5 geodesics from i to j.) (There are 8 paths from i to j.) (a is 3 times on the geodesics from i to j.) (The link ab is 3 times on the geodesics from i to j.)



Case 2
(There are 5 geodesics from i to j.) (There are 8 paths from i to j.)(b is 2 times on the geodesics from i to j.) (The link ab is 2 times on the geodesics from i to j.)



Case 3:
(There are 8 geodesics from i to j.) (There are 13 paths from i to j.) (There are 2 geodesics from i to a.) (There are 2 geodesics from b to j.) (The link ab is 2×2 = 4 times on the geodesics from i to j.)

Case 4: (There is 1geodesic from i to j.)(There is 1 path from i to j.)(The link ab is 1 time on the geodesic from i to j.)

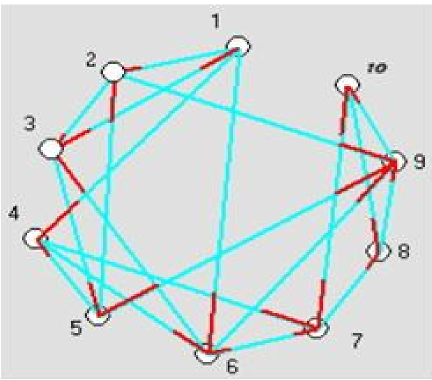
Figure (4): The four possible cases for link ab appearance on the geodesics between nodes i and j.

iii- If a≠i and b≠j, then for the link ab to appear on the geodesics from node i to node j, each of these geodesics must pass through nodes a and b simultaneously.

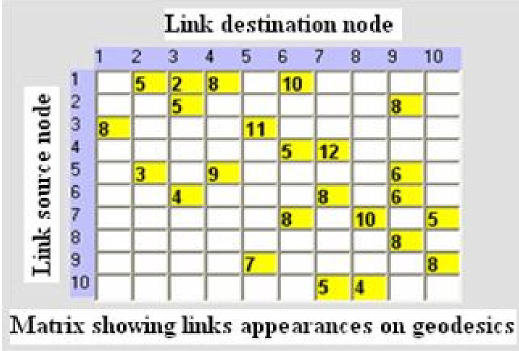
Hence, the number of appearances of link ab depends on the number of times node a appears on the geodesics from node i to node j with the condition of passing through node b, and on the number of times node b appears on the geodesics from node i to node j with the condition of passing through node a. Based on this mutual dependability, the number of times link ab appears on the geodesics from node i to node j is given by the number of appearances of node a on the geodesics between node i and node b, multiplied by the number of appearances of node b on the geodesics between node a and node j, or: NAG(i,b,a) × NAG(a,j,b).

iv- If a=i, this means that node b is directly linked to node i (i.e. distance from node b to node i is 1), and so the link ab will appear on the geodesics from i to j as much as node b appears on these geodesics, because all the geodesics passing from node i to node j through node b have to pass through the unique geodesic of length one, namely link ab. This is given by: NAG (i, j, b).

Figure (5-b) shows the links number of appearances on geodesics matrix for the network of Figure (5-a).



5-a



Link destination node

Link source node	1	2	3	4	5	6	7	8	9	10
1		5	2	8		10				
2			5						8	
3	8				11					
4						5	12			
5		3		9					6	
6			4				8		6	
7						8		10		5
8									8	
9					7					8
10							5	4		

Matrix showing links appearances on geodesics

5-b

Figure (5): A 10 nodes network with a matrix showing the number of appearances of each link on the geodesics for the network.

5- Conclusions

Two new important mathematical models related to network topology have been introduced here. The models are used to compute the number of appearances of links and nodes on the geodesics between nodes in a given network topology. The two models have the following advantages:

- They could be used for many design purposes such as, fair load distribution among nodes and links, improving network reliability, and improving network efficiency.
- Although the work concentrated on the usefulness of the models for fair load distribution among nodes and links in communication networks, the models could be used for any other kind of networks which could be represented by nodes and links, and traffic networks is just an example.
- The proposed properties could be used when developing topology objectives for topology optimization purpose.
- The models have been developed using graph theory approach and so they are backed up and powered by the tools, ideas, and theorems of this field.
- The models are quite suitable for programming purposes.

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Appendix A: Finding the Distance Matrix[3]

Let $N(D) = [d_{ij}]$ be the distance matrix of a given graph D, where i = 1,2,…,n, and j = 1,2,…,n. Then;

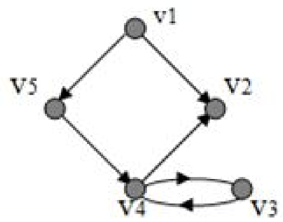
1. Every diagonal entry d_{ij} is 0,
2. $d_{ij} = \infty$ if there is no path from i to j and so i cannot reach j, and otherwise, d_{ij} is the smallest power y to which A must be raised so that $a^{y}_{ij} > 0$, that is, so that the i,j entry of $A^{y}\#$ is 1. (Note: The $A^{y}\#$ stands for the y_{th} logic power of the adjacency matrix A. The logic power is made by logic multiplication

rather than arithmetic multiplication, and as an example: $1\times1+1\times0+1\times1=1$).

3. N(D) only tells the length of the geodesic between two points, and does not tell how many geodesics are there between any given two points.

The procedure for constructing the distance matrix N(D) from the adjacency matrix will be illustrated for the graph of Figure (6).

D:



$$A = \begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ v1 & 0 & 1 & 0 & 0 & 1 \\ v2 & 0 & 0 & 0 & 0 & 0 \\ v3 & 0 & 0 & 0 & 1 & 0 \\ v4 & 0 & 1 & 1 & 0 & 0 \\ v5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

$$A^2\# = \begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ v1 & 0 & 0 & 0 & 1 & 0 \\ v2 & 0 & 0 & 0 & 0 & 0 \\ v3 & 0 & 1 & 1 & 0 & 0 \\ v4 & 0 & 0 & 0 & 1 & 0 \\ v5 & 0 & 1 & 1 & 0 & 0 \end{array}$$

$$A^3\# = \begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ v1 & 0 & 1 & 1 & 0 & 0 \\ v2 & 0 & 0 & 0 & 0 & 0 \\ v3 & 0 & 0 & 0 & 1 & 0 \\ v4 & 0 & 1 & 1 & 0 & 0 \\ v5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

$$A^4\# = \begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ v1 & 0 & 0 & 0 & 1 & 0 \\ v2 & 0 & 0 & 0 & 0 & 0 \\ v3 & 0 & 1 & 1 & 0 & 0 \\ v4 & 0 & 0 & 0 & 1 & 0 \\ v5 & 0 & 1 & 1 & 0 & 0 \end{array}$$

$$N(D) = \begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ v1 & 0 & 1 & 3 & 2 & 1 \\ v2 & \infty & 0 & \infty & \infty & \infty \\ v3 & \infty & 2 & 0 & 1 & \infty \\ v4 & \infty & 1 & 1 & 0 & \infty \\ v5 & \infty & 2 & 2 & 1 & 0 \end{array}$$

Figure (6): Constructing the distance matrix N(D) for graph D.