Fuzzy α-Separated Sets and Fuzzy p-Separated Sets

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Abstract:

The aim of this papar is to introduce fuzzy α-open (α-closed) sets, fuzzy p-open (p-closed) sets and study some properties and theorems on them and study the relationship between them .Fuzzy α-separated sets , fuzzy weak α-separated sets , fuzzy p-separated sets and fuzzy weak p-separated sets are introduced and some of their remarks and theorems are studied and study the relationship between them.

1. Introduction

Change in 1968 used the fuzzy set theory for defining and introduction fuzzy topological space while wong in 1973 discussed and converted the properties of fuzzy topological space, [5] , [16]. The notion of fuzzy topology on fuzzy sets were introduction by Chakrabarty and Ahsanullah [6] , Njastad , O . in 1965 introduced the concepts of fuzzy α-open sets.The notions of fuzzy α-closure and fuzzy α-interior of a fuzzy set in fuzzy topological space were introduced by Singal and Rajvansh in 1990 [15] , Othman and Latha in 2009 [11],respectively . Zhen-Guo and Fu-Gui Shi in 2007 were introduced the concepts of fuzzy pre-open sets,fuzzy pre-closed sets,fuzzy pre-interior and fuzzy pre-closure and some properties on them in fuzzy toplogical space on fuzzy set . The concepts of fuzzy α-separated sets were introduced and prove some theorems of fuzzy α-separated sets in fuzzy topological space on fuzzy set, [10] , [13] , [17].

2. Fuzzy Topology On Fuzzy Set

Definition (2.1) , [14] :

in X is $\lambda(y)$ A fuzzy point a fuzzy set with membership , defined by: $X_\lambda(y)$ function

$$X_\lambda(y) = \begin{cases} \lambda, & for\ x = y \\ 0, & x \neq y \end{cases}$$

Where $0 < \lambda < 1$, x is called the support of $\lambda(y)$ and λ its value.

Definition(2.2),[8]:

Let $\tilde{B}$, $\tilde{C} \in P(\tilde{A})$ then $\tilde{B}$ and $\tilde{C}$ are said to be quasi-coincident to $\tilde{A}$ if there exist $x \in X$, such that $\mu_{\tilde{A}(x)} > \mu_{\tilde{B}(x)} + \mu_{\tilde{C}(x)}$

IF $\tilde{B}$ and $\tilde{C}$ are not quasi-coincident refered to $\tilde{A}$,we denoted for this $\tilde{B}$ $\tilde{C}$ $\not{q}$

Proposition (2.1),[12] :

Let X be a non-empty set and $\tilde{A}, \tilde{B} \in I^x$, then:

1. If $\tilde{A} \cap \tilde{B} = \tilde{\emptyset}$, then $\tilde{A}\ \not{q}\ \tilde{B}$.

2. $\tilde{A}\ \not{q}\ \tilde{B}$ if and only if $\tilde{A} \subseteq \tilde{B}^c$.

Definition (2.3),[8] :

A collection $\tilde{T}$ of fuzzy subsets of $\tilde{A}$ is said to be fuzzy topology on $\tilde{A}$, if:

a. $\tilde{\emptyset}, \tilde{A} \in \tilde{T}$.

b. $\tilde{B}, \tilde{C} \in \tilde{T} \Rightarrow \tilde{B} \cap \tilde{C} \in \tilde{T}$.

c. $\tilde{B}_j \in \tilde{T}$, $\forall$ j $\in$ J, where J is any index set $\Rightarrow \bigcup_{j \in J} \tilde{B}_j \in \tilde{T}$.

) is said to be a fuzzy $\tilde{T}$, $\tilde{A}$(
($\tilde{A}$ topological space on fuzzy set
). $\tilde{A}$ or f.t.s on

3. The Relation Between Fuzzy α-open (α-closed) Sets and Fuzzy p-open (p-closed) Sets

Definition (3.1),[1],[3] :

A fuzzy set $\tilde{B}$ in a f.t.s ($\tilde{A}$, $\tilde{T}$) is called:

1. Fuzzy α-open set if $\tilde{B} \subseteq$ int(cl(int($\tilde{B}$))).

2. Fuzzy α-closed set if cl(int(cl($\tilde{B}$))) $\subseteq \tilde{B}$.

Definition(3.2),[4]:

Let $\tilde{B}$ be a fuzzy subset of a fuzzy topological space ($\tilde{A}$, $\tilde{T}$) then $\tilde{B}$ is said to be:

1. Fuzzy pre-open set (or Fuzzy p-open set for short) if $\tilde{B} \subseteq int(cl(\tilde{B}))$.
2. Fuzzy pre-closed set (or Fuzzy p-closed set for short) if $cl(int(\tilde{B}))\subseteq\tilde{B}$.

The fuzzy subset $\tilde{B}$ of $\tilde{A}$ is called fuzzy p-closed if its complement $\tilde{B}^c$ is fuzzy p-open in $\tilde{A}$.

3.Fuzzy pre-closure(or p-cl($\tilde{B}$)) is smallest fuzzy p-closed set of $\tilde{A}$ that contains $\tilde{B}$

4.Fuzzy pre-interior (or p-int) is largest fuzzy p-open set of $\tilde{A}$ that is contained in $\tilde{B}$.

Definition(3.3),[7]:

A fuzzy set $\tilde{B}$ in a f.t.s $(\tilde{A}, \tilde{T})$ is called fuzzy pre-neighborhood (or F-p-nbd for short) of fuzzy point $p^r_{x_0}$ if and only if there exists a fuzzy p-open set $\tilde{G}$ in $\tilde{A}$ such that $p^r_{x_0} \in \tilde{G} \subseteq \tilde{B}$.

Proposition (3.1):

If $(\tilde{A}, \tilde{T})$ be a f.t.s, $p^r_{x_0}$ fuzzy point in $\tilde{A}$ and let $\cup(p^r_{x_0})$ be the set of all F-p-nbd of a fuzzy point $p^r_{x_0}$, then:

1. If $\tilde{C} \in \cup(p^r_{x_0})$, then $p^r_{x_0} \in \tilde{C}$.
2. If $\tilde{C}_j \in \cup(p^r_{x_0})$, for $j \in J$, then $\bigcup_{j\in J} \tilde{C}_j \in \cup p^r_{x_0}$.
3. If $\tilde{C} \subseteq \tilde{D}$ and $\tilde{C} \in \cup(p^r_{x_0})$, then $\tilde{D} \in \cup(p^r_{x_0})$.

Proof:

1. Since $\tilde{C}$ is F-p-nbd of $p^r_{x_0}$

Implies that, $p^r_{x_0} \in \tilde{G} \subseteq \tilde{C}$

Hence, $p^r_{x_0} \in \tilde{C}$

2. Since $\tilde{C}_j$ are F-p-nbd of $p^r_{x_0}$

Then, $p^r_{x_0} \in \tilde{G}_j \subseteq \tilde{C}_j$

Since $\tilde{G}_j \subseteq \tilde{C}_j$ for $j \in J \Rightarrow \bigcup_{j\in J} \tilde{G}_j \subseteq \bigcup_{j\in J} \tilde{C}_j$

Since $\tilde{G}_j$ are fuzzy p-open sets in $\tilde{A}$

Then $\bigcup_{j\in J} \tilde{G}_j$ is fuzzy p-open set in $\tilde{A}$

Since $p^r_{x_0} \in \tilde{G}_j \Rightarrow p^r_{x_0} \in \bigcup_{j\in J} \tilde{G}_j$

Hence, $p^r_{x_0} \in \bigcup_{j\in J} \tilde{G}_j \subseteq \bigcup_{j\in J} \tilde{C}_j$

Therefore , $\cup \tilde{C}_j \in \cup(p^r_{x_0})$

3. Since $\tilde{C}$ is a F-p-nbd of $p^r_{x_0}$

Implies that , $P^r_{x_0} \in \tilde{G} \subseteq \tilde{C}$

Since $\tilde{C} \subseteq \tilde{D}$

Then, $P^r_{x_0} \in \tilde{G} \subseteq \tilde{D}$

Hence, $\tilde{D} \in \cup(p^r_{x_0})$. ■

Proposition(3.2):

Let $\tilde{C}$ be a fuzzy set in f.t.s ($\tilde{A}$, $\tilde{T}$), then the following statements are equivalent:

1. $\tilde{C}$ is a fuzzy p-open set in $\tilde{A}$.

2. For each fuzzy point $p^r_{x_0} \in \tilde{C}$, then $\tilde{C}$ is F-p-nbd of $p^r_{x_0}$.

3. For each fuzzy point $p^r_{x_0} \in \tilde{C}$, there exists a F-p-nbd $\tilde{D}$ of $p^r_{x_0}$ such that $p^r_{x_0} \in \tilde{D} \subseteq \tilde{C}$.

Proof:

($1 \Rightarrow 2$) Hold .

($2 \Rightarrow 3$) Since that for each fuzzy point $p^r_{x_0} \in \tilde{C}$, $\tilde{C}$ is F-p-nbd of $p^r_{x_0}$

Then , $p^r_{x_0} \in \tilde{D} \subseteq \tilde{C}$

Since , $\tilde{D}$ is a fuzzy p-open set in $\tilde{A}$

By ($1 \Rightarrow 2$) then $\tilde{D}$ is F-p-nbd of $p^r_{x_0}$.

($3 \Rightarrow 1$) Suppose that $p^r_{x_0} \in \tilde{D} \subseteq \tilde{C}$

Since , $\tilde{D}$ a F-p-nbd of $p^r_{x_0}$

Implies that, $p^r_{x_0} \in \tilde{G}_j \subseteq \tilde{D} \subseteq \tilde{C}$

Hence , $p^r_{x_0} \in \tilde{G}_j \subseteq \tilde{C}$

Then $\cup p^r_{x_0} \in \bigcup_{j \in J} \tilde{G}_j \subseteq \tilde{C}$

Since, $\cup p^r_{x_0} = \tilde{C}$

$\Rightarrow \tilde{C} = \cup p^r_{x_0} \in \bigcup_{j \in J} \tilde{G}_j \subseteq \tilde{C}$

So, $\tilde{C} = \bigcup_{j \in J} \tilde{G}_j$

Since , $\tilde{G}_j$ are fuzzy p-open sets in $\tilde{A}$

Then , $\bigcup_{j \in J} \tilde{G}_j$ is fuzzy p-open set in $\tilde{A}$

Hence , $\tilde{C}$ is a fuzzy p-open set . ■

Theorem (3.1):

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s, $\tilde{C}$ is a fuzzy set in $\tilde{A}$ and $p^r_{x_0}$ fuzzy point in $\tilde{A}$, then $p^r_{x_0} \in$ p-int($\tilde{C}$) if and only if $p^r_{x_0}$ has a F-p-nbd $\tilde{D}$ in $\tilde{A}$, such that $\tilde{D} \subseteq \tilde{C}$.

Proof:

($\Rightarrow$) Suppose that $p^r_{x_0} \in$ p-int($\tilde{C}$)

Then $p^r_{x_0} \in$ p-int($\tilde{C}$) $\subseteq \tilde{C}$.

By proposition (3.2) then p-int($\tilde{C}$) is a F-p-nbd of $p^r_{x_0}$ and suppose that p-int($\tilde{C}$) = $\tilde{D}$ then $p^r_{x_0} \in \tilde{D} \subseteq \tilde{C}$

Hence, $p^r_{x_0}$ has a F-p-nbd $\tilde{D}$ in $\tilde{A}$, such that $\tilde{D} \subseteq \tilde{C}$.

($\Leftarrow$) Suppose that $p^r_{x_0}$ has a F-p-nbd $\tilde{D}$ in $\tilde{A}$, such that $p^r_{x_0} \in \tilde{D} \subseteq \tilde{C}$

Suppose that: $p^r_{x_0} \notin$ p-int($\tilde{C}$) = $\cup\{\tilde{G} : \tilde{G}$ is a fuzzy p-open set and $\tilde{G} \subseteq \tilde{C}$ }

$p^r_{x_0} \notin$ p-int($\tilde{C}$) $\Rightarrow$ $p^r_{x_0} \notin \cup\tilde{G} \Rightarrow$ $p^r_{x_0} \notin \tilde{G}$, $\forall\ \tilde{G} \subseteq \tilde{C}$

Since $\tilde{D}$ is a F-p-nbd of $p^r_{x_0}$ in $\tilde{A}$

Then, there exists fuzzy p-open set $\tilde{B}$ in $\tilde{A}$, such that : $p^r_{x_0} \in \tilde{B} \subseteq \tilde{D} \subseteq \tilde{C}$

Hence $p^r_{x_0} \in \tilde{B} \subseteq \tilde{C}$, which is a contradiction

Then , $p^r_{x_0} \in$ p-int($\tilde{C}$) . ■

Lemma(3.1)[9]:

Every fuzzy α-open set [resp.fuzzy α-closed set] is fuzzy p-open set [resp. fuzzy p-closed set] in a f.t.s $(\tilde{A}, \tilde{T})$.

Proof:

Since int($\tilde{B}$) $\subseteq \tilde{B}$

Implies that cl(int($\tilde{B}$)) $\subseteq$ cl($\tilde{B}$)

$\Rightarrow$ int(cl(int($\tilde{B}$))) $\subseteq$ int(cl($\tilde{B}$))

Since $\tilde{B}$ is fuzzy α-open set in $\tilde{A}$
$\Rightarrow$ $\tilde{B} \subseteq$ int(cl(int($\tilde{B}$)))

Then , $\tilde{B} \subseteq$ int(cl(int($\tilde{B}$))) $\subseteq$ int(cl($\tilde{B}$))

$\Rightarrow$ $\tilde{B} \subseteq$ int(cl($\tilde{B}$))

Hence , $\tilde{B}$ is fuzzy p-open set in $\tilde{A}$.

And also to show that every fuzzy α-closed set is fuzzy p-closed set in $\tilde{A}$

Since $\tilde{B} \subseteq$ cl($\tilde{B}$)

Then int($\tilde{B}$) $\subseteq$ int(cl($\tilde{B}$))

$\Rightarrow$ cl(int($\tilde{B}$)) $\subseteq$ cl(int(cl($\tilde{B}$)))

Since , $\tilde{B}$ is a fuzzy α-closed set in $\tilde{A}$

$\Rightarrow$ cl(int(cl($\tilde{B}$))) $\subseteq \tilde{B}$

Then , cl(int($\tilde{B}$)) $\subseteq$ cl(int(cl($\tilde{B}$))) $\subseteq$ $\tilde{B}$

$\Rightarrow$ cl(int($\tilde{B}$)) $\subseteq$ $\tilde{B}$

Hence , $\tilde{B}$ is a fuzzy p-closed set in $\tilde{A}$.

Example(3.1):

Let X = {a, b} and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ are fuzzy sets defined as follows :

$\tilde{A}$ = {(a, 0.8), (b, 0.8)}

$\tilde{B}$ = {(a, 0.5), (b, 0.5)}

$\tilde{C}$ = {(a, 0.6), (b, 0.6)}

$\tilde{D}$ = {(a, 0.8), (b, 0.3)}

$\tilde{E}$ = {(a, 0), (b, 0.5)}

Let $\tilde{T}$ = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$} be a fuzzy topology on $\tilde{A}$, then $\tilde{D}$ is fuzzy p-open set, but not fuzzy α-open set and also $\tilde{E}$ is fuzzy p-closed set , but not fuzzy α-closed set.

Remark(3.1):

We explain the relation between fuzzy open set , fuzzy α-open set and fuzzy p-open set by fig.1

Fuzzy open ⇒ Fuzzy α-open ⇒ Fuzzy p-open

4 . The Relation Between Fuzzy α - Separated Sets and Fuzzy p- Separated Sets.

Definition(4.1),[10],[13]:

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s and $\tilde{B}$, $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are said to be:

1. Fuzzy α-separated if and only if α-cl($\tilde{B}$) ∩ $\tilde{C}$ = $\tilde{\emptyset}$; and α-cl($\tilde{C}$) ∩ $\tilde{B}$ = $\tilde{\emptyset}$.

2. Fuzzy weak α-separated if and only if α-cl($\tilde{B}$) $\not q$ $\tilde{C}$; and α-cl($\tilde{C}$) $\not q$ $\tilde{B}$.

Definition(4.2),[2],[18]:

If ($\tilde{A}$, $\tilde{T}$) be a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are said to be:

1. Fuzzy pre-separated (or fuzzy p-separated for short) if p-cl($\tilde{B}$) ∩ $\tilde{C}$ = $\tilde{\emptyset}$ and p-cl($\tilde{C}$) ∩ $\tilde{B}$ = $\tilde{\emptyset}$.

2. Fuzzy weak pre-separated (or fuzzy weak p-separated for short) if $\tilde{B}$ $\not q$ p-cl($\tilde{C}$) and $\tilde{C}$ $\not q$ p-cl($\tilde{B}$).

Theorem(4.1):

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated.

Proof:

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$ then :

p-cl($\tilde{B}$) ∩ $\tilde{C}$ = $\tilde{\emptyset}$ and p-cl($\tilde{C}$) ∩ $\tilde{B}$ = $\tilde{\emptyset}$.

By proposition (2.1) $\tilde{B} \not q$ p-cl($\tilde{C}$) and $\tilde{C} \not q$ p-cl($\tilde{B}$).

Hence , $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated. ■

Remark(4.1):

The convers of theorem (4.1) is not true in general .

Example(4.1):

Let X = {a, b} and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$ are fuzzy sets defined as follows:

$\tilde{A}$= {(a, 0.5), (b, 0.7)}

$\tilde{B}$= {(a, 0), (b, 0.5)}

$\tilde{C}$= {(a, 0.5), (b, 0)}

$\tilde{D}$= {(a, 0.5), (b, 0.5)}

$\tilde{E}$= {(a, 0.3), (b, 0.2)}

$\tilde{F}$= {(a, 0), (b, 0.2)}

Let $\tilde{T}$= {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$} be a fuzzy topology on $\tilde{A}$, then $\tilde{E}$ and $\tilde{F}$ are fuzzy weak p-separated in $\tilde{A}$, but not fuzzy p-separated in $\tilde{A}$

$\tilde{T}^c$= {$\tilde{A}$, $\tilde{\emptyset}$, $\tilde{B}^c$, $\tilde{C}^c$, $\tilde{D}^c$}

Such that:

$\tilde{B}^c$= {(a, 0.5), (b, 0.2)}

$\tilde{C}^c$= {(a, 0), (b, 0.7)}

$\tilde{D}^c$= {(a, 0), (b, 0.2)}

p-cl($\tilde{E}$) $= \tilde{B}^c$ and p-cl($\tilde{F}$) $= \tilde{D}^c$

Since p-cl($\tilde{E}$) $\not q$ $\tilde{F}$ and p-cl($\tilde{F}$) $\not q$ $\tilde{E}$

Then, $\tilde{E}$ and $\tilde{F}$ are fuzzy weak p-separated in $\tilde{A}$, but :

p-cl($\tilde{E}$) $\cap \tilde{F} \neq \tilde{\emptyset}$ and p-cl($\tilde{F}$) $\cap \tilde{E} \neq \tilde{\emptyset}$.

Hence, $\tilde{E}$ and $\tilde{F}$ are not fuzzy p-separated in $\tilde{A}$. ■

Theorem(4.2):

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$ and $\tilde{D}$ is fuzzy set in $\tilde{A}$, then $\tilde{B} \cap \tilde{D}$ and $\tilde{C} \cap \tilde{D}$ are fuzzy p-separated in $\tilde{A}$.

Proof:

Since p-cl($\tilde{B} \cap \tilde{D}$) $\subseteq$ p-cl($\tilde{B}$) $\cap$ p-cl($\tilde{D}$)

Then, p-cl($\tilde{B} \cap \tilde{D}$) $\cap$ ($\tilde{C} \cap \tilde{D}$) $\subseteq$

[p-cl($\tilde{B}$) $\cap$ p-cl($\tilde{D}$)] $\cap$ ($\tilde{C} \cap \tilde{D}$)

= (p-cl($\tilde{B}$) $\cap \tilde{C}$) $\cap$ (p-cl($\tilde{D}$) $\cap \tilde{D}$)

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$ then :

,p-cl($\tilde{B}$) $\cap$ $\tilde{C} = \tilde{\emptyset}$ and p-cl($\tilde{C}$) $\cap$ $\tilde{B} = \tilde{\emptyset}$

$\Rightarrow$ p-cl($\tilde{B}\cap\tilde{D}$) $\cap$ ($\tilde{C}\cap\tilde{D}$) $\subseteq \tilde{\emptyset} \cap$ (p-cl($\tilde{D}$) $\cap\tilde{D}$) $\subseteq \tilde{\emptyset}$

$\Rightarrow$ p-cl($\tilde{B}\cap\tilde{D}$) $\cap$ ($\tilde{C}\cap\tilde{D}$) = $\tilde{\emptyset}$.

Similarly : p-cl($\tilde{C}\cap\tilde{D}$) $\cap$ ($\tilde{B}\cap\tilde{D}$) = $\tilde{\emptyset}$.

Hence, $\tilde{B}\cap\tilde{D}$ and $\tilde{C}\cap\tilde{D}$ are fuzzy p-separated in $\tilde{A}$. ■

Theorem(4.3):

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$ and $\tilde{D}$ is fuzzy set in $\tilde{A}$ then $\tilde{B}\cap\tilde{D}$ and $\tilde{C}\cap\tilde{D}$ are fuzzy weak p-separated in $\tilde{A}$.

Proof:

Suppose that $\tilde{B}$, $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$

Suppose that $\tilde{B}\cap\tilde{D}$, $\tilde{C}\cap\tilde{D}$ are not fuzzy weak p-separated in $\tilde{A}$

Then p-cl($\tilde{B}\cap\tilde{D}$) q ($\tilde{C}\cap\tilde{D}$) and

p-cl($\tilde{C}\cap\tilde{D}$) q ($\tilde{B}\cap\tilde{D}$)

Since $\tilde{C}\cap\tilde{D} \subseteq \tilde{C} \Rightarrow \mu_{\tilde{C}\cap\tilde{D}}(x) \leq \mu_{\tilde{C}}(x)$

Since $\tilde{B}\cap\tilde{D} \subseteq \tilde{B}$

Then p-cl($\tilde{B}\cap\tilde{D}$) $\subseteq$ p-cl($\tilde{B}$)

$\Rightarrow \mu_{p-cl(\tilde{B}\cap\tilde{D})}(x) \leq \mu_{p-cl(\tilde{B})}(x)$

Since p-cl($\tilde{B}\cap\tilde{D}$) q ($\tilde{C}\cap\tilde{D}$)

Then,

$\mu_{p-cl(\tilde{B}\cap\tilde{D})}(x)+\mu_{\tilde{C}\cap\tilde{D}}(x) \geq \mu_{\tilde{A}}(x)$

$\Rightarrow \mu_{p-cl(\tilde{B})}(x)+\mu_{\tilde{C}}(x) \geq \mu_{\tilde{A}}(x)$

Hence : p-cl($\tilde{B}$) q$\tilde{C}$ ………………..…(1)

Similarly: p-cl($\tilde{C}$) q$\tilde{B}$… ……………...(2)

From (1) and (2), we get $\tilde{B}$ and $\tilde{C}$ are not fuzzy weak p-separated in $\tilde{A}$, which is a contradiction

Hence, $\tilde{B}\cap\tilde{D}$ and $\tilde{C}\cap\tilde{D}$ are fuzzy weak p-separated in $\tilde{A}$. ■

Theorem(4.4):

If ($\tilde{A}$, $\tilde{T}$) is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$, such that $\tilde{M}\subseteq\tilde{B}$ and $\tilde{N}\subseteq\tilde{C}$ then $\tilde{M}$ and $\tilde{N}$ are fuzzy p-separated in $\tilde{A}$.

Proof:

Since $\tilde{M}\subseteq\tilde{B}$, then p-cl($\tilde{M}$) $\subseteq$ p-cl($\tilde{B}$)

$\Rightarrow \mu_{p-cl(\tilde{M})}(x) \leq \mu_{p-cl(\tilde{B})}(x)$

Since $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$

Then Min{$\mu_{p-cl(\tilde{M})}(x)$, $\mu_{\tilde{N}}(x)$} $\leq$ Min{ $\mu_{p-cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$ }

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$

Then, $\text{Min}\{\mu_{p-cl(\tilde{B})}(x),\ \mu_{\tilde{C}}(x)\} = 0$ and

$$\text{Min}\{\mu_{p-cl(\tilde{C})}(x),\ \mu_{\tilde{B}}(x)\} = 0$$

$\Rightarrow \text{Min}\{\mu_{p-cl(\tilde{M})}(x),\ \mu_{\tilde{N}}(x)\} = 0$

Hence: p-cl($\tilde{M}$) $\cap \tilde{N} = \tilde{\emptyset}$……………(1)

Similarly: p-cl($\tilde{N}$) $\cap \tilde{M} = \tilde{\emptyset}$…………..(2)

From (1) and (2) we get, $\tilde{M}$ and $\tilde{N}$ are

fuzzy p-separated in $\tilde{A}$. ■

Theorem(4.5):

If $(\tilde{A}, \tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$, such that $\tilde{M} \subseteq \tilde{B}$ and $\tilde{N} \subseteq \tilde{C}$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy weak p-separated in $\tilde{A}$.

Proof:

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$

We must to prove $\tilde{M}$ and $\tilde{N}$ are fuzzy weak p-separated in $\tilde{A}$

Since $\tilde{M} \subseteq \tilde{B}$ then p-cl($\tilde{M}$) $\subseteq$ p-cl($\tilde{B}$)

$\Rightarrow \mu_{p-cl(\tilde{M})}(x) \le \mu_{p-cl(\tilde{B})}(x)$

Since $\mu_{\tilde{N}}(x) \le \mu_{\tilde{C}}(x)$

Then

$$\mu_{p-cl(\tilde{M})}(x) + \mu_{\tilde{N}}(x) \le \mu_{p-cl(\tilde{B})}(x) + \mu_{\tilde{C}}(x)$$

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$

Then, p-cl($\tilde{B}$) $\not{q}$ $\tilde{C}$ and p-cl($\tilde{C}$) $\not{q}$ $\tilde{B}$

Implies that $\mu_{p-cl(\tilde{B})}(x) + \mu_{\tilde{C}}(x) \le \mu_{\tilde{A}}(x)$ and $\mu_{p-cl(\tilde{C})}(x) + \mu_{\tilde{B}}(x) \le \mu_{\tilde{A}}(x)$

Then

$$\mu_{p-cl(\tilde{M})}(x) + \mu_{\tilde{N}}(x) \le \mu_{p-cl(\tilde{B})}(x) + \mu_{\tilde{C}}(x) \le \mu_{\tilde{A}}(x)$$

$$\Rightarrow \mu_{p-cl(\tilde{M})}(x) + \mu_{\tilde{N}}(x) \le \mu_{\tilde{A}}(x)$$

Hence:

p-cl($\tilde{M}$) $\not{q}$ $\tilde{N}$ ……………....(1)

Similarly:

p-cl($\tilde{N}$) $\not{q}$ $\tilde{M}$ …………….(2)

From (1) and (2), $\tilde{M}$ and $\tilde{N}$ are fuzzy weak p-separated in $\tilde{A}$. ■

Theorem(4.6):

If $(\tilde{A}, \tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$ if and only if there exists fuzzy p-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that $\tilde{B}\subseteq\tilde{G}$, $\tilde{C}\subseteq\tilde{H}$, $\tilde{B}\not{q}\tilde{H}$ and $\tilde{C}\not{q}\tilde{G}$.

Proof:

(⇒) Since $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$

Then p-cl($\tilde{B}$) $\not{q}$ $\tilde{C}$ and p-cl($\tilde{C}$) $\not{q}$ $\tilde{B}$

By proposition (2.1) then $\tilde{C}\subseteq$ [p-cl($\tilde{B}$)]^c and $\tilde{B}\subseteq$ [p-cl($\tilde{C}$)]^c

Suppose that [p-cl($\tilde{B}$)]^c $=\tilde{H}$ and [p-cl($\tilde{C}$)]^c $=\tilde{G}$, $\tilde{G}$ and $\tilde{H}$ are fuzzy p-open sets in $\tilde{A}$

Then $\tilde{C}\subseteq\tilde{H}$ and $\tilde{B}\subseteq\tilde{G}$

$\subseteq$ p-$\tilde{B}$ Since

$\mu_{p-cl(\tilde{B})(x)} \leq \Rightarrow \mu_{\tilde{B}(x)}$) $\tilde{B}$ cl(

$\mu_{\tilde{A}(x)}+\mu_{p-cl(\tilde{B})(x)} \leq \mu_{\tilde{B}(x)}+\mu_{\tilde{A}(x)}$ Then

$$\Rightarrow \mu_{\tilde{B}}(x)+\mu_{\tilde{A}}(x)-\mu_{p-cl(\tilde{B})}(x)\leq\mu_{\tilde{A}}(x)$$

$$\Rightarrow \mu_{\tilde{B}}(x)+\mu_{[p-cl(\tilde{B})]^c}(x)\leq\mu_{\tilde{A}}(x)$$

$$\Rightarrow \mu_{\tilde{B}}(x)+\mu_{\tilde{H}}(x)\leq\mu_{\tilde{A}}(x)$$

$\Rightarrow \tilde{B}\not{q}\tilde{H}$

Similariy: $\tilde{C}\not{q}\tilde{G}$

(⇐) Suppose that there exists fuzzy p-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that $\tilde{B}\subseteq\tilde{G}$, $\tilde{C}\subseteq\tilde{H}$, $\tilde{B}\not{q}\tilde{H}$ and $\tilde{C}\not{q}\tilde{G}$

Since $\tilde{B}\not{q}\tilde{H}$

By proposition (2.1) then $\tilde{B}\subseteq\tilde{H}^c$

Implies that, p-cl($\tilde{B}$) $\subseteq$ p-cl($\tilde{H}^c$) = $\tilde{H}^c$

Then p-cl($\tilde{B}$) $\subseteq\tilde{H}^c$

By proposition (2.1) then p-cl($\tilde{B}$) $\not{q}$ $\tilde{H}$

Therefore,

$\mu_{p-cl(\tilde{B})}(x)+\mu_{\tilde{H}}(x)\leq\mu_{\tilde{A}}(x)$

Since $\tilde{C}\subseteq\tilde{H}$, then: $\mu_{\tilde{C}(x)}\leq\mu_{\tilde{H}(x)}$

$\mu_{\tilde{C}(x)}+\mu_{p-cl(\tilde{B})(x)}\leq\mu_{\tilde{H}(x)}+\mu_{p-cl(\tilde{B})(x)}$

$\leq$

$\leq\mu_{\tilde{A}}(x)$

Then, $\mu_{\tilde{C}}(x)+\mu_{p-cl(\tilde{B})}(x)\leq\mu_{\tilde{A}}(x)$

Hence: $\tilde{C}\not{q}$ p-cl($\tilde{B}$)…………………...(1)

Similarly: $\tilde{B}\not{q}$ p-cl($\tilde{C}$)…………..……(2)

$\tilde{C}$ and $\tilde{B}$ From (1) and (2) we get . $\tilde{A}$ are fuzzy weak p-separated in

■

Theorem(4.7):

Two fuzzy sets $\tilde{B}$ and $\tilde{C}$ in ($\tilde{A}$, $\tilde{T}$) are fuzzy weak p-separated if there exist fuzzy p-open sets $\tilde{G}$ and

$\tilde{H}$, such that $\tilde{B}\subseteq\tilde{G}$, $\tilde{C}\subseteq\tilde{H}$, $\tilde{B}\cap\tilde{H}=\tilde{\emptyset}$ and $\tilde{C}\cap\tilde{G}=\tilde{\emptyset}$.

Proof:

Since $\tilde{B}\cap\tilde{H}=\tilde{\emptyset}$ and $\tilde{C}\cap\tilde{G}=\tilde{\emptyset}$

Then by proposition (2.1) $\tilde{B}\,\not{q}\,\tilde{H}$ and $\tilde{C}\,\not{q}\,\tilde{G}$

Implies that, by theorem (4.6) $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$. ■

Theorem(4.8):

If $(\tilde{A},\tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated sets in $\tilde{A}$ if and only if there exist fuzzy p-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$, such that $\tilde{B}\subseteq\tilde{E}$, $\tilde{C}\subseteq\tilde{F}$, $\tilde{B}\cap\tilde{F}=\tilde{\emptyset}$ and $\tilde{C}\cap\tilde{E}=\tilde{\emptyset}$.

Proof:

(⇒) Since $\tilde{B}\subseteq$ p-cl$(\tilde{B})$ and $\tilde{C}\subseteq$p-cl$(\tilde{C})$,

Suppose that p-cl$(\tilde{B})=\tilde{E}$ and p-cl$(\tilde{C})=\tilde{F}$

Then $\tilde{B}\subseteq\tilde{E}$ and $\tilde{C}\subseteq\tilde{F}$

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$

Then p-cl$(\tilde{C})\cap\tilde{B}=\tilde{\emptyset}$ and

p-cl$(\tilde{B})\cap\tilde{C}=\tilde{\emptyset}$

Hence, $\tilde{B}\subseteq\tilde{E}$, $\tilde{C}\subseteq\tilde{F}$, $\tilde{B}\cap\tilde{F}=\tilde{\emptyset}$ and $\tilde{C}\cap\tilde{E}=\tilde{\emptyset}$

(⇐) Since $\tilde{B}\cap\tilde{F}=\tilde{\emptyset}$ and $\tilde{C}\cap\tilde{E}=\tilde{\emptyset}$,

And suppose that $\tilde{F}=$ p-cl$(\tilde{C})$ and $\tilde{E}=$ p-cl$(\tilde{B})$

Then $\tilde{B}\cap$ p-cl$(\tilde{C})=\tilde{\emptyset}$ and

$\tilde{C}\cap$ p-cl$(\tilde{B})=\tilde{\emptyset}$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$. ■

Theorem(4.9):

If $(\tilde{A},\tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then:

1. If $\tilde{B}\,\not{q}\,\tilde{C}$ and either $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$ or $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$.
2. If either $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$ or $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$, then $\tilde{B}\cap\tilde{C}^c$ and $\tilde{C}\cap\tilde{B}^c$ are fuzzy weak p-separated in $\tilde{A}$.

Proof:

1. (a) Suppose that $\tilde{B}\,\not{q}\,\tilde{C}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$.

Since $\tilde{B} \not q \tilde{C}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$

By theorem (4.6) then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$

(b) Suppose that $\tilde{B} \not q \tilde{C}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$.

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$.

Then, p-cl($\tilde{B}$) $=\tilde{B}$ and p-cl($\tilde{C}$)$=\tilde{C}$

Since $\tilde{B} \not q \tilde{C}$

Then, p-cl($\tilde{B}$) $\not q\ \tilde{C}$ and p-cl($\tilde{C}$) $\not q\ \tilde{B}$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$.

2. (a) Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$.

Then, $\tilde{B}^c$ and $\tilde{C}^c$ are fuzzy p-closed sets

in $\tilde{A}$

Then, p-cl($\tilde{B}^c$) $=\tilde{B}^c$ and p-cl($\tilde{C}^c$) $=\tilde{C}^c$

Since, p-cl($\tilde{B}\cap\tilde{C}^c$) $\subseteq$ p-cl($\tilde{B}$) $\cap$ p-cl($\tilde{C}^c$)

Then p-cl($\tilde{B}\cap\tilde{C}^c$) $\subseteq$ p-cl($\tilde{B}$) $\cap\tilde{C}^c$

$\Rightarrow \mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x) \le \mu_{p-cl(\tilde{B})\cap\tilde{C}^c}(x)$

Then $\mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x)+\mu_{\tilde{C}\cap\tilde{B}^c}(x)$

$\le \mu_{p-cl(\tilde{B})\cap\tilde{C}^c(x)}+\mu_{\tilde{C}\cap\tilde{B}^c(x)}$

Since, $\mu_{p-cl(\tilde{B})\cap\tilde{C}^c}(x) \le \mu_{\tilde{C}^c}(x)$ and $\mu_{\tilde{C}\cap\tilde{B}^c}(x) \le \mu_{\tilde{C}}(x)$

Then $\mu_{p-cl(\tilde{B})\cap\tilde{C}^c(x)}+\mu_{\tilde{C}\cap\tilde{B}^c(x)} \le$

$$\mu\tilde{C}^c(x) + \mu_{\tilde{C}(x)} = \mu_{\tilde{A}(x)}$$

Then $\mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x)+\mu_{\tilde{C}\cap\tilde{B}^c}(x) \le$

$\mu_{p-cl(\tilde{B})\cap\tilde{C}^c}(x)+\mu_{\tilde{C}\cap\tilde{B}^c}(x) \le \mu_{\tilde{A}}(x)$

Hence, $\mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x)+\mu_{\tilde{C}\cap\tilde{B}^c}(x) \le \mu_{\tilde{A}}(x)$

Therefore: p-cl($\tilde{B}\cap\tilde{C}^c$) $\not q$ ($\tilde{C}\cap\tilde{B}^c$).....(1)

Similarly: p-cl($\tilde{C}\cap\tilde{B}^c$) $\not q$ ($\tilde{B}\cap\tilde{C}^c$)......(2)

From (1) and (2), we get $\tilde{C}\cap\tilde{B}^c$ and $\tilde{B}\cap\tilde{C}^c$ are fuzzy weak p-separated in $\tilde{A}$

(b) Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$

Then $\tilde{B}$ = p-cl($\tilde{B}$) and $\tilde{C}$ = p-cl($\tilde{C}$)

Since p-cl($\tilde{B}\cap\tilde{C}^c$) $\subseteq$ p-cl($\tilde{B}$) $\cap$ p-cl($\tilde{C}^c$)

Then, p-cl($\tilde{B}\cap\tilde{C}^c$) $\subseteq \tilde{B}\cap$ p-cl($\tilde{C}^c$)

$\Rightarrow \mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x) \le \mu_{\tilde{B}\cap p-cl(\tilde{C}^c)}(x)$

Then $\mu_{p-cl(\tilde{B}\cap\tilde{C}^c)(x)}+\mu_{\tilde{C}\cap\tilde{B}^c(x)} \leq$

$\mu_{\tilde{B}\cap p-cl(\tilde{C}^c)(x)}+\mu_{\tilde{C}\cap\tilde{B}^c(x)}$

$\leq \mu_{\tilde{B}}(x)+\mu_{\tilde{B}^c}(x)=\mu_{\tilde{A}}(x)$

Then

$\mu_{p-cl(\tilde{B}\cap\tilde{C}^c)}(x)+\mu_{\tilde{C}\cap\tilde{B}^c}(x)\leq\mu_{\tilde{A}}(x)$

Therefore: $p\text{-}cl(\tilde{B}\cap\tilde{C}^c)\, \not{q}\, (\tilde{C}\cap\tilde{B}^c)$…..(1)

Similarly: $p\text{-}cl(\tilde{C}\cap\tilde{B}^c)\, \not{q}\, (\tilde{B}\cap\tilde{C}^c)$…...(2)

From (1) and (2), $\tilde{C}\cap\tilde{B}^c$ and $\tilde{B}\cap\tilde{C}^c$ are fuzzy weak p-sparated in $\tilde{A}$. ■

Theorem(4.10):

If $(\tilde{A}, \tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy

If $(\tilde{A}, \tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then:

1. If $\tilde{B}\cap\tilde{C}=\tilde{\emptyset}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$.
2. If $\tilde{B}\cap\tilde{C}=\tilde{\emptyset}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$.

Proof:

1.Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-closed sets in $\tilde{A}$

Then $\tilde{B}$ = p-cl($\tilde{B}$) and $\tilde{C}$ = p-cl($\tilde{C}$)

$\tilde{\emptyset}=\tilde{C}\cap\tilde{B}$ Since

p-cl($\tilde{B}$) $\cap\tilde{C}=\tilde{\emptyset}$ and p-cl($\tilde{C}$) $\cap\tilde{B}=\tilde{\emptyset}$

Hence $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$.

2. Since $\tilde{B}\cap\tilde{C}=\tilde{\emptyset}$

Then by proposition (2.1) $\tilde{B}\, \not{q}\, \tilde{C}$

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy p-open sets in $\tilde{A}$ and $\tilde{B}\, \not{q}\, \tilde{C}$.

Implies that by theorem (4.9) ,

$\tilde{B}$ and $\tilde{C}$ are fuzzy weak p-separated in $\tilde{A}$. ■

Remark(4.2):

The convers of theorem(4.10) is not true in general .

Example(4.2):

Let X = {a, b} and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\tilde{H}$, $\tilde{I}$ are fuzzy sets defined as follows:

$\tilde{A}$= {(a, 0.9), (b, 0.9)}

$\tilde{B}$= {(a, 0.3), (b, 0)}

$\tilde{C} = \{(a, 0), (b, 0.9)\}$

$\tilde{D} = \{(a, 0.3), (b, 0.9)\}$

$\tilde{E} = \{(a, 0.9), (b, 0.1)\}$

$\tilde{F} = \{(a, 0), (b, 0.1)\}$

$\tilde{G} = \{(a, 0.3), (b, 0.1)\}$

$\tilde{H} = \{(a, 0.4), (b, 0)\}$

$\tilde{I} = \{(a, 0), (b, 0.4)\}$

Let $\tilde{T} = \{\tilde{\emptyset}, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}, \tilde{G}\}$ be a fuzzy topology on $\tilde{A}$ then $\tilde{H}$ and $\tilde{I}$ are fuzzy p-separated in $\tilde{A}$, but not fuzzy p-closed sets in $\tilde{A}$ and also $\tilde{H}$ and $\tilde{I}$ are fuzzy weak p-separated in $\tilde{A}$ but not fuzzy p-open sets in $\tilde{A}$

$\tilde{T}^c = \{\tilde{A}, \tilde{\emptyset}, \tilde{B}^c, \tilde{C}^c, \tilde{D}^c, \tilde{E}^c, \tilde{F}^c, \tilde{G}^c\}$ such that:

$\tilde{B}^c = \{(a, 0.6), (b, 0.9)\}$

$\tilde{C}^c = \{(a, 0.9), (b, 0)\}$

$\tilde{D}^c = \{(a, 0.6), (b, 0)\}$

$\tilde{E}^c = \{(a, 0), (b, 0.8)\}$

$\tilde{F}^c = \{(a, 0.9), (b, 0.8)\}$

$\tilde{G}^c = \{(a, 0.6), (b, 0.8)\}$

$p\text{-}cl(\tilde{H}) = \tilde{B}^c$, $p\text{-}cl(\tilde{I}) = \tilde{E}^c$

$p\text{-}cl(\tilde{H}) \cap \tilde{I} = \tilde{\emptyset}$ and $p\text{-}cl(\tilde{I}) \cap \tilde{H} = \tilde{\emptyset}$

Hence, $\tilde{H}$ and $\tilde{I}$ are fuzzy p-separated sets in $\tilde{A}$, but not fuzzy p-closed sets.

Since $p\text{-}cl(\tilde{H}) \not\subseteq \tilde{I}$ and $p\text{-}cl(\tilde{I}) \not\subseteq \tilde{H}$

Then, $\tilde{H}$ and $\tilde{I}$ are fuzzy weak p-separated in $\tilde{A}$, but not fuzzy p-open sets in $\tilde{A}$

Theorem(4.11):

If $(\tilde{A}, \tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated.

Proof:

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated in $\tilde{A}$

Then $\alpha\text{-}cl(\tilde{B}) \cap \tilde{C} = \tilde{\emptyset}$ and

$\alpha\text{-}cl(\tilde{C}) \cap \tilde{B} = \tilde{\emptyset}$

Since $p\text{-}cl(\tilde{B}) \subseteq \alpha\text{-}cl(\tilde{B})$

Implies that $p\text{-}cl(\tilde{B}) \cap \tilde{C} \subseteq \alpha\text{-}cl(\tilde{B}) \cap \tilde{C}$

$\Rightarrow p\text{-}cl(\tilde{B}) \cap \tilde{C} \subseteq \tilde{\emptyset}$

$\Rightarrow p\text{-}cl(\tilde{B}) \cap \tilde{C} = \tilde{\emptyset}$ ………………….... (1)

Similarly: $p\text{-}cl(\tilde{C}) \cap \tilde{B} = \tilde{\emptyset}$ …………... (2)

From (1) and (2) we get $\tilde{B}$ and $\tilde{C}$ are fuzzy p-separated in $\tilde{A}$. ■

Remark(4.3):

The convers of theorem(4.11) is not true in general .

Example(4.3):

Let X = {a, b} and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ are fuzzy sets defined as follows:

$\tilde{A}$= {(a, 0.8), (b, 0.8)}

$\tilde{B}$= {(a, 0.5), (b, 0.5)}

$\tilde{C}$= {(a, 0.6), (b, 0.6)}

$\tilde{D}$= {(a, 0), (b, 0.5)}

$\tilde{E}$= {(a, 0.4), (b, 0)}

} be a fuzzy $\tilde{C}$, $\tilde{B}$, $\tilde{A}$, $\tilde{\emptyset}$= {$\tilde{T}$Let

are $\tilde{E}$ and $\tilde{D}$ topology on Ã, then fuzzy p-separated in Ã but not fuzzy $\tilde{A}$ α-separated in

$\tilde{T}^c$= {$\tilde{A}$, $\tilde{\emptyset}$, $\tilde{B}^c$, $\tilde{C}^c$} such that:

$\tilde{B}^c$= {(a, 0.3), (b, 0.3)}

$\tilde{C}^c$= {(a, 0.2), (b, 0.2)}

$\tilde{D}$)=$\tilde{D}$, p-cl($\tilde{E}$)=$\tilde{E}$ p-cl(

$\tilde{E}$) ∩ $\tilde{D}$, and p-cl($\tilde{\emptyset}$ = $\tilde{D}$) ∩ $\tilde{E}$ p-cl(

are fuzzy p-$\tilde{E}$ and $\tilde{D}$ Hence, $\tilde{\emptyset}$= separated in Ã , but

, then $\tilde{A}$)=$\tilde{E}$, α-cl($\tilde{A}$)=$\tilde{D}$ α-cl(

; and $\tilde{\emptyset}$ ≠ $\tilde{D}$) ∩ $\tilde{E}$ α-cl(

$\tilde{\emptyset}$≠$\tilde{E}$) ∩ $\tilde{D}$ α-cl(

are not fuzzy $\tilde{E}$ and $\tilde{D}$ Hence, α-separated in A

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