



## HEAT TRANSFER FROM COMPOSITE MATERIAL EXTENDED SURFACES HEAT SINK

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### ABSTRACT

The heat transfer from two dimensional unsteady state of composite straight extended surfaces heat sink of different materials is analyzed here by using finite element technique. Galerkin method with linear rectangular elements is used in the present analysis. The effects of conductivity ratio ( $K_R$ ) on the efficiency and the effectiveness of the heat sink are examined ( the efficiency & the effectiveness of the heat sink are increased with increasing of ( $K_R$ ), also the present and non-present of the heat generation through the heat sink is discussed. The effect of the conductivity ratio on the steady state time is discussed here. It can be noted that the increasing of the conductivity ratio (from less than one to greater than one) will reduce the time for reach the steady state.

Keywords: Composite Material, Extended Surfaces, Heat Sink, FEM.

### الخلاصة

لقد تم دراسة انتقال الحرارة من السطوح الممتدة المركبة باستعمال الطرق العددية (Numerical Solution) وقد تم استعمال طريقة العناصر المحددة (Finite Element) ذات العناصر الرباعية الشكل (Rectangular Elements) باستعمال طريقة كليركن (Galerkin Method). لقد تم دراسة تأثير نسبة التوصيلية (Conductivity Ratio) (النسبة بين التوصيلية الحرارية للمادة الاساس و التوصيلية الحرارية للمادة المغطاة) على كل من فاعلية (Effectiveness) وكفاءة (Efficiency) للسطوح المزعفة المغطاة وكذلك تأثير هذه النسبة على كل من انتقال الحرارة والزمن اللازم للوصول الى حالة الاستقرار. وكذلك لقد تم دراسة وجود مصدر للحرارة المتولدة (Heat Generation) خلال المادة الاساس وتأثيرها على توزيع درجات الحرارة. ويمكن ملاحظة ان الزيادة في نسبة التوصيل (من اقل من واحد الى اعلى من واحد) يؤدي الى تقليل الزمن اللازم للوصول الى حالة الاستقرار وكذلك زيادة الفاعلية والكفاءة للسطوح الممتدة.

### INTRODUCTION

Heat sink consists from surface with plate fin extended from its surface. The heat sink with air as coolant fluid are used in more engineer applications like air conditioning system, air heater, electronic package,.....etc.

The heat sink extended surface can be constructed by coated or laminated a second material on the base material to protection it from a corrosive environment or an environment of extremely high temperature.

Most of the studies were restricted to steady state heat transfer .In practice; heat sink is frequently employed under unsteady state condition.

The study for two dimensional plates was reported by Barker, [1], where the exact solution is obtained in the form of an infinite series. Feijoo et al, [2], are using Greens function to convert the boundary value problem of composite solid into Fredholm integral equation of the second kind. Hung and Chung, [3], extended the Green function method to the unsteady laminated composite problem with or without Contact resistance at the interface. Hsin-Sen Chu et al, [4], presented the Laplace

transformation with respect to the time variable to calculate the heat transfer from composite straight fin.

In the present study, the effect of the conductivity ratio & heat generation has been studied by using Galerkin finite element method with rectangular elements. In our studied we are used aluminum & bronze materials for a & b materials respectively.

R. N. Alkaby, [5], discussed the transient heat transfer from the fin arrays by using finite element. She assumed the fin arrays material is homogenous & constant thermal conductivity. Hong-Sen Kou et al, [6], used a new approach (recursive formulation) to calculate the thermal performance of singular fin with variable thermal properties, the singular fin divided into many sections. The recursive formulas for both conditions with & without heat transfer on fin tip are derived. Donald,[7], used ANSYS finite element model to study a composite fins (that has different materials along it's length) of varying shape (rectangular & tapered fin shape) and materials

### MATHEMATICAL FORMULATION

Consider the heat sink consisting from unsteady state rectangular straight extended surface which compose abase material coated with a face material of different thermal characteristics, as shown schematically in Fig. (1) .The analysis is based on the following assumptions:

- Two dimension heat transfer.
- All the physical properties are assumed to be constant.
- The material of each region is isotropic & homogenous for each material.
- There is a perfect contact between the wall and the extended surfaces.

The differential equation for the transient behavior of temperature distribution is formulated from a consideration of heat balance over the differential element ( $\Delta x$ ,  $\Delta y$ ), taken into account the assumptions of the problem. Hence, the differential equation for a two dimensional unsteady-state (transient) heat flows with heat generation can be written as following, [6]:

$$\rho_i C_i \frac{\partial T_i}{\partial t} = k_i \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right) + \dot{q}_i \quad \dots\dots\dots (1)$$

Where  $i = \mathbf{a, b}$

Then, the subscript (i) refers to the region a & b respectively.

The Initial & boundary conditions for this situation as following:

The initial condition is given by

$$T_{init.} = T_o = T_{ba} \quad \text{at} \quad t=0 \quad \dots\dots\dots (2)$$

In the present analysis, we take the initial temperature equal to the base temperature (in the present study, the base temperature equal is 500 K, [11].

So, equation (1) can be written for surface (a) as following:

$$\frac{\partial^2 T_a}{\partial x^2} + \frac{\partial^2 T_a}{\partial y^2} + \frac{\dot{q}}{k_a} = \frac{1}{\alpha_a} \frac{\partial T_a}{\partial t} \quad \dots\dots\dots (3)$$

So, equation (1) can be written for surface (b) as following:

$$\frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2} = \frac{1}{\alpha_b} \frac{\partial T_b}{\partial t} \dots\dots\dots (4)$$

From the symmetrical of the system, the boundary conditions are:

$$\text{At } y = 0 \quad -k_b \frac{\partial T_b}{\partial y} = h(T_b - T_\infty) \dots\dots\dots (5)$$

$$\text{At } y = \delta_b \quad T_a = T_b \quad \& \quad k_b \frac{\partial T_b}{\partial y} = k_a \frac{\partial T_a}{\partial y} \dots\dots\dots (6)$$

$$\text{At } y = \delta_b + \delta_a \quad \frac{\partial T_a}{\partial y} = 0 \dots\dots\dots (7)$$

$$\text{At } x = 0 \quad T = T_a = T_b = T_o \dots\dots\dots (8)$$

$$\text{At } x = L \quad -k_b \frac{\partial T_b}{\partial x} = h(T_b - T_\infty) \quad \& \quad -k_a \frac{\partial T_a}{\partial x} = h(T_a - T_\infty) \dots\dots\dots (9)$$

**FINITE ELEMENT FORMULATION**

To calculate the temperature distribution along the composite extended surfaces & the heat flux, the weighted residual finite element based on Galerkin method with linear rectangular elements [8] had been used to solve the governing differential equations (3) & (4) that mentioned above compound with the initial & boundary conditions.

The residual transpose for each rectangular element is:

$$\{R^{(e)}\} = - \int_A [N]^T \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} + \frac{q_i}{k_i} - \frac{1}{\alpha_i} \frac{\partial T_i}{\partial t} \right) dA \dots\dots\dots (10)$$

Then

$$\{R^{(e)}\} = - \int_A [N]^T \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} + \frac{q_i}{k_i} \right) dA - \int_A [N]^T \left( \frac{1}{\alpha_i} \frac{\partial T_i}{\partial t} \right) dA \dots\dots\dots (11)$$

$$\{R^{(e)}\} = \{R_s^{(e)}\} + \{R_{us}^{(e)}\} \dots\dots\dots (12)$$

Where,  $\{R_s^{(e)}\}$  and  $\{R_{us}^{(e)}\}$  represent the steady state and unsteady state components of residual equation respectively.

The numerical solution of Eq. (12) is solved in two basic steps, as following:

- In the first step, the steady state component is a determined based on the boundary conditions of the system.
- In the second step, the solution of the unsteady state component is evaluated based on the initial conditions of the case study.

**For steady state component**, the solution can be obtained

$$\left\{ R_s^{(e)} \right\} = - \int_A [N]^T \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right) + \frac{q_i}{k_i} dA \dots\dots\dots(13)$$

Where,

$$T = N_i T_i + N_j T_j + N_k T_k + N_m T_m \dots\dots\dots(14)$$

and,

$$[N]^T = \begin{bmatrix} N_i \\ N_j \\ N_k \\ N_m \end{bmatrix} \dots\dots\dots(15)$$

Where,

$$\left. \begin{aligned} N_i &= \left(1 - \frac{S}{2bb}\right) \left(1 - \frac{\tau}{2aa}\right) \\ N_j &= \frac{S}{2bb} \left(1 - \frac{\tau}{2aa}\right) \\ N_k &= \frac{S\tau}{4aabb} \\ N_m &= \frac{\tau}{2aa} \left(1 - \frac{S}{2bb}\right) \end{aligned} \right\} \dots\dots\dots(16)$$

Where, (2 aa) & (2 bb) representing the dimension of rectangular elements.

The details of solving the Eq. (12) was performed in references [5], and the final equation described the steady state component can be expressed as:

$$\left\{ R_s^{(e)} \right\} = \left[ K^{(e)} \right] \left\{ T^{(e)} \right\} \dots\dots\dots(17)$$

Where,

$$\left[ K^{(e)} \right] = \left[ K_M^{(e)} \right] + \left[ K_S^{(e)} \right] \dots\dots\dots(18)$$

and,

$$\left[ K_S^{(e)} \right] = \frac{aa}{6bb} \begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{bb}{6aa} \begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \dots\dots\dots(19)$$

$$\left[ K_M^{(e)} \right] = \frac{MDx}{6} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots(20)$$

Where, the (Dx) is the length of the respective side of the element that subject to the boundary condition. There are three other results of the  $\left[ K_M^{(e)} \right]$ , one for each element side.

**For unsteady-state component**, the lumped formulation, [8] is used to define the variation in  $\frac{\partial T}{\partial t}$ , when

$$\frac{\partial T}{\partial t} = \left[ N^* \right] \left\{ \dot{T}^{(e)} \right\} \dots\dots\dots(21)$$

Then, the unsteady state component can be written as:

$$\left\{ R_{us}^{(e)} \right\} = \frac{1}{\alpha A} \left[ N^* \right]^T \left[ N^* \right] \left\{ \dot{T}^{(e)} \right\} dA \dots\dots\dots(22)$$

Since,

$$\left[ c^{(e)} \right] = \frac{A}{4\alpha} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(23)$$

When  $\left[ c^{(e)} \right]$  is combined with the other element matrices and summed overall the elements using the direct stiffness procedure, the final result is a system of first- order differential equations given by

$$\left[ C \right] \left\{ \dot{T} \right\} + \left[ K \right] \left\{ T \right\} = 0 \dots\dots\dots(24)$$

Where

$\left[ C \right]$ - is global capacitance matrix

$\left[ K \right]$ - is global stiffness matrix

The linear first- order differential equations in the time domain will produce in finite element solution with time-dependent heat transfer problems. These equations must be solved before the variation of (T) in space and time. A several procedures can be used for numerically solving of the Eq. (24).

To solve the time domain, the finite difference approximation will be used to solve a numerical solution.

Given a function T(t) and the interval  $[V1, V2]$ , we can use the mean value theorem for differentiation to develop an equation for T(t). The mean value theorem states that there is a value of (t), denoted by ( $\zeta$ ), such that

$$T(V2) - T(V1) = (V2 - V1) \frac{dT}{dt} (\zeta) \dots\dots\dots(25)$$

Rearranging gives

$$\frac{dT}{dt} (\zeta) = \frac{T(V2) - T(V1)}{\Delta t} \dots\dots\dots(26)$$

Where,

$\Delta t = (V2 - V1)$ , is the length of the interval.

$$T(V1) = T(\zeta) - (\zeta - V1) \frac{dT}{dt}(\zeta) \dots\dots\dots(27)$$

Rearranging gives

$$T(\zeta) = T(V1) + (\zeta - V1) \frac{dT}{dt}(\zeta) \dots\dots\dots(28)$$

The ratio ( $\theta$ ) may be defined as

$$\theta = \frac{(\zeta - V1)}{\Delta t} \dots\dots\dots(29)$$

Then eq. (39) will be written

$$T(\zeta) = (1 - \theta)T(V1) + \theta T(V2) \dots\dots\dots(30)$$

Define  $\{T\}_{v1}$  and  $\{T\}_{v2}$  as the vectors containing the nodal values at times  $v1$  and  $v2$ , then

$$\frac{d\{T\}}{dt} = \frac{\{T\}_{v2} - \{T\}_{v1}}{\Delta t} \dots\dots\dots(31)$$

and,

$$\{T\} = (1 - \theta)\{T\}_{v1} + \theta\{T\}_{v2} \dots\dots\dots(32)$$

After rearranged the previous equations, then the result is

$$([C] + \theta \Delta t [K])\{T\}_{v2} = ([C] - (1 - \theta) \Delta t [K])\{T\}_{v1} + \Delta t (\theta \{T\}_{v2}) \dots\dots\dots(33)$$

The value of ( $\theta$ ) must also be specified. In present analysis we choice the central difference method where

$$\theta = \frac{1}{2}, \quad \zeta = \frac{\Delta t}{2}$$

Then, the final equation becomes

$$\left([C] + \frac{\Delta t}{2} [K]\right)\{T\}_{v2} = \left([C] - \frac{\Delta t}{2} [K]\right)\{T\}_{v1} \dots\dots\dots(34)$$

Or,

$$[A]\{T\}_{v2} = [P]\{T\}_{v1} \dots\dots\dots(35)$$

Where,

$$[A] = [C] + \frac{\Delta t}{2} [K] \dots\dots\dots(36)$$

$$[P] = [C] - \frac{\Delta t}{2} [K] \dots\dots\dots(37)$$

The actual temperature distribution is found by solving Eq. (24) using Gaussian Elimination method [8]. Finally in order to calculate the efficiency & Effectiveness of heat sink, the heat lost from it must be obtained.

The total heat lost from the composite extended surface heat sink equal the summation of heat lost from each element that subjected to the boundary condition.

$$q_{ct} = \sum_{c=1}^{NDBC} h A_c (T_c - T_\infty) \dots\dots\dots(38)$$

$$A_{cr} = \Delta n \cdot Z \dots\dots\dots(39)$$

Where

Z- The depth of extended surface, which is unity for two-dimension.

**Heat sink extended surface effectiveness( $\epsilon$ )**, is defined as the ratio of actual heat transfer (lost) by the extended surface to the heat transfer from the bared surface without extended surfaces, [1].

$$q_{nf} = h A_o (T_{ba} - T_\infty) \dots\dots\dots(40)$$

$$A_o = L_{hs} Z \dots\dots\dots(41)$$

Then, the heat exchanger effectiveness

$$\epsilon_c = \frac{\sum_{c=1}^{NDBC} h (\Delta n)_c Z (T_c - T_\infty)}{L_{hs} Z (T_{ba} - T_\infty)} \dots\dots\dots(42)$$

**Heat sink extended surface efficiency( $\eta$ )**, is defined the ratio of actual heat transfer (lost) by the extended surface to the heat transfer lost from the same extended surface but it's surfaces temperature are constant and equal to the base temperature, [1], as following:

$$q_{fb} = \sum_{c=1}^{NDBC} h A_c (T_{ba} - T_\infty) \dots\dots\dots(43)$$

Then, the extended surface efficiency

$$\eta_c = \frac{\sum_{c=1}^{NDBC} h (\Delta n)_c Z (T_c - T_\infty)}{\sum_{c=1}^{NDBC} h (\Delta n)_c Z (T_b - T_\infty)} \dots\dots\dots(44)$$

FORTRAN computer program was written to calculate the temperature distribution [8], this program is modified o calculate heat transfer, efficiency & Effectiveness of extended surfaces.

The nodals temperature during the execution of the program are assumed to be reached to the steady-state values when the difference of the nodals temperature between two time adjusting steps (n) and (n+1) satisfies the equation

$$\left| \frac{T_i^{n+1} - T_i^n}{T_i^n} \right| \leq 10^{-7}$$

The convergence of the nodals temperature will be checked, and if convergence had not occurred, the nodals temperature for the previous time step will replace with the newly calculated nodals temperature & the program will continued with new time increment. If

the converging is occurred, the time increment loop will stop & the program will calculate the heat transfer rate, the efficiency & the effectiveness of the heat sink.

## RESULTS & DISCUSSION

For checking the accuracy of the following computer program, the temperature distribution of the following program (for the case of  $K_R=1$ ) is compared with two dimensional heat transfer from single fin (homogenous material) with same properties, [4], the results shown in Fig (2). We can note from this figure, the accuracy of the present program is acceptable & the maximum range of error is around (0.6%).

Then, the effect of thermal conductivity ratio on heat transfer & steady state time will be discussed as following:

In the case,  $k_1 > k_2$ , i.e.,  $KR (k_2/k_1) < 1$ , is considered since the conductivity of the coated material is generally smaller than that of the base material. Figs. (2) & (3) shows the counter temperature lines through the coated extended surface area. It is shown that when thermal conductivity & heat transfer coefficient is fixed, the temperature difference between the center region & surface temperature region will increase as the conductivity ratio decrease than one (the difference between  $k_1$  &  $k_2$  are large). This phenomenon is reasonable, because the small value of ( $KR$ ) means that the internal resistance to conduction heat flow in region 2 (material A) must be high, so that the heat conduction from region 1 (material B) to the surface is impeded. This causes large quantity of heat stored in the extended surface material so that the extended surface temperature will be higher. It is noted that the conductivity ratio also plays an important role in affecting the time to reach the steady state, the larger the conductivity ratio; the earlier the steady state can be reached. These contour lines can be compared with the contour lines for the case of ( $KR=1$ ) as shown in Fig. (5).

The effect of the conductivity ratio on the efficiency & the effectiveness of composite extended surfaces are shown in Figs.(6 & 7). We can observe from these figures the effect of  $KR$  on both the steady state time, values of efficiency & effectiveness especially when the difference between the thermal conductivities of base & coated materials are large, Also we can note the effectiveness & efficiency of composite extended surfaces will increase & faster to reach the steady state for the case of  $KR$  less than 1 (for the same reasons that mentioned before). At the time equal zero, all the materials at same temperature (initial temperature) equal to the base temperature, then the heat transfer from the heat sink equal to the maximum heat transfer & the efficiency of the heat sink equal 100%. The increasing of the time will increase the temperature drop in the heat sink & then decrease the efficiency. The increasing of the  $K_R$  will decrease the time for reaching to steady state condition & also the efficiency of the heat sink increased as  $K_R$  increased.

In order to express the effect of the  $KR$  on the steady state time, efficiency & effectiveness, we graphed a matching contour chart among  $KR$ , steady state time, efficiency & effectiveness as shown if Figs.(8 & 9).

Finally, as shown in Fig.(10), we discussed the effect of the heat generation on the temperature distribution for the case of ( $KR=1$ ). These contour lines show a greater difference between the center lines & the surface lines, this phenomenon is reasonable



because all the heat generated constructed in the base material & then it will increase the internal energy of the base material.

Fig.(11), the comparison between the efficiency of the composite fins with heat generation & without heat generation cases. The efficiency nearly equal at the initial time (time =0) because the fin at constant & same temperature, the difference between the efficiencies (with & without heat generation) increased as the time increased because the effect of the heat generation become more sensible due to the temperature drop through the fin with time.

## CONCLUSION

The partial differential equations derived for the transient state heat balance from composite extended surface heat sink has been solved numerically by using finite element method. From the present work, it is shown that the conductivity ratio plays in an important role on both the heat transfer rate & the time to reach the steady state. The effect of heat generation is very significant in the composed fins.

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### NOMENCLATURES

Symbol	Description	Unit	Symbol	Description	Unit
<b>A</b>	Cross section area	m <sup>2</sup>	<b>[C]</b>	Elemental capacity matrix	-
<b>Bi</b>	Biot number	-	<b>[N]</b>	Shape function vector	-
<b>K</b>	Thermal conductivity	W/m °C	<b>[K]</b>	Elemental stiffness matrix	-
<b>T</b>	Temperature	K	<b>[N*]</b>	Unsteady shape function vector	-
<b>T</b>	Time	Sec	<b>{R}</b>	Residual vector	-
<b>K<sub>R</sub></b>	Conductivity ratio (k <sub>1</sub> /k <sub>2</sub> )	-	<b>[W]</b>	Weighted function	-
<b>S</b>	Local coordinate in longitudinal direction	m	<b>[K<sub>m</sub>]</b>	Elemental stiffness matrix due to boundary condition	-
<b>Y</b>	Transverse coordinate (y- axis)	-	<b>x</b>	Longitudinal coordinate (x- axis)	m
<b>Y</b>	Dimensionless y-axis	m	<b>X</b>	Dimensionless x-axis	-
<b>H</b>	The average heat transfer coefficient	W/m <sup>2</sup> °C	<b>L</b>	Extended surface length	m
<b>q̇</b>	Heat generation for unit volume	W/m <sup>3</sup>	<b>q</b>	Heat transfer rate	W
<b>Greek Symbols</b>					
<b>A</b>	Thermal diffusivity	m <sup>2</sup> /s	<b>η</b>	Overall efficiency of extended surface	-
<b>E</b>	Extended surface effectiveness	-	<b>δ</b>	Material Thickness	m
<b>Subscript</b>					
<b>A</b>	The a- material		<b>cr</b>	Subject to boundary condition	
<b>B</b>	The b-material		<b>∞</b>	Fluid	
<b>F</b>	Fin		<b>ba</b>	Base of Wall	
<b>C</b>	Average value between two adjustment values		<b>o</b>	Initial	

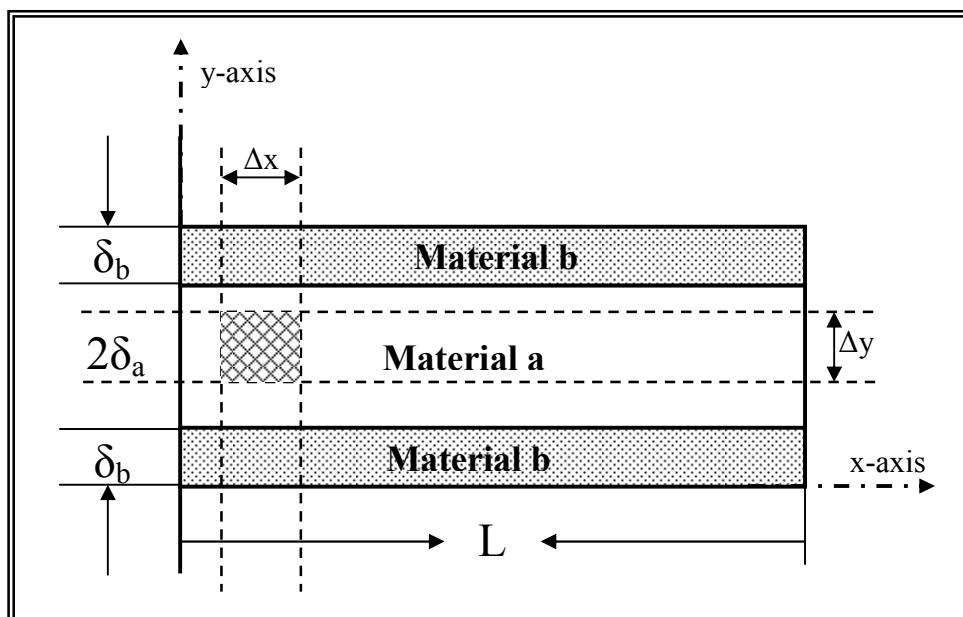


Fig. (1): Composite Rectangular Extended Surfaces

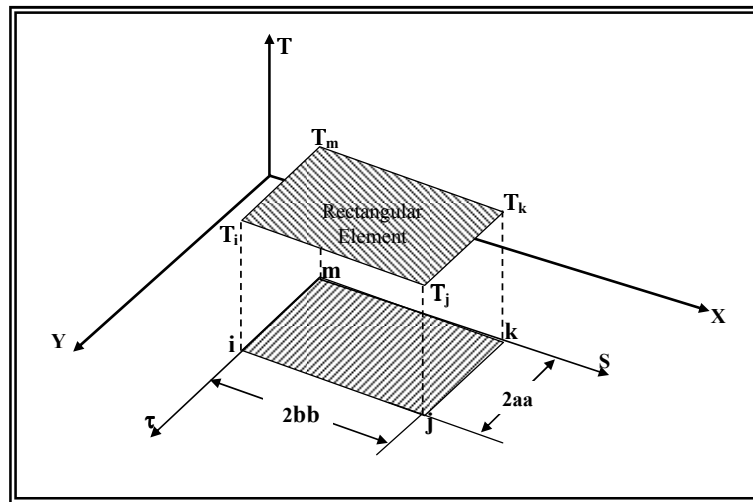


Fig. (2): Rectangular Element

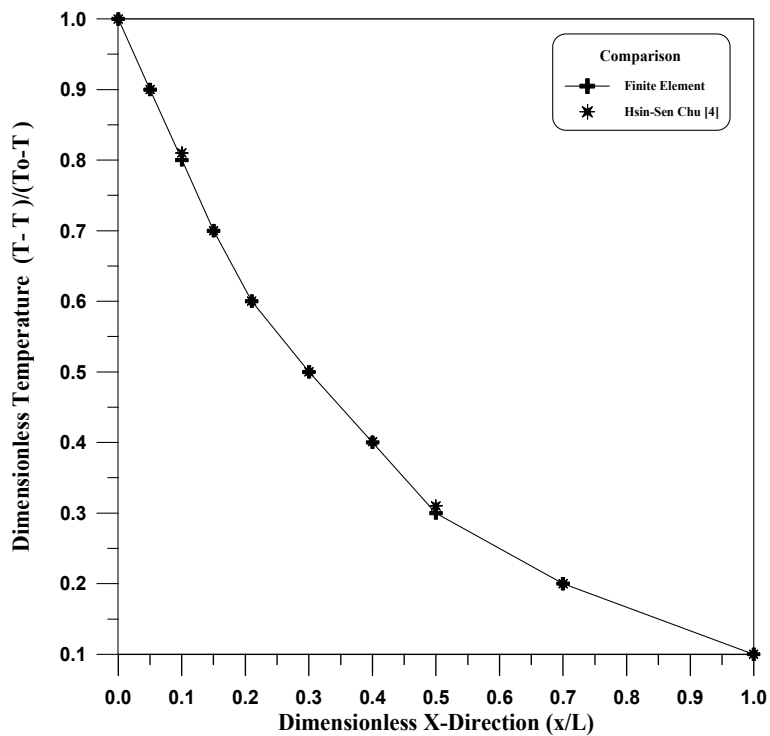


Fig.(2), Accuracy of the Present Analysis for Dimensionless Temperature  $(T - T_{\infty}) / (T_0 - T_{\infty})$  Profile of Composite Fin ( $\alpha=0.2$ ,  $KR=.2$ ,  $Bi=2$ ,  $\delta_a=.0045$  and  $\delta_b=.5$ ) with Dimensionless X ( $x/L$ ).

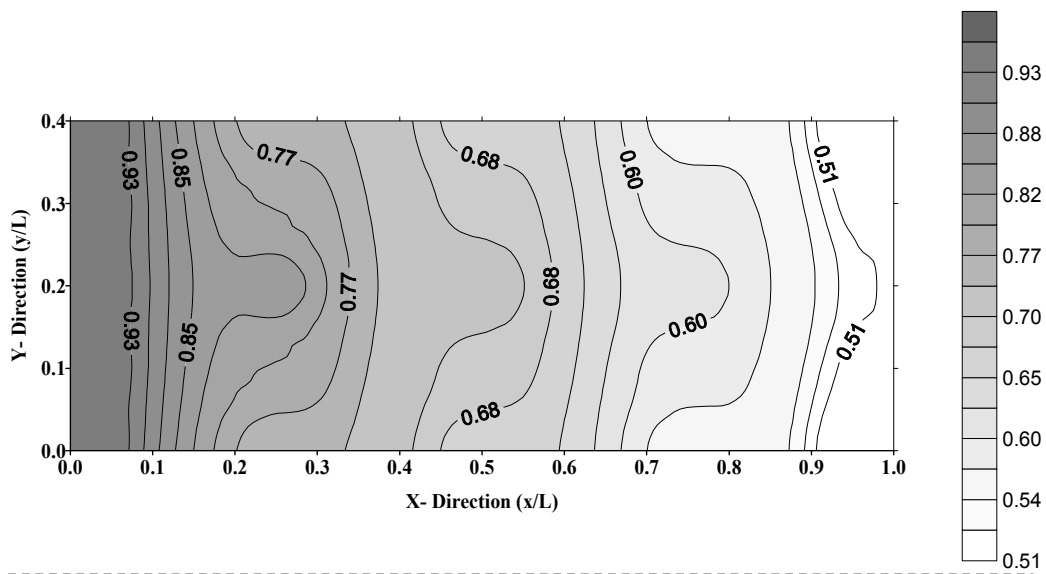


Fig.(3), Pattern of Contour Lines  $( (T - T_{\infty}) / (T_0 - T_{\infty}) )$  for the Composite Heat Sink with  $KR = 0.5$ .

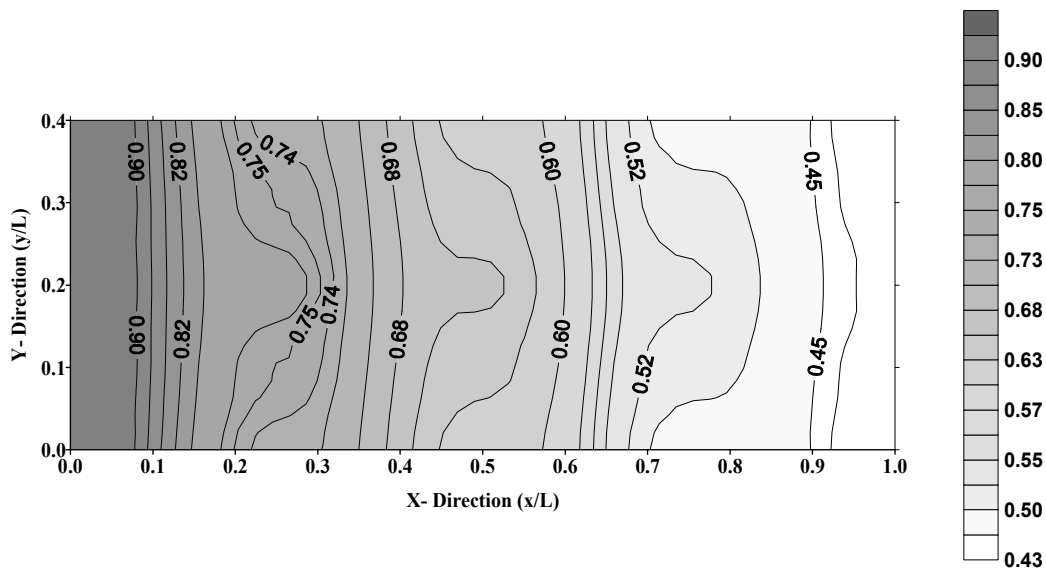


Fig.(4), Pattern of Contour Lines  $( (T - T_{\infty}) / (T_0 - T_{\infty}) )$  for the Composite Heat Sink with  $KR = 0.75$ .

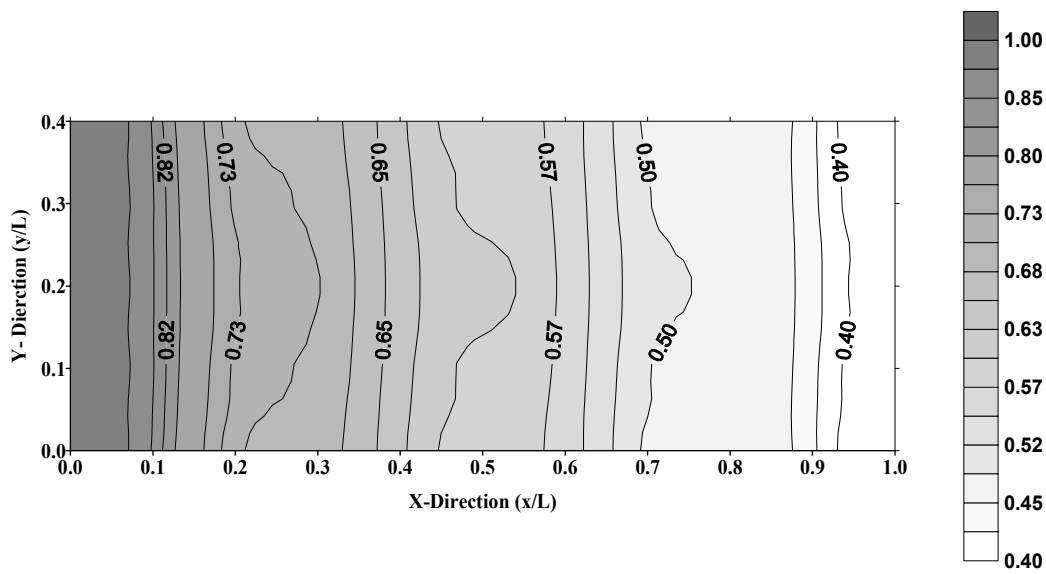


Fig.(5), Pattern of Lines  $(T - T_{\infty}) / (T_0 - T_{\infty})$  for the Composite Heat Sink with  $KR = 1$ .

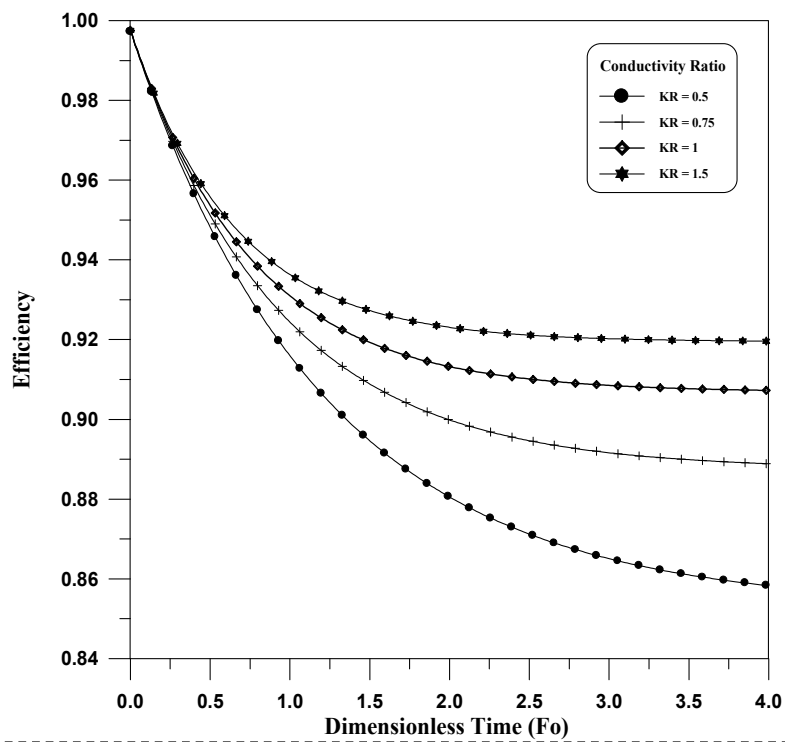


Fig. (6), The Efficiency of Composite Extended Surfaces of Different Values of KR with Dimensionless Time  $(\alpha L^2/t)$  (Fourier No.).

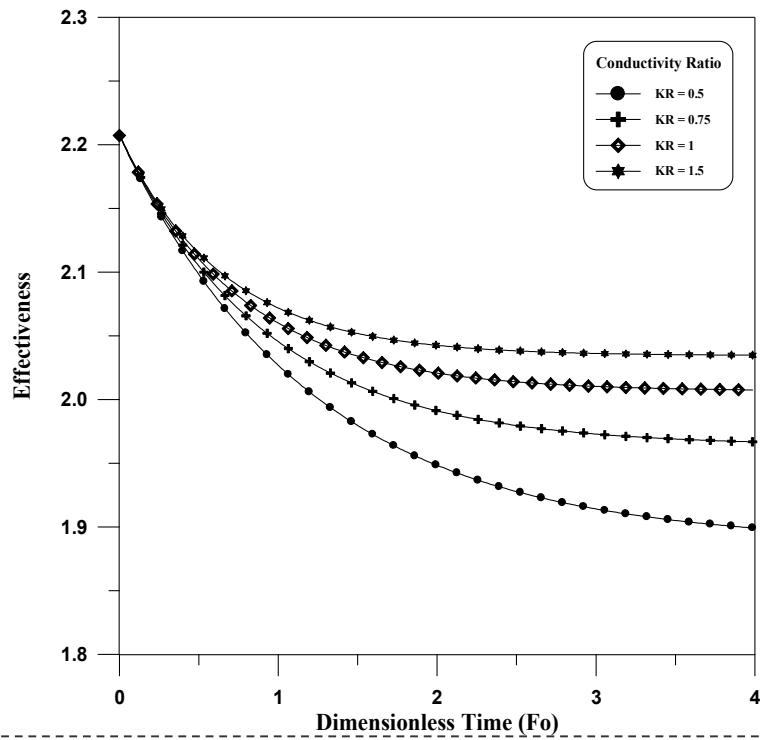


Fig.(7), The Effectiveness of Composite Extended Surfaces of Different Values of KR with  $(\alpha L^2/t)$  (Fourier No.).

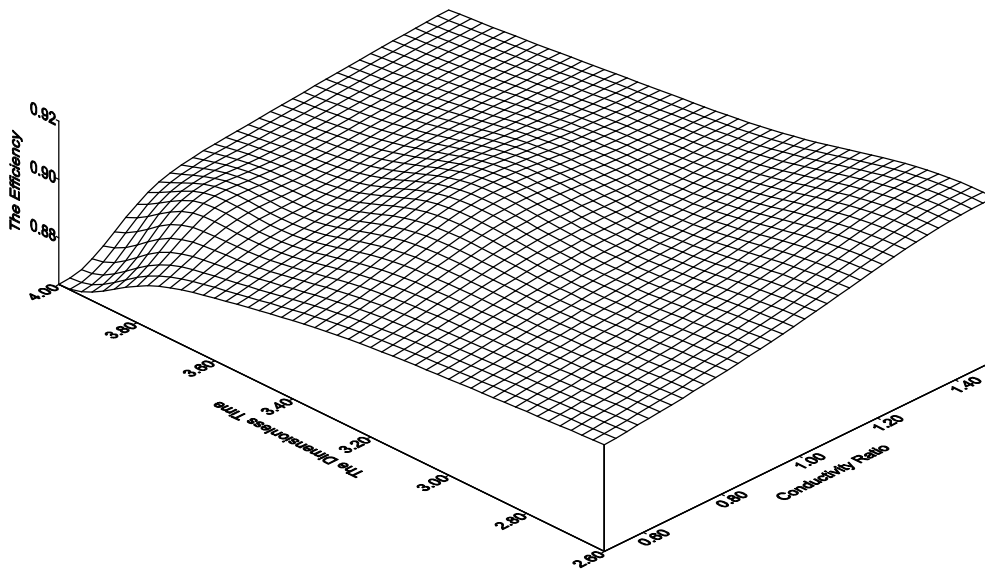


Fig.(8), A Contour Matching Chart for Efficiency of Composite Extended Surfaces.

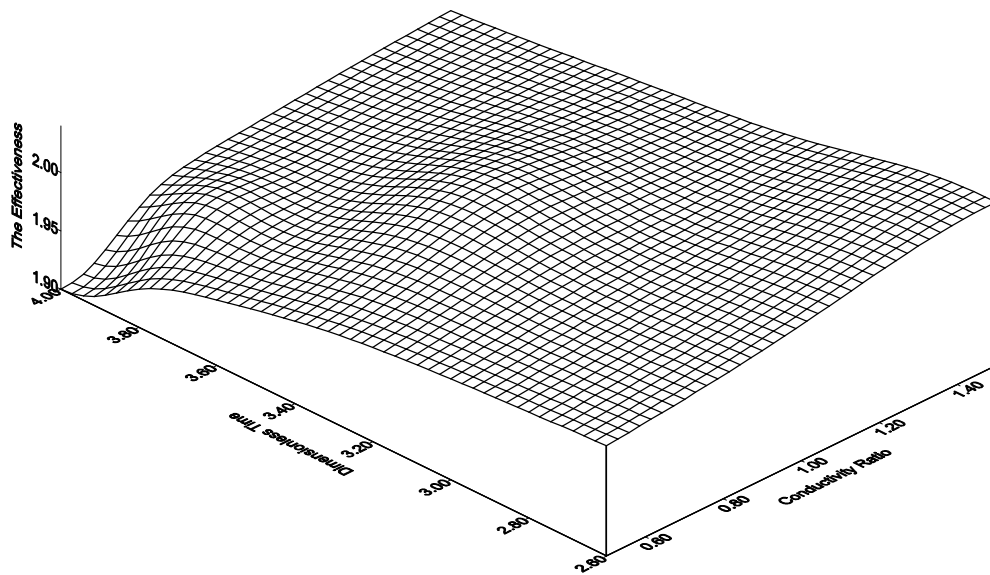


Fig.(9), A Contour Matching Chart for Effectiveness of Composite Extended Surfaces.

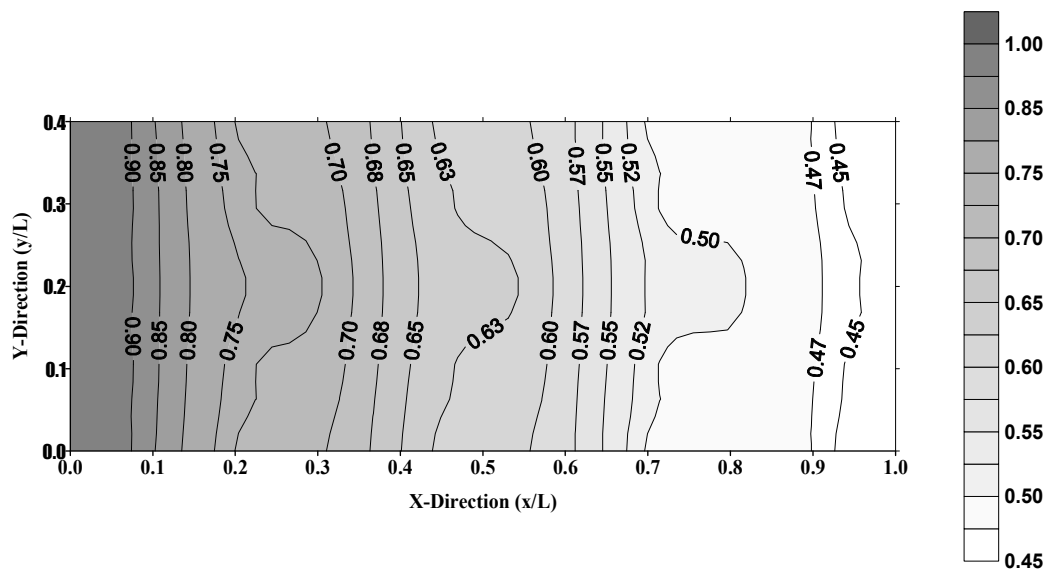


Fig.(10), A Contour Lines  $(T - T_{\infty}) / (T_0 - T_{\infty})$  for The Composite Heat Sink with Heat Generation.

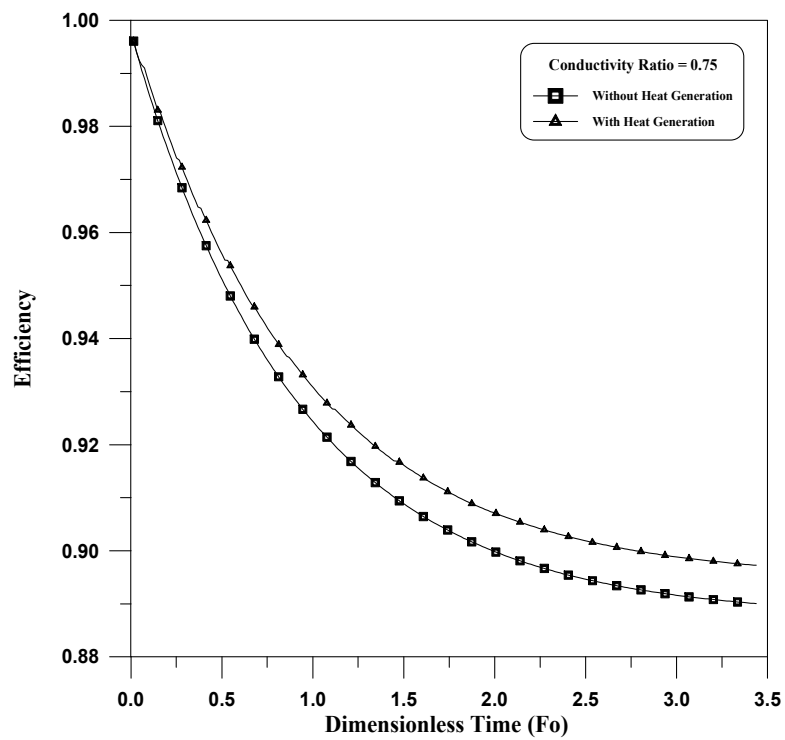


Fig.(11), The Efficiency of Composite Extended Surfaces without & with Heat Generation ( $q_0 = 0.5 \text{ MW/m}^3$ ) against Dimensionless Time ( $\alpha L^2/t$ ) (Fourier No.) for  $KR = 0.75$ .