EXPERIMENTAL AND THEORETICAL ANALYSIS EFFECT OF HOLE ON THE DEFLECTION AND MAXIMUM STRESS OF PLATE BENDING [+]

التحليل العملي و النظري لتأثير الفجوة على انحراف وأقصى اجهاد الصفيحة المنحنية

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Abstract:

The effect of hole made in plate (subjected to concentrated force) on the deflection and maximum stress is studied. Different hole sizes and distance from the plate center(x) are tested experimentally and compared with a reference plate without hole. The specimens are classified in to two groups; the first represents the specimens which contain hole leis in the x-axis of the plate and the second group represents the specimens which contain two half hole leis at the edge of plate. The results are analyzed analytically, which give a good agreement with the experimental deflection data. The first and second group with the same diameter gave a lower deflection than that for the plate without hole for the same cross sectional area. Increasing hole diameter gave a reduction in the deflection values for the specimens in each group. The second group of specimens gave a higher deflection values for a wide range of (x) as comparing with those of specimens in the first group. The maximum stress (σx) is higher in the specimens of the two groups as comparing with the reference plate for wide range of (x). When this distance (x) increased, the value of maximum stress is reduced for the specimens of the two groups. The same behavior is found when increasing the hole diameter.

المستخلص:

تم دراسة تأثير الفجوة المصنوعة في الصفيحة المسلط عليها حمل متمركزعلى انحرافها وأقصى اجهاد فيها. اختبرنا عمليا احجام فجوة ومسافات مختلفة عن مركز الصفيحة ومقارنتها مع صفيحة اساس لا تحوي تجويف. العينات صنفت لمجوعتين؛ الاولى تمثل العينات التي تحوي فجوة واقعة على المحور السيني للعينة والثانية تحوي نصفي الفجوة واقعة عند الحافات. النتائج حللت نظريا ، والتي اعطت مطابقة جيدة مقارنة مع النتائج العملية للانحراف. المجموعة الاولى والثانية من العينات بنفس القطر اعطت مقدار انحراف اقل مقارنة مع العينة بدون فجوة لنفس مساحة المقطع. زيادة قطر الفجوة اعطى نقصان بقيم الانحراف للعينات بكل مجموعة.المجموعة الثانية من العينات اعطت قيم انحراف اعلى لمدى واسع من(x) مقارنة مع عينات المجموعة الاولى.

[+] **Received on 2/10/2011 , Accepted on 13/2/2012 .**

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أقصى أجهاد بالاتجاه (x) يكون أعلى في عينات المجموعتين مقارنة مع تلك بدون فجوة لمدى واسع من(x). وعند زيادة تلك المسافة فأن أقصى أجهاد يقل لعينات المجموعتين. ونفس التصرف لوحظ عند زيادة قطر الفجوة.

Introduction:

In order to link the concept of stress concentration with the elasticity theory, it necessary to determine the stress field around the hole in an infinite plate. The analysis of stress concentration factor (SCF) of adjacent holes in a spherical pressure vessel can be approached by considering a thin plate undergoing hydrostatic stresses [1]. The von Mises stress is considered to determine the SCF. Various arrangements of adjacent holes are investigated i.e., two, three, four, and five adjacent holes are taken into account. The SCF curves with respect to the ratio of the distance between adjacent holes to the diameter of hole, L/d, and for a certain ratio of the diameter of hole to the plate thickness, d/t, are then plotted. The results show that the decreasing of L/d will affect the increasing of SCF, while for the case of five adjacent holes configuration, the increasing of d/t doesn't make any significant effect to the increasing of SCF. C. Pickthall, et al [2] examines the efficiency of an adhesively bonded reinforcement patch in reducing the stress concentration around a hole in a plate, as a function of hole size. A stress concentration factor (SCF) is defined as the tangential stress in the plate at the hole boundary, compared to that far away in the plate but still under the reinforcement. Nitin [3] studied the distributions of stresses and deflection in rectangular isotropic and orthotropic plates with central circular hole under transverse static loading by using finite element method. The D/A ratio (where D is hole diameter and A is plate width) is varied from 0.01 to 0.9. The analysis is done for plates of isotropic and two different orthotropic materials. The results are obtained for three different boundary conditions. The variations of SCF and deflection with respect to D/A ratio are presented in graphical form and discussed. The finite element formulation is carried out in the analysis section of the ANSYS package. Chyanbin [4] researched the case when an anisotropic plate contains a triangular, oval, or square opening, the only solution available in the literature is an approximate solution for orthotopic plates with openings. The solutions presented here have only one simple unified expression for various openings such as the ellipse, circle, crack, triangle, oval, and square. Two special loading conditions are considered. The results show that the effect of anisotropy on the stress concentration is totally determined through the fundamental elasticity matrices. Rao [5] used a shear loaded square panel representing a segment of composite rib. He observed that reinforcement of hole is necessary to reduce stress in vicinity of the hole.

The determination of the appropriate boundary conditions for a two-dimensional theory of elastic flat plates (and shells) consistent with the expected order of accuracy of the theory is both critical and challenging. The reciprocal theoremof elasticity will be applied in a novel way to obtain the appropriate stress boundary conditions for plate bending accurate to all order (with respect to the usual dimensionless thickness parameter) for plates of general edge geometry and loading [6].

calculate the perturbation to the total stress tensor due to ocean loading in coastal regions. Our stress calculation is fully 3-D and makes use of a semianalytic model to efficiently calculate stresses within a thick elastic plate overlying a viscoelastic or fluid half‐space. The 3-D stress perturbation is resolved into normal and shear stresses on plate boundary fault planes of known orientation so that Coulomb stress perturbations can be calculated [7].

Experimental work:

(1)Material used in this work:

An Aluminum sheet material is used to made the plates of bending tests. The chemical composition is listed in the following table:

Table (1) Chemical compositions of Aluminum

Si: 0.173	Fe: 0.225	Cu: 0.0125	Mg: 0.0045	Mn: 0.0215	Cr: 0.0043	Ni: 0.0051
Ca: 0.00045	Cd: 0.00035	Co: 0.001	Hg: 0.0021	La: 0.005	Li: 0.0002	Na: 0.00099
Zn: 0.0113	Ti: 0.0163	Ag:0.00062	B: 0.0028	Bi: 0.001	Zr: 0.0003	V: 0.005
P: 0.001	Pb: 0.0027	Sn: 0.001	Sr: 0.0001	Al: 99.5		

the thickness of sheet is (t=0.9mm).
A tensile test has been done to find the material tensile properties from standard tensile specimens. From the experimental tensile ، the yield stress for this material is (σy.s= MPa.).

(2)Preparation the reference plate bending:

The bending plate used in this work has the following dimensions:
W=25 mm   (width of plate).
L =80 mm   (length of plate).
t =0.9 mm   (thickness of plate).
Figure (1) shows the dimensions of this plate

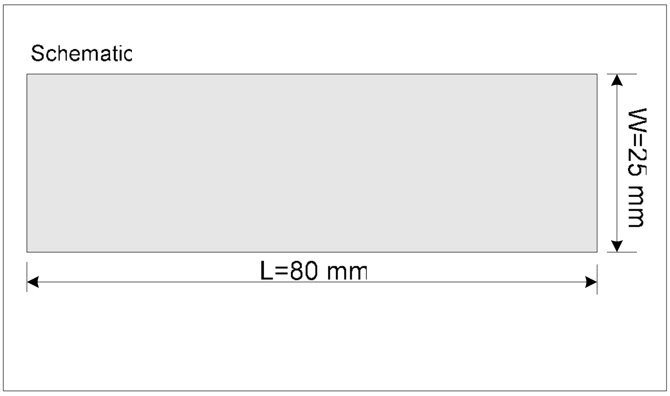

Fig.(1) Reference plate dimensions

The cross sectional area of plate is:

$$A=W*t=25*0.9=22.5\ mm^2 \qquad (1)$$

The value of cross sectional area will be assumed as a reference (constant for all specimens) to compare the results.

(3)Preparation of holed plate:

Two sets of holed plate are prepared such that the cross sectional area (A=22.5 mm^2) remains the same as in the reference plate:

1-Set A: figure (2)

To explain the effect of central hole on the values of stress and deflection, the plate is drilled with central hole along the longitudinal center line. Hence the width of reference plate (W=25mm); a five specimens is made such that the net width remains the same and results in equal cross sectional area. The center of hole is the same that the center of plate, that means the distance (x=0.0 mm) in fig.(2) .

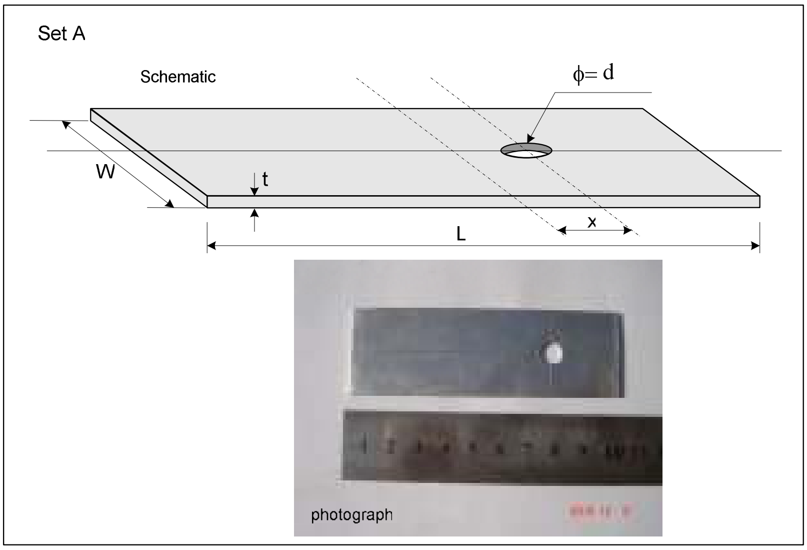

Fig. (2) Shape of set A specimen

The net cross sectional area will be, figure, (3):

$$A= (W-d)*t \qquad (2)$$

Where:
d: diameter of hole

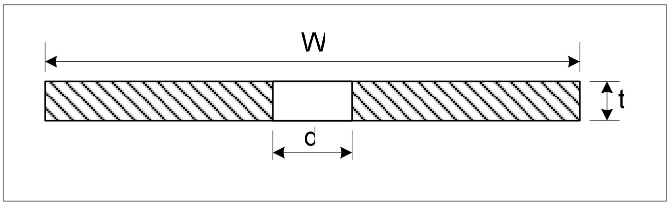

Fig. (3) Cross sectional area of set A specimen

Therefore, five width and hole diameter values are assumed as in table(2)

Table (2) Dimensions of set A specimens

Specimen No.	W(mm)	d(mm)	t(mm)	$A= (W-d)*t$
1	27	2	0.9	22.5
2	29	4	0.9	22.5
3	31	6	0.9	22.5
4	33	8	0.9	22.5
5	35	10	0.9	22.5

To know the effect of hole size on the deflection and stress of plate; another five specimens is made for each width value of the above table as follow:
For W=27mm, d=2mm:
x=0, 5, 10, 15, 20 and 25 mm.
The same experimental procedure is done for the other widths of specimens (W=27, 29, 31, 33 and 35 mm). The total number of specimens in set A is 30 specimens.

2-Set B: figure (4)

The same specimen dimensions mentioned in set A are made for set B. but the hole is divided in two half circle at the edges of plate to predict and compare the effect of side half holes with that the same of central hole.

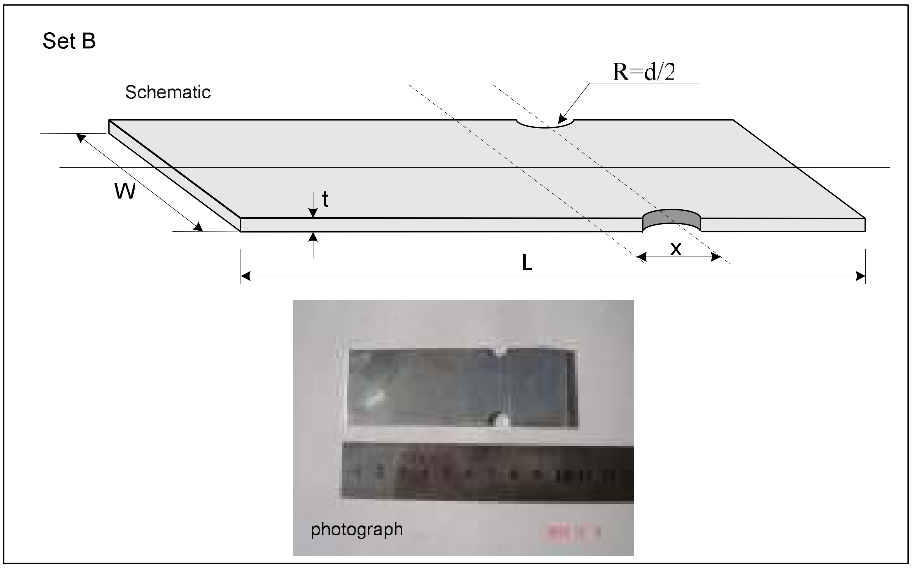

Fig. (4) Shape of set B specimen

The cross sectional area is shown in figure (5)

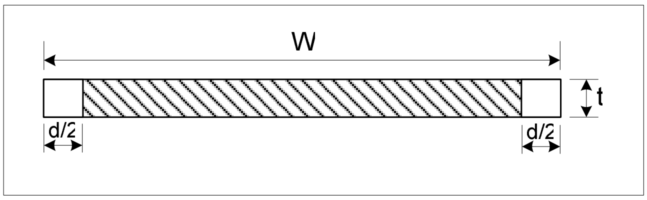

Fig. (5) Cross sectional area of set B specimen

(4)Experimental test procedure:

A concentrated force is applied along the center line of plate parallel to the width of plate. This force will give a maximum deflection in the center line of plate figure (6):

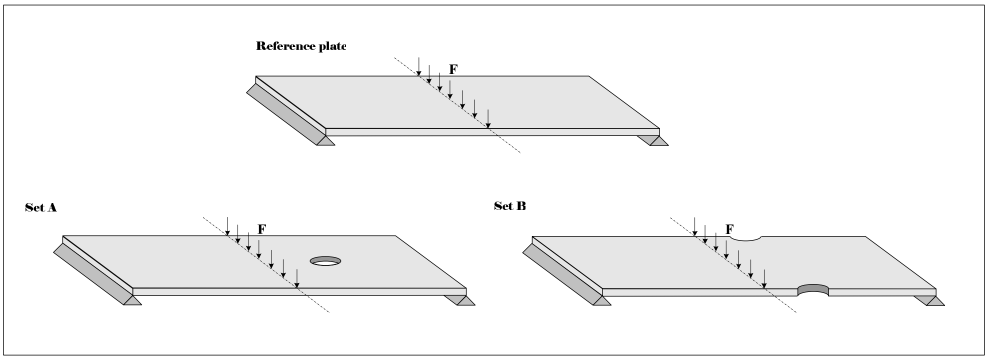

Fig. (6) Boundary conditions and concentrated force of specimens

In this work; the value of concentrated force used to compare the results is (F=10 N). As well as the same boundary conditions are used for each specimen:
Two opposite edges simply supported and the other free as shown in the above figure.
Bending equipment is used to test the deflection produce under the applied load, figure (7).
The concentrated load is increased gradually from zero value (deflection =0.0) until reach the value of (F=10 N) while the deflections of plate were recorded.

Fig. (7) Photograph of bending equipment

Theoretical analysis:

The general strain energy for plate subjected to distributed load (P) and simply supported for each edge [8] is:

$$U=\frac{1}{2}D\int_0^W\int_0^L (w_{xx}+w_{yy})^2\,dxdy \qquad (3)$$

Where:

$$D=\frac{Et^3}{12(1-v^2)} \qquad (4)$$

E: modulus of elasticity.
v: poisson's ratio.
W: plate width.

L: plate length.
w: deflection of plate (mm)
w_{xx}: second differentiation of deflection w.r.t x.
w_{yy}: second differentiation of deflection w.r.t y.
The deflection is given by the following series:

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{W} \tag{5}$$

The potential energy is:

$$\Omega=-\int_0^W\int_0^L Pw\ dxdy \tag{6}$$

For this work, the assumption will be as follow:

The deflection is a function of x only (the plate is free along x-axis and simply supported along y-axis):

$$w=\sum_{m=1}^{\infty}a_m\sin\frac{m\pi x}{L} \tag{7}$$

The applied load is concentrated and constant; therefore the potential energy will be constant:

$$\Omega=-Pw=-P\sum_{m=1}^{\infty}a_m\sin\frac{m\pi x}{L} \tag{8}$$

The second differentiation of eq.(7):

$$w_{xx}=-\sum_{m=1}^{\infty}(\frac{m\pi}{L})^2a_m\sin\frac{m\pi x}{L} \tag{9}$$

Sub eq.(9) in (3) and evaluate the integration :

$$U=\frac{WD\pi^4}{4L^3}\sum_{m=1}^{\infty}m^4a^2{}_m \tag{10}$$

The total energy (potential and strain energy) is

$$V=U+\Omega \tag{11}$$

The coefficient (a_m) is determined by the principle of stationary total potential energy:

$$\frac{\partial V}{\partial a_m}=\frac{\partial U}{\partial a_m}+\frac{\partial\Omega}{\partial a_m}=0 \tag{12}$$

Differentiate Eqs. (8&10) w.r.t. a_m and sub in Eq.(12)

$$\frac{\partial V}{\partial a_m}=\frac{WD\pi^4}{2L^3}\sum_{m=1}^{\infty}m^4a_m-P\sum_{m=1}^{\infty}\sin\frac{m\pi x}{L}=0 \tag{13}$$

From Eq.(13), the value of a_m can be evaluate as :

$$a_m=\frac{2L^3P}{WD\pi^4}\frac{\sin\frac{m\pi x}{L}}{m^4} \tag{14}$$

Sub Eq.(14) in Eq.(7); the general deflection equation for plate bending in this work will be as follow:

$$w = \frac{2L^3 P}{WD\pi^4} \sum_{m=1}^{\infty} \frac{\sin^2 \frac{m\pi x}{L}}{m^4} \qquad (15)$$

Now, to evaluate the central deflection (*x=L/2*), Sub this value in Eq. (15) and extend the summation:

$$w = \frac{2L^3 P}{WD\pi^4}\left[\frac{\sin^2 \frac{\pi}{2}}{1^4} + \frac{\sin^2 \pi}{2^4} + \frac{\sin^2 \frac{3\pi}{2}}{3^4} + \ldots\ldots\ldots\ldots\ldots\right] \qquad (16)$$

Or:

$$w = \frac{2L^3 P}{WD\pi^4} * 1.0147 \qquad (17)$$

Sub Eq.(4) in Eq.(15):

$$w = \frac{24.36L^3 P(1-v^2)}{WEt^3\pi^4} \qquad (18)$$

Finite element model:

The stress and deflection of plates are analyzed with finite element method using (ANSYS 11) code. The following steps are done for each plate:

- Define the element type (elastic 4 nodes 63).
- Define the real constant for the above element (using thickness t=0.9mm).
- Define the material property (E=70000 N/mm^2, *v*=0.3).
- Molding area (drawing the specimens set A, B and the reference plate).
- Meshing the specimen with fine mesh.
- Applying the boundary conditions (two edges simply supported and other free).
- Applying the concentrated force (F=10N).
- Solve the current model and
- Plot the results of stress and deflection for each specimen.

Results:

A sample variation of deflection with the applied load is record for the reference plate with wide range of force-deflection to compare those results with the theoretical and FEM model, figure (8).

Hence the theoretical analysis and the FEM from the ANSYS 11 for the deflection values give a good agreement as comparing with the experimental results

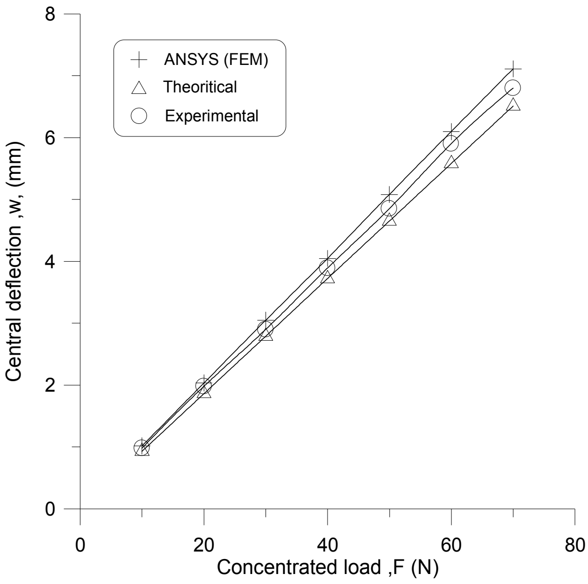

Fig. (8) Experimental, theoretical and FEM deflection results of reference plate

The results of this work can be classified into:

- Central deflection results (include the experimental and the FEM results which represents by the ANSYS 11 program).
- Results of maximum stress in plate (σx) (calculated by ANSYS11).

to compare the results between all specimens (set A&B) with the reference plate, the same cross sectional area ,the applied concentrated load (F=10N) and the boundary conditions are used as mentioned in the experimental work. Hence, the central deflection of reference plate was (w=1mm).

Figure (9) represents the variation of central deflection with the horizontal distance for five widths of specimens (W=27-35mm), set A.

As comparing with the reference plate, the deflection of plates for set A is lower than that for the reference plate.

That means; making a central hole in the plate will result in decreasing the deflection (for the same cross sectional area, force and B.C.).

Also for the same hole diameter, the deflection will decrease as the horizontal distance (x) increase for each specimen.

Increasing the hole diameter for the same value of (x) results in decreasing the value of central deflection.

Figure (10) represents the behavior of deflection with the distance (x) for five specimen widths (w=27-35mm), set B.

Where the deflection for all specimens is lower than that of the reference plate and decrease with increasing hole diameter for the same distance (x).

The experimental data along the horizontal distance may be above or below the theoretical data with small error which may be due to the error resulted due to the recorded data from the deflection equipment, figure (9&10).

It has been shown that the deflection increase gradually for each specimen width with increasing the distance from (x=0 to 5mm) and reach maximum value at (x=5mm).After this, the deflection decrease with increasing (x). This is due to the fact that stress concentration gives a higher deflection as the hole become near the applied concentrated load (deflection record data). This resulted in a weak area at this region.

Figures (11-15) represent a comparison of deflection for a similar plate dimensions between set A&B.

In general, at (x=0.0 mm) the specimen of set A gives a higher deflection than that for the other specimen in set B which have two sided half holes.

Increasing (x) results in decreasing the deflection for set A and decreasing it for set B until the two curves intersect at a point (x <5mm) in which the deflection of specimens in set A&B have equal value.

As shown from the above figures, the intersection point (z) decrease with increasing the width of specimen (W); and table (3) gives those values:

Table(3) Intersection deflection point for set A&B for each width

Width (W mm)	27	29	31	33	35
z (mm)	3.75	3	2.5	1.75	1.5

After the above intersection point the deflection is higher for set B and lower for set A and to approach one other with increasing the distance (x).

Figure (16) represents the variation of maximum stress (*σx*) with the horizontal distance for five width of specimens (W=27-35mm), set A.

As it has been observed, the stress decreases with increasing the distance (x). The increase in width (W) results in decreasing the maximum stress for the same value of (x).

The maximum stress in the reference plate (*σx=61.4 MPa*) which is small as comparing with specimens (set A) for the range of (0<x<15mm) for specimens (W=29,31,33 and 35 mm) and for the range (0<x<20mm) for specimen (W=27 mm) due to the stress concentration in the hole.

After the above ranges, the stress is modified gradually with increasing the (x). where, for the specimen (W=35mm) at (x=25mm), the stress reaches a value of (*σx=45 MPa*).

Approximately, the same behavior is observed for Set B, figure (17).

The curves in figures (18-22) represent a comparison of maximum stress (*σx*) for similar plate dimensions between set A&B.

In general, the specimens of set A gives a smaller stress values than that for set B (Fig. 19-22) and the stress approaches to an equal values for the range (x>20mm).

In figure (18) the stress alternate about the value of (x=10 mm) and equate at (x<20mm).

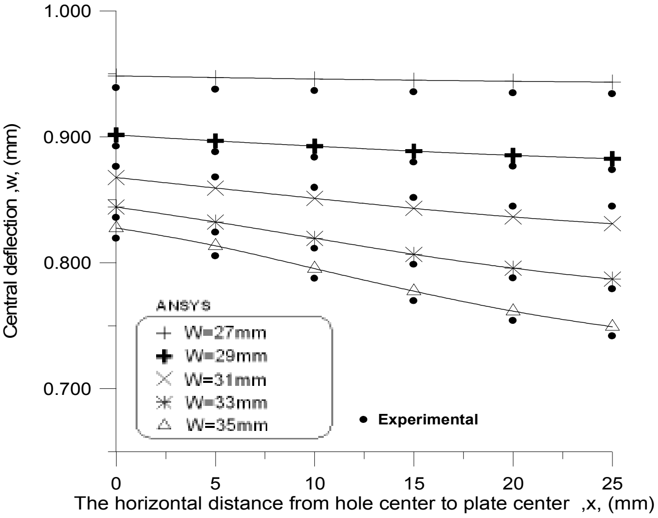

Fig. (9) Variation of central deflection with x for set A of specimens

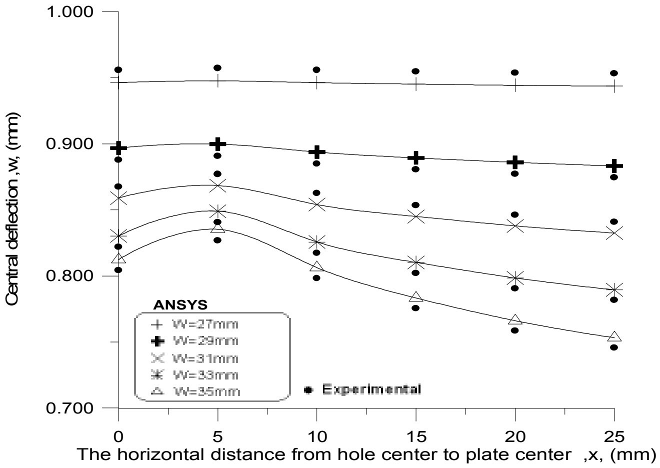

Fig. (10) Variation of central deflection with x for set B of specimens

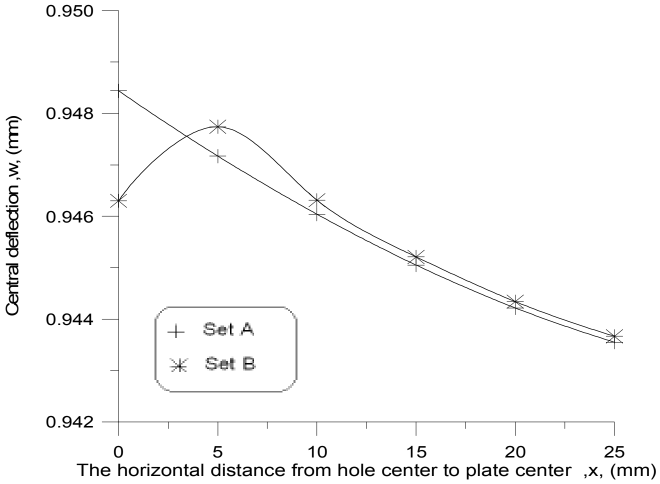

Fig. (11) Variation of central deflection with x for set A&B, (W=27mm)

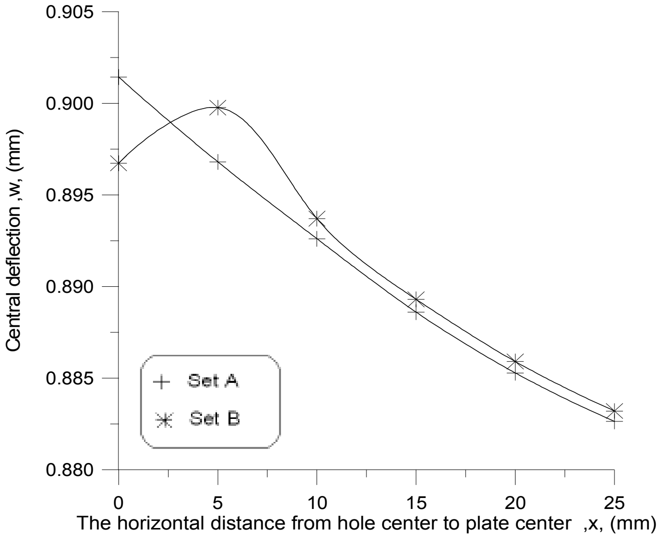

Fig. (12) Variation of central deflection with x for set A&B, (W=29mm)

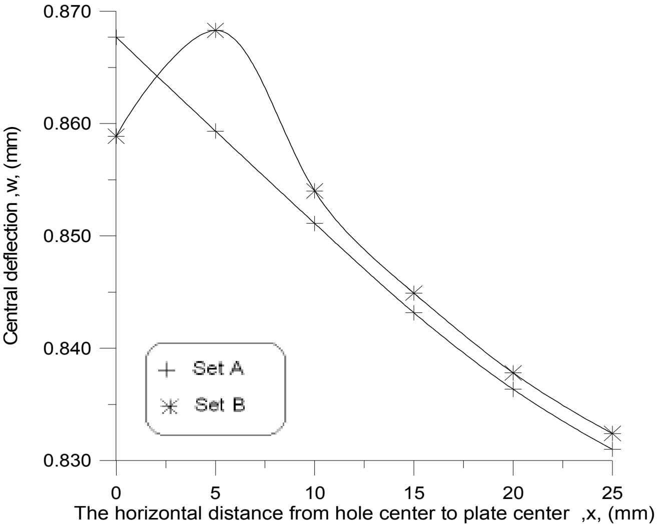

Fig. (13) Variation of central deflection with x for set A&B, (W=31mm)

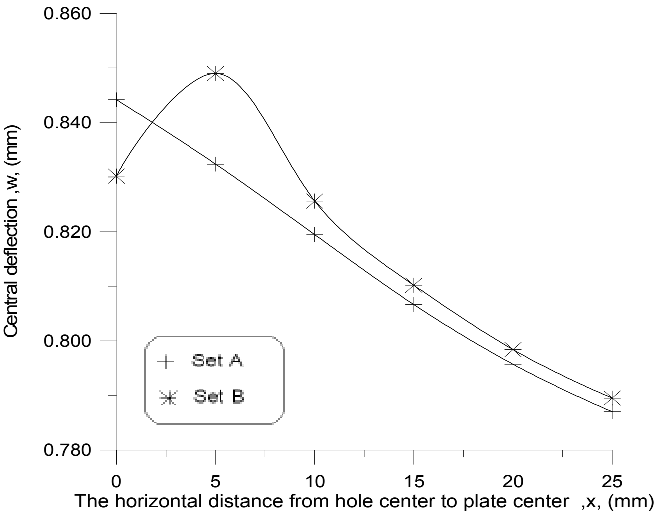

Fig. (14) Variation of central deflection with x for set A&B, (W=33mm)

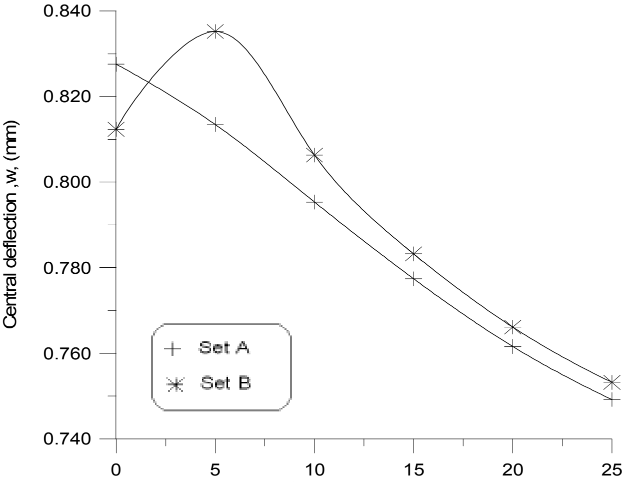

Fig. (15) Variation of central deflection with x for set A&B, (W=35mm)

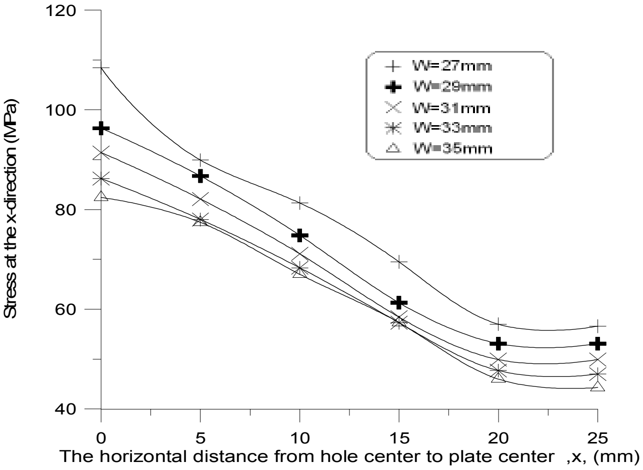

Fig. (16) Variation of $\sigma x)_{max}$ with x for set A of specimens

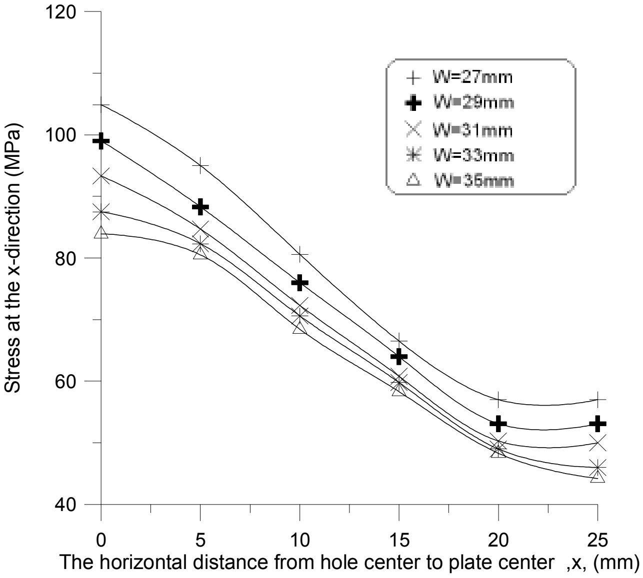

Fig. (17) Variation of $\sigma x\,)_{max}$ with x for set B of specimens

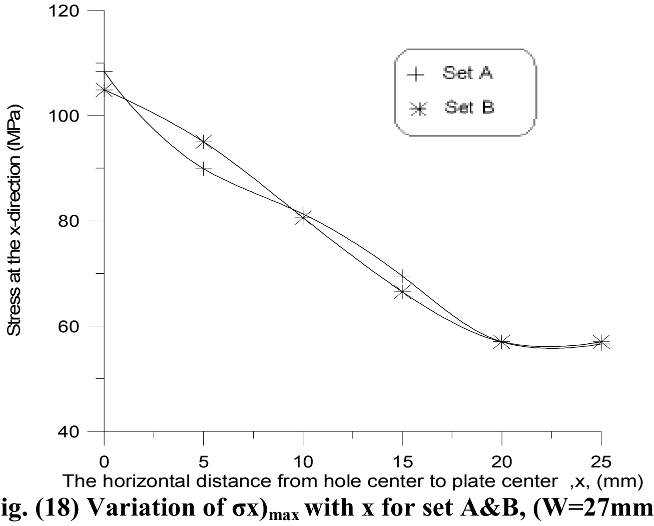

Fig. (18) Variation of $\sigma x)_{max}$ with x for set A&B, (W=27mm)

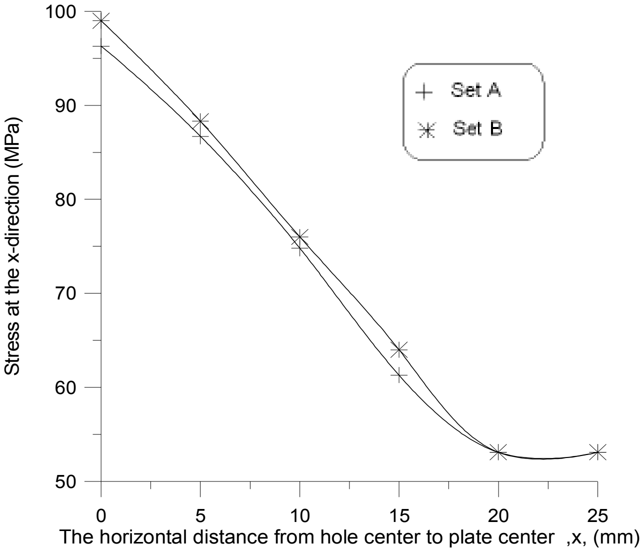

Fig. (19) Variation of $\sigma x)_{max}$ with x for set A&B, (W=29mm)

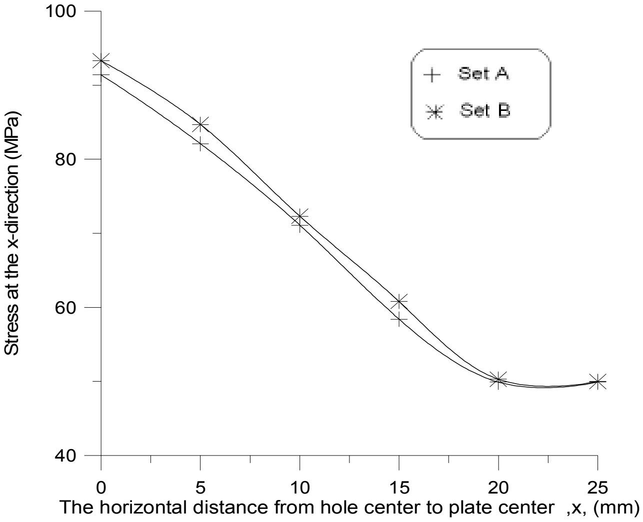

Fig. (20) Variation of $\sigma x)_{max}$ with x for set A&B, (W=31mm)

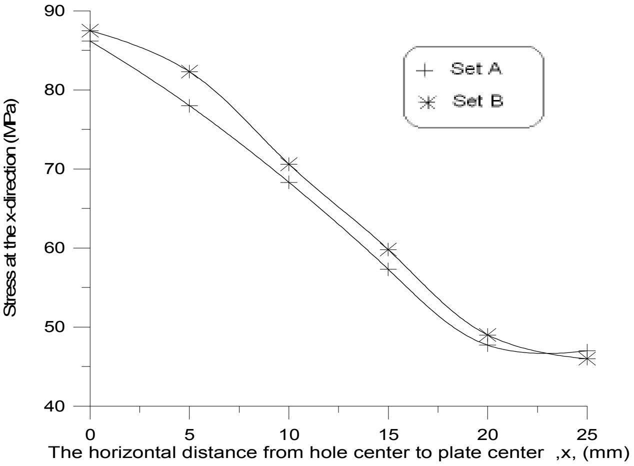

Fig. (21) Variation of $\sigma x)_{max}$ with x for set A&B, (W=33mm)

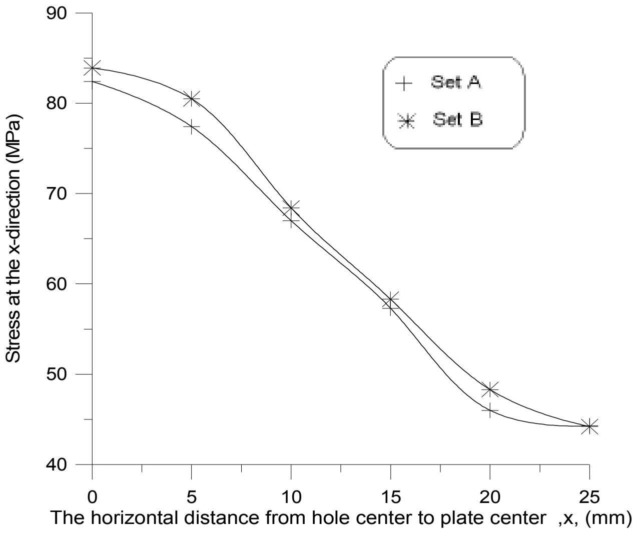

Fig. (22) Variation of $\sigma x)_{max}$ with x for set A&B, (W=35mm)

Conclusions:

A good agreement was found as comparing the deflection results from the theoretical, ANSYS11 program with the experimental data.
For the same cross sectional area, applied load and the boundary conditions of plate bending, the following conclusions are presented:

1-The specimens which have hole (set A) or side hole (set B) gave small deflection compared with that of the plate without hole. Hence, the value of central deflection is decreased with increasing the distance (x) for set A&B except the range (0<x<5mm) for set B where the deflection increase.
2-Set B of specimens give a higher deflection for a wide range of (x) as comparing with those of hole specimens (set A). where, Increasing hole diameter gives a reduction in the deflection.
3-Increasing the distance (x) and hole diameter (d) reduce the value of maximum stress for set A&B
4- The maximum stress (σx) is higher in set A&B as comparing with the reference plate for wide range of (x). This stress is appear around the hole of specimen for the range (0<x<20mm) and at the center of specimen for the range (x>15mm).
5-Maximum stress is found only at the center of specimen for the value of (x=25mm) and transfer from the location of hole surrounding to the plate center as the distance (x) increase.

References:

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