Fitting a Two Parameters of Weibull Distribution
Using Goodness of Fit Tests

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ABSTRACT
Goodness of fit test is used to determine how well the observed sample data fits with a proposed model. The test is based on the empirical distribution function, these test statistics are a variable when the hypothesized distribution is completely specified and the parameters of the distribution are unknown and must be estimated from the sample data. In this paper, some numerical results were obtained through a simulation study to obtain critical values of Kolmogorov-Smirnov D test, Cramer-Von Mises W² test and Anderson-Darling A² test. For weibull distribution with unknown parameters, this simulation was carried out using Monte Carlo techniques to create table of critical values for such tests. Furthermore, the power comparison between D test, W² test and A² test is investigated for a three alternative distributions.

Key words: Goodness-of-fit test, empirical distribution function, critical values, power studies.

INTRODUCTION
The weibull distribution has been widely used as a model in many areas of application specifically in the studies of failure components and as a model for product life. The two parameters form, the density function is:

\[ f(x; \alpha, \beta) = \begin{cases} \alpha \beta x^{\alpha-1}e^{-\beta x^\alpha}, & -\infty < x < \infty \text{ where } \alpha, \beta > 0 \\ 0 & \text{e.w.} \end{cases} \]

Smith and Brain T statistic was based on the sample correlation between
the order statistics of the sample and the expected value of the order
statistics under the assumption that the sample comes from two
parameter weibull distribution. For complete samples and two levels of
censoring, they provided critical values for the samples containing 8, 20,
40, 60, or 80 observations. Stephens (1977) [3] produced tables of
theasymptoticcritical values of the Anderson-Darling $A^2$ statistic and the
Cramer-von Mises $w^2$ statistics for various significance levels. Littell
and others (1979) [4] compared the Mann, Scheuer, and Fertig S
statistic, the Smith and Bain T statistic, the modified Kolmogorov-
Smirnov D statistic, the modified Cramer-von Mises $w^2$ statistic, and
modified Anderson- darling $A^2$ statistics though aseries of power studies
for sample size $n=10$ to 40. They also calculated critical values for the
D, $W^2$, and $A^2$ statistics for $n = 10, 15... 40$. Chandra and others (1981)
[5] calculated critical values for the Kolmogorov–Simonov D statistic
for $n = 10, 20, 50$, and infinity for three situations.

1. **Goodness-of-fit technique**

   Goodness-of-fit technique means the methods of examining how
well a sample of data agree with assumed distribution as its population
the important goodness-of-fit techniques are:

   1. Tests of chi- square types.
   2. Moment ratio techniques.
   3. Tests based on correlation.
   4. Tests based on empirical distribution function.

   Most of these test statistics suffer from serious limitations. In general
test of chi-square type have less power due to loss of information
caused by grouping. The distribution theory of chi-square statistics is a
large sample theory. The higher order moments are usually under
estimated and this fact prevents the use of moment ratio techniques and
so would be the case with correlation type tests. Several power studies
have revealed empirical distribution function (EDF) tests to be more
powerful than other tests of fit for wide range of sample sizes[8][3].

   We Consider tests of fit based on the empirical distribution
function (EDF). The EDF is a step function, calculated from the sample,
which estimates the population distribution function. EDF statistics are
measures of the discrepancy between the EDF and a given distribution
function and are used for testing the fit of the sample to the distribution
this may be completely specified or may contain parameters which must
be estimated from the sample.
2. Empirical distribution function [8]

Suppose a given random sample of size \( n \) is \( X_1, X_2, \ldots, X_n \) and let \( X_{(1)} < X_{(2)} < \cdots < X_{(n)} \) be the order statistics; and also suppose that the cumulative distribution function of \( X \) is \( F(x) \). and we assume this distribution to be continuous. The empirical distribution function (EDF) is defined as that proportion of the sample having a value less than or equal to \( x \). \[8\]

\[
F_n(x) = \sum_{i=1}^{n} f(x_i), \quad \text{where} \quad f(x_i) = \frac{1}{n} \quad \text{for} \quad x_i \leq x, \quad 0 \text{ e. w.}
\]

More precisely, the definition is

\[
F_n(x) = \begin{cases} 
0 & x < X_{(1)} \\
\frac{i}{n} & X_{(i)} < x < X_{(i+1)}; \ i = 1,2,\ldots,n-1 \\
1 & X_{(n)} \leq n
\end{cases}
\]

Thus, \( F_n(x) \) is a step function, calculated from the data; as \( x \) increase it take a step up of height \( 1/n \) as each sample observation is reached. For any \( x \), \( F_n(x) \) records the population less than or equal to \( x \), while \( F(x) \) is probability of an observation less than or equal to \( x \) \{\( [F(x) = P(X \leq x) \}] \). We can expect \( F_n(x) \) to estimate \( F(x) \), and it is in fact a consistent estimator of \( F(x) \); as \( x \to \infty \), \( |F_n(x) - F(x)| \) decreases to zero with probability one.


A statistic measuring the difference between \( F_n(x) \) and \( F(x) \) will be called an empirical distribution function (EDF) statistic. We shall concentrate on several EDF statistics which have attracted most attention. They are based on the vertical differences between \( F_n(x) \) and \( F(x) \), and are conveniently divided into two classes, "the supremum class" and "the quadratic class". The supremum statistics include Kolmogorov D and Kuiper V statistics and quadratic statistics are Cramer-von Mises \( W^2 \), Anderson-Darling \( A^2 \) and Watson \( U^2 \) statistics.

3.1 The supremum statistics. [10] The first two EDF statistics, \( D^+ \) and \( D^- \), are respectively, largest vertical difference when \( F_n(x) \) is greater than \( F(x) \) and the largest vertical difference when \( F_n(x) \) is smaller than \( F(x) \); formally,

\[
D^+ = \sup_{x} \{F_n(x) - F(x)\}
\]

And \( D^- = \sup_{x} \{F(x) - F_n(x)\} \)

The most well-known EDF statistics is \( D \), introduced by Kolmogorov-Simonov(1933):

\[
D = \sup_{-\infty \leq x \leq \infty} |F_n(x) - F(x)| = \max_{x}(D^+, D^-)
\]

\[139\]
3.2 The quadratic statistics. [11] A second and wide class of measures of discrepancy which is given by Anderson-Darling (1954) as:

$$W_n^2 = n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \Psi(x) dF(x)$$

Where $\Psi(x)$, $0 \leq x \leq 1$ is a suitable function which gives weights to the squared difference $\{F_n(x) - F(x)\}^2$ where $\Psi(x) = 1$ the statistic is Cramer-Von Mises statistic, now usually called $W^2$, and when $\Psi(x) = \left\{F_n(x)\right\}^{-1}$ the statistic is the Anderson-Darling (1954) statistic, called $A^2$. A modification of $W^2$ is the Watson (1961) statistic $U^2$ defined by:

$$U^2 = n \int_{-\infty}^{\infty} \left\{F_n(x) - F(x) - \int_{-\infty}^{F_n(x)} [F_n(x) - F(x)] dF(x)\right\}^2 dF(x)$$

Clearly $U^2 = W^2 - (W)^2$

4. Computing formulas

Form the basic definitions of the supremum statistics and the quadratic statistic given above in section (3) suitable computing formula must be found. To each observation $x_i$, on the random variable $X$, the Probability Integral Transformation (PIT) is applied to give a new observation $z_i$ on a random variable $Z$. It may be defined as:

$$z(x) = \int_{-\infty}^{x} f(x) dx$$

where $f(x)$ is the probability density function of the random variable $X$.

Let $Z = F(X)$; when $F(x)$ is the true distribution of $X$, the new random variable $Z$ is uniformly distributed between 0 and 1 [9]. Then $Z$ has the distribution function $F^*(z) = z, 0 \leq z \leq 1$.

Suppose that a sample $X_1, X_2, ..., X_n$ gives the values $Z_i = F(x_i), i = 1, 2, ..., n$, and let $F^*_n(z)$ be the EDF of the values $Z_i$. EDF statistics can now be calculated from a comparison of $F^*_n(z)$ with the uniform distribution for $Z$. It easy to show the values $Z$ and $x$ related by $z = F(x)$, the corresponding vertical difference in the EDF diagrams for $X$ and $Z$ are equal; that is,

$$F_n(x) - F(x) = F^*_n(z) - F^*(z) = F^*_n(z) - z$$

Consequently EDF statistics calculated from the EDF of the $Z_i$ compare with the Uniform distribution will take the same values as if they were calculated from the EDF of the $x_i$ compare with $F(x)$. This leads to the following formulas [9] for calculating EDF statistics from the $Z$-values, the formulas involve the $Z$-values arranged in ascending order let $Z_{(1)} < Z_{(2)} < \cdots < Z_{(n)}$. Then, with $\bar{Z} = \frac{\sum_{i=1}^{n} z_i}{n}$,


$$D^+ = \max_{1 \leq i \leq n} \left\{ \frac{i}{n} - Z_{(i)} \right\}; D^- = \max_{1 \leq i \leq n} \left\{ Z_{(i)} - \frac{(i-1)}{n} \right\}$$
\[ D = \max_{i \leq t \leq n} (D^+, D^-) \]

b) The Cramer-von Mises statistic \( W^2 \).

\[ W^2 = \sum_{i=1}^{n} \left\{ z_i - \frac{2i - 1}{2n} \right\}^2 + \frac{1}{12n} \]

c) The Anderson-Darling (1952-1954) statistic \( A^2 \).

\[ A^2 = \frac{-n(\sum_{i=1}^{n}(2i-1)(\ln z_i + \ln(1 - z_{n+1-i}))}{n} - n \]

5. The test statistic based on the EDF

Suppose the sample \( x_1, x_2, \ldots, x_n \) has been obtained on the random variable \( X \). We wish to test the null hypothesis is: \( X \sim F(x, \theta) \), i.e. \( H_0 \): a random sample of \( n \) \( X \)-values comes from \( X \sim F(x, \theta) \) where \( X \sim F(x, \theta) \) is a continuous distribution and \( \theta \) is a vector of parameters. When \( \theta \) is fully specified, i.e. the parameters are known. Then \( z_{(i)} = F(x_{(i)}; \theta) \) gives a set \( Z_{(i)} \) which, on \( H_0 \), are ordered uniform [9] and equations in section (4,a,b,c) are used to give EDF statistics. On the other hand, \( F(x, \theta) \) may be defined only as a member of a family of distributions, but all or part of the vector \( \tilde{\theta} \) may be known.

When \( \tilde{\theta} \) is known, distribution theory of EDF statistics is well-developed, even for finite samples, and tables are available for some time. When \( \tilde{\theta} \) contains one or more unknown parameters, these parameters may be replaced by estimates, to give \( \hat{\theta} \) as the estimate of \( \tilde{\theta} \). Then formulas in section (4,a,b,c) may still be used to calculate EDF statistics, with \( Z_{(i)} = F(x_{(i)}; \hat{\theta}) \). However, even when \( H_0 \) is true, the \( Z_{(i)} \) will now not be an ordered uniform sample, and the distributions of EDF statistics will be very different from those when \( \tilde{\theta} \) is known, they will depend on the distribution tested, the parameters estimated, and the method of estimation, as well as on the sample size. New points should then be used for the appropriate test, even for large samples; otherwise a serious error in significance level will result.

5.1 EDF statistics with unknown parameters

The EDF statistic will not depend on true values of unknown parameters, when unknown parameters are estimated by appropriate method. Therefore percentage points for EDF tests depend only on the sample size \( n \). Since, the exact distributions of EDF statistics are very difficult to find, Monte Carlo studies have been extensively used to find points for finite \( n \). Fortunately, for the quadratic statistics \( W^2 \), \( U^2 \) and \( A^2 \), asymptotic theory is available; the percentage points of these statistics
for finite $n$ converge rapidly to the asymptotic points. No asymptotic theory for the statistics, $D^+$, $D^-$, $D$ and $V$ except in case $F(x)$ is continuous and completely specified, the asymptotic points must be estimated.

David and Johnson (1948) [12] showed that if the parameters estimated are parameters of scale or location, and the estimators satisfy certain general conditions, then when one applies the probability integral transformation, the joint distribution of the transformed variables will not depend on the true parameter values.

6. EDF test for Weibull distribution

6.1 Estimation of unknown parameters

This section is concerned with the MLE of unknown parameters $\alpha$ and $\beta$ for Weibull distribution which is given by

$$f(x; \alpha, \beta) = \begin{cases} \alpha \beta x^{\alpha-1} e^{-\beta x^\alpha}, & -\infty < x < \infty \text{ where } \alpha, \beta > 0 \\ e.w. \end{cases}$$

And whose distribution function takes the form

$$F(x; \alpha, \beta) = 1 - e^{-\beta x^\alpha} \quad 0 < x < \infty$$

When the parameters $\alpha$ and $\beta$ are unknown and are estimated from a random sample. Let $x_1, x_2, ..., x_n$ be a random sample of size $n$ from Weibull distribution with parameters $\alpha$ and $\beta$.

It is well known that maximum likelihood estimators of $\alpha$ and $\beta$ are obtained by solving [13]

$$\frac{-n}{\hat{\alpha}} + \sum_{i=1}^{n} \ln x_i - \hat{\beta} \sum_{i=1}^{n} x_i^{\hat{\alpha}} \ln x_i = 0 \quad (1)$$

And

$$\frac{n}{\hat{\alpha}} - \sum_{i=1}^{n} x_i^{\hat{\alpha}} = 0 \quad (2)$$

Solution for $\hat{\alpha}$ and $\hat{\beta}$ cannot found analytically from the non-linear equations (1) and (2). An approximate solution for $\hat{\alpha}$ and $\hat{\beta}$ from equation (1) and (2) can be made iteratively by using Newton-Raphson method for solving non-linear equations as follows:

Let

$$f_1 = f_1(\hat{\alpha}, \hat{\beta}) = \frac{n}{\hat{\alpha}} + \sum_{i=1}^{n} \ln x_i - \hat{\beta} \sum_{i=1}^{n} x_i^{\hat{\alpha}} \ln x_i$$

And

$$f_2 = f_2(\hat{\alpha}, \hat{\beta}) = \frac{n}{\hat{\alpha}} - \sum_{i=1}^{n} x_i^{\hat{\alpha}}$$

Suppose that $(\hat{\alpha}^{(s)}, \hat{\beta}^{(s)})$ represent the approximate solution of $(\hat{\alpha}, \hat{\beta})$ at stage (s). Then the approximate solution at stage (s+1) for $(\hat{\alpha}^{(s)}, \hat{\beta}^{(s)})$ is:
\[ \hat{\alpha}^{(s+1)} = \hat{\alpha}^{(s)} + \delta_1 \]  
\[ \hat{\beta}^{(s+1)} = \hat{\beta}^{(s)} + \delta_2 \]  
(3)

(4)

Solving (3) and (4), we get:

\[ \hat{\alpha}^{(s+1)} = \hat{\alpha}^{(s)} - \frac{1}{ac - b^2}(cf_1 - bf_2) \]
\[ \hat{\beta}^{(s+1)} = \hat{\beta}^{(s)} - \frac{1}{ac - b^2}(-bf_1 + af_2) \]

Let \( y_i = \ln x_i; i = 1, 2, ..., n \) and \( z_i = F(x_i; \hat{\alpha}, \hat{\beta}) \) where \( x_i \) is the ith order statistics.

### 6.2 Calculation of test statistics

A goodness-of-fit test is used to test null hypothesis.

**H\(_0\):** the random sample \( x_1, x_2, ..., x_n \) comes from the Weibull distribution \( f(x; \alpha, \beta) \) with unknown parameters \( \alpha \) and \( \beta \).

We will construct on the following main test for goodness-of-fit for the Weibull distribution.

1. **Modified Kolmogorov-Smirnov statistic D.**

   \[ D = \sup |F(x_i; \hat{\alpha}, \hat{\beta}) - F_n(x)| = \sup |(1 - e^{-\beta x^\alpha}) - F_n(x)| \]

   Where \( F_n(x) = \sum_{i=1}^{n} f(x_i), f(x_i) = \frac{1}{n} \) for \( x_i \leq x, 0 \) e.w.

   Is the empirical distribution function of the sample.

   This is equivalent to \( D = \max(D^+, D^-) \) where \( D^+ = \max_{1 \leq i \leq n} \{ \frac{i}{n} - z_i \} \)

   \( D^- = \max_{1 \leq i \leq n} \{ z_i - \frac{i-1}{n} \} \)

   Where \( z_i = F(x_i; \hat{\alpha}, \hat{\beta}) \) and \( x_i \) is the ith order statistics.

2. **Modified Cramer-Von Mises statistics \( W^2 \).**

   \[ W^2 = n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 dF(x) \]

   Where \( F_n(x) = \sum_{i=1}^{n} f(x_i). \) Put \( Z = F(x) \) where

   \[ Z_n = \begin{cases} 
   i/n & z < Z_i \leq Z_{i+1} \\
   1 & Z_{(n)} < z 
   \end{cases} \]

   \( Z_0 = 0 \) and \( Z_{(n+1)} = 1 \). Then

   \[ W^2 = n \int_{0}^{1} \{z_n(n) - z\}^2 dz = n \sum_{i=1}^{n} \int_{z_{(i)}}^{Z_{(i+1)}} \left[ \frac{i}{n} - z \right]^2 dz \]

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\[ \frac{1}{3} n \sum_{i=1}^{n} \left[ \left( z_{i+1} - \frac{1}{n} \right)^3 - \left( z_i - \frac{1}{n} \right)^3 \right]^2 \]

\[ = \sum_{i=1}^{n} \left\{ z_i - \frac{2i - 1}{2n} \right\}^2 + \frac{1}{12n} \]

\[ = \sum_{i=1}^{n} \left\{ 1 - e^{-\beta x^\alpha} - \frac{2i - 1}{2n} \right\} + \frac{1}{12n} \].

3. Modified Anderson-Darling statistic \( A^2 \).

\[ A^2 = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1 - F(x))} \frac{1}{dF(x)} \]

\[ = n \int_{0}^{1} \frac{(z_n(n) - z)^2}{z(1 - z)} \frac{1}{dz} \]

\[ = -n - \frac{1}{n} \sum_{i=1}^{n} (2i - 1) \left[ \ln z(i) + \ln \left\{ 1 - z_{n+1-i} \right\} \right]. \]

The results of David and Johnson (1948) [12] imply that the distributions of \( D, W^2 \) and \( A^2 \) do not depend upon the values \( \alpha \) and \( \beta \). Therefore without loss of generality, the distributions of \( D, W^2 \) and \( A^2 \) can be obtained assuming \( \alpha = \frac{1}{2} \) and \( \beta = 1 \). (See [4] for results may be used to arrive at the same conclusion, specific to the Weibull distribution).

**RESULT AND DISCUSSION**

The main objective of the research reported here in were two things: first to show the goodness-of-fit test for weibull distribution by using the test \( D, W^2 \) and \( A^2 \) for the sample of size \( n \), and second to compare between the power functions of test statistics \( D, W^2 \) and \( A^2 \) for several distributions. To calculate the critical values for proposed test statistics for weibull distribution with unknown parameters, by Monte Carlo simulation which is carried out via Mathcad (2001) package [14]. The following steps are used in calculating critical values for the proposed test statistics:

1st step A random sample \( X_1, X_2, \ldots, X_n \) from weibull distribution was generated. Firstly a random sample \( Z_1, Z_2, \ldots, Z_n \) of \( n \) order statistics from a uniform \((0,1)\) distribution was generated, then the \( i \)th order statistic from the \( W(\alpha, \beta) \) with \( \alpha = \frac{1}{2} \) and \( \beta = 1 \) will obtained as follows

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\[ X_{(i)} = \frac{-1}{\beta} \left\{ \ln(Z_{(i)}) \right\} ^{1/\alpha} \]

2nd step  This random sample was used to estimate the unknown parameters by method of MLE in section (6.1).
3rd step  The resulting maximum likelihood estimator of the unknown parameters is used to determine the hypothesized cumulative distribution function for the weibull distribution.
4th step  Selected sample size as \( n = 10(10)100 \). The appropriate test statistics mentioned in section (6.2) was calculated for the given values of \( n \).
5th step  This procedure was repeated 10000 times, thus generating 10000 independent values of the appropriate test statistics. These 10000 values were then ranked, and the values of these test statistics at five significance levels, i.e., \( \alpha= 0.01, 0.05, 0.10, 0.15, \) and \( 0.20 \) are calculated.

Table (1) list the critical values for the statistics test \( D, W^2 \) and \( A^2 \), using Monte Carlo method.

The power of a goodness-of-fit test is defined as the probability that a statistic will lead to the rejection of the null hypothesis, \( H_0 \), when it is false, i.e. when a sample is not from the hypothesized population but an alternative population [1].

To evaluate our results we conducted a power study of the three statistics \( D, W^2 \) and \( A^2 \) using three different distributions.

i. The gamma distribution with density \( (\Gamma(t))^{-1}x^{t-1}e^{-x} \) denoted by \( \Gamma(t) \).

ii. The exponential distribution with density \( te^{-tx} \) denoted by \( \exp(t) \).

iii. The chi-square distribution with density \( \frac{1}{2^{n/2}}\Gamma\left(\frac{n}{2}\right)x^{n/2-1}e^{(-x/2)} \) denoted by \( \chi_n^2 \).

We generate 1000 pseudo-random samples of size \( n \) from each of the three alternative distributions considered. We then calculated each of the three test statistics and compared them to its respective critical values from Table (1). In this case we counted the number of rejections of the null hypothesis. We repeated this procedure for sample size \( n=10(10)40 \).

Results of power study are presented in Table (2) which show that the Anderson-Darling \( A^2 \) is general has a larger power than (superior to) both the Kolmogorov-Smirnov \( D \), and Cramer-Von Mises \( W^2 \) statistics for the alternatives presented. Neither \( D \) nor \( W^2 \) appears to be very powerful across this group of alternative distributions.
CONCLUSIONS

From the results of the critical points appears to be very good for the range of sample sizes \( n=10(10)100 \) at \( \alpha = 0.01, 0.05, 0.10, 0.15 \) and \( 0.20 \) significance levels for the sample sizes. And for \( n=10(10)40 \) at significance levels considered the Anderson-Darling \( A^2 \) statistic appears to be the best of EDF test statistics for testing the goodness of fit of a two parameter weibull distribution to a set of data because the larger power it displayed in the power studies. The Kolmogorov-Smirnov D tends to be the least powerful among the three EDF tests considered here. So it may advisable to use the Anderson-Darling \( A^2 \) and Cramer-Von Mises \( W^2 \) test statistics for testing the two parameter weibull distribution.

Table 1: Two weibull critical values for D, W^2 and A^2 when \( n=10(10)100 \).

<table>
<thead>
<tr>
<th>Sample size n</th>
<th>Test statistics</th>
<th>Significance levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>( A^2 )</td>
<td>0.3109</td>
</tr>
<tr>
<td></td>
<td>( W^2 )</td>
<td>0.9522</td>
</tr>
<tr>
<td>20</td>
<td>( A^2 )</td>
<td>0.3162</td>
</tr>
<tr>
<td></td>
<td>( W^2 )</td>
<td>0.9529</td>
</tr>
<tr>
<td>30</td>
<td>( A^2 )</td>
<td>0.3241</td>
</tr>
<tr>
<td></td>
<td>( W^2 )</td>
<td>0.9532</td>
</tr>
<tr>
<td>40</td>
<td>( A^2 )</td>
<td>0.3295</td>
</tr>
<tr>
<td></td>
<td>( W^2 )</td>
<td>0.9534</td>
</tr>
<tr>
<td>50</td>
<td>( A^2 )</td>
<td>0.3309</td>
</tr>
<tr>
<td></td>
<td>( W^2 )</td>
<td>0.9537</td>
</tr>
<tr>
<td>60</td>
<td>( A^2 )</td>
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</tr>
<tr>
<td></td>
<td>( W^2 )</td>
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</tr>
<tr>
<td>70</td>
<td>( A^2 )</td>
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</tr>
<tr>
<td></td>
<td>( W^2 )</td>
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<td>80</td>
<td>( A^2 )</td>
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</tr>
<tr>
<td></td>
<td>( W^2 )</td>
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<td></td>
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<td>100</td>
<td>( A^2 )</td>
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<td></td>
<td>( W^2 )</td>
<td>0.9556</td>
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Table -2: powers of tests for weibull distribution when n=10(10)40.

<table>
<thead>
<tr>
<th>Sample size n</th>
<th>Significance levels</th>
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<td></td>
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<td>D</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>0.0032</td>
</tr>
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<td></td>
<td>0.05</td>
<td>0.0048</td>
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<tr>
<td></td>
<td>0.10</td>
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</tr>
<tr>
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<td>0.20</td>
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</tr>
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<tr>
<td></td>
<td>0.10</td>
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REFERENCES


