The Effect of Mhd on Unsteady Flow of A Second Grade Fluid Film Over an Unsteady Stret Ching Sheet

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Abstract

The aim of this paper is the study of the influence of magnetic field on unsteady flow of the second-grade fluid with constant viscosity. The equations which controlled this type of fluid flow are complicated, so finding an analytical solution is not easy, because it is a system of partial differential equations.We obtained an expression for the velocity by using homotopy analysis method HAM.

It is found that the equations motion are controlled by many dimensionless parameter, namely magnetic field parameter M and material constant α, dimensionless film thickness β and unsteadiness parameter S.We have been studied the influence of all the physical parameters, that mentioned above on the velocity field, also a comparison study among unsteady flow and unsteady flow under the influence of the magnetic field had been done.This study is done through drawing about 75 graph by using the Mathematica package.

Keywords:Magnetic Field ; Unsteady Flow ; Second-grade Fluid With Constant Viscosity

تأثير الحقل المغناطيسي على الجريان اللامستقر لمائع من الرتبة الثانية تعلوه غشاوة لصفيحة مطاطية غير مستقرة

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الخلاصة

الهدف من هذا البحث هو دراسة تأثير الحقل المغناطيسي على جريان غير مستقر لمائع ثابت اللزوجة من الدرجة الثانية .ان المعادلات التي تحكم هذا النوع من المسائل تكون ذات طبيعة معقدة , وبالتالي إيجاد الحل التحليلي لها غير سهل وذلك لكونها نظام من المعادلات التفاضلية الجزئية .لقد حصلنا على تعبير للسرعة بأستخدام طريقة هوموتوبي التحليلية HAM .لقد تبين أن معادلة الحركة تحكمها بعض المعلمات اللابعدية , مثل معلمة الحقل المغناطيسي M والثوابت المادية α , β و s, قمنا بدراسة تأثير كل من

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الاعداد الفيزيائية المذكورة اعلاه على السرعة , كذلك قد تمت دراسة المقارنة بين الجريان اللامستقر والجريان اللامستقر تحت تأثير المجال المغناطيسي .هذه الدراسة قد تمت من خلال رسم حوالي 75 بيان بأستخدام البرنامج الجاهز Mathematica .

1. Introduction

Magneto fluid dynamics (MFD) is that branch of applied mathematics which deals with the flow of electrically conducting fluids in electric and magnetic fields. It unified in a common framework the electromagnetic and fluid-dynamic theories to yield a description of the concurrent effects of the magnetic field on the flow and the flow on the magnetic field.

The magneto hydro dynamic (MHD) phenomenon is characterized by an interaction between the hydrodynamic and boundary layer and the electromagnetic field. The studies of boundary layer flows of viscous and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy and chemical engineering, for example, in the extrusion of polymer sheet from a dye or in the drawing of plastic films. Such investigations of magneto hydro dynamic (MHD) flows are very important industrially and have applications in different areas of researches such as petroleum production and metallurgical processes,
it is now well known that in technological applications the non-Newtonian fluids are more appropriate than the Newtonian fluids. The non-Newtonian fluids finding increasing applications in industry such as the cooling of metallic plate in cooling bath, wire drawing, hot rolling etc. Glass blowing, fiber production, crystal growing and paper production also involves the flow due to a stretching surface. Many applications in industry has mentioned in [1-4].

In view of the differences between the non-Newtonian fluids and Newtonian fluids, several models of non-Newtonian fluids have been proposed, which cannot be described simply as of Newtonian fluids. Such fluids have received special status from the researchers in the field, as the second grade for which one can hope to obtain an analytic solution. Since the pioneering work of Sakiadis and Schlichting [5-7] shows various aspects of the stretching problem including Newtonian and non-Newtonian fluids which have been studied by several researchers.

Hang Xu. [8], used an analytic technique, namely the homotopy analysis method (HAM), to study the flow and heat transfer characteristics in an electrically conducting fluid near an isothermal sheet. The sheet is linearly stretched in the presence of a uniform free stream of constant velocity and temp-rature. The effects of free convection and internal heat generation or absorption are also considered. Within the framework of boundary layer approximations, he explicit, totally analytic and niformly valid solutions governed by a set of three fully coupled, highly non-linear equations are obtained.

Hayat and Sajid [9], studied the problem of laminar flow and heat transfer of a second grade fluid over a radially stretching sheet is considered. The axisymmetric flow of a second grade fluid is induced due to linear stretching of a sheet. The heat transfer analysis has been carried out for two heating processes, namely (i) with prescribed surface temperature (PST-case) and (ii) prescribed surface heat flux (PHF-case). Introducing the dimensionless quantities the governing partial differential equations are transformed to ordinary differential equations. The developed non-linear differential equations are solved analytically using (HAM).

Kayvan Sadeghy et.al. [10], in their paper the flow of an upper-convected Maxwell (UCM) fluid is studied theoretically above a rigid plate moving steadily in an otherwise quiescent fluid. It is assumed that the Reynolds number of the flow is high enough for boundary layer approximation to be valid. Assuming a laminar, two-dimensional flow above the plate, the concept of stream function coupled with the concept of similarity solution is utilized to reduce the governing equations into a single third-order ODE. It is concluded that the fluid's elasticity destroys similarity between velocity profiles; thus an attempt was made to find local similarity solutions. Three different methods will be used to solve the governing equation: (i) the perturbation method, (ii) the fourth-order Runge-Kutta method, and (iii) the finite-difference method. The velocity profiles obtained using the latter two methods are shown to be virtually the same at corresponding Deborah number. The velocity profiles obtained

using perturbation method, in addition to being different form those of the other two methods, and are dubious in that they imply some degree of reverse flow. The wall skin friction coefficient is predicted to decrease with an increase in the Deborah number for Sakiadis flow of a UCM fluid.

Z. Abbas et.al. [1] studied the flow problem in a thin liquid film of second grade fluid over an unsteady stretching surface is investigated. By means of suitable transf-rmations, the governing nonlinear partial differential equation has been reduced to the nonlinear ordinary differential equation. The developed nonlinear equation is solved analytically by using the homotopy analysis method (HAM). An expression for analytic solution is derived in the form of series. The convergence of the obtained series is shown explicitly through numerical computations. The effects of various parameters on the velocity components are shown through graphs and discussed. The values of the skin friction coefficient for different emerging parameters are also tabulated.

The effects of the various parameters of interest for the velocity are pointed out.

The homotopy analysis method (HAM) can be used to solve the nonlinear problems, this method is a very powerful technique developed by Liao [11-13] to obtain the expression for velocity fields and characteristic by:

- Are valid strongly nonlinear problems even if a given nonlinear problems does not contain any small /large parameters.
- Provide us with a convenient way to adjust the convergence region and rate of approximate.
- Provide us with a freedom to use different basis functions to approximate a nonlinear problem.

Our problem has many applications in different sciences for example:

- Oil supply system.
- In medical science Magnetic Resonance Imaging .

2. A mathematical Formulation

Let (*x, y, z*) denote the Cartesian coordinates, tangential and axial directions, V = (*u, v, w*) the velocity components in these directions, and *t* is the time.

Consider the MHD unsteady flow of a second-grade fluid in a pipe the fluid is electrically conducting in the presence of an applied magnetic field β_0. The electric and induced magnetic fields are neglected. The viscosity of the fluid is constant.

It is well known that the second grade fluid has Cauchy stress tensor T of the following form [1]:

$$T = -PI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2,$$

where *P* is the hydrostatic pressure, I is the identity tensor, μ is the dynamic viscosity, $\alpha_i \ (i = 1,2)$ are material constants.

Moreover, thermodynamics imposes the follo-wing constrains, [14]:

$$\mu \geq 0 \ , \quad \alpha_1 \geq 0 \ , \quad \alpha_1 + \alpha_2 = 0 .$$

The Rivlin-Ericksen tensors are given by, [14]:

$$A_1 = (\nabla \vec{v}) + (\nabla \vec{v})^T$$

and

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1}(\nabla \vec{v}) + (\nabla \vec{v})^T A_{n-1}$$

Where $\frac{DA_n}{Dt} = \frac{\partial A_n}{\partial t} + (\vec{v} \cdot \nabla) A_n$

We need in our problem to calculate A_n when n=2, thus we start with:

$$\frac{D A_1}{Dt} = \frac{\partial A_1}{\partial t} + (\vec{v} \cdot \nabla) A_1$$

We should be noted that the equations which govern the unsteady boundary layer were flow and satisfy $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, we get:

$$divT = \nabla \cdot T = \mu u_{yy} + \alpha_1 [u_{yty} + uu_{yxy} + vu_{yyy} + u_x u_{yy} + u_y u_{yx}] \quad .. (1)$$

The momentum equation [15] is given by:

$$\rho[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}] = div\, T + J \times B \qquad .. (2)$$

Where ρ is density, *divT* is the divergence of stress tensor, and the Lorentz force is given by using Ohm's low [16]:

$$J = \sigma(E + V \times B)$$

Where E is electricity strength, V is the velocity vector and B is the applied magnetic field since there is no electric strength then E=0 we have:

$$J = \sigma(V \times B)$$

$$V \times B = \begin{bmatrix} i & j & k \\ u & v & 0 \\ 0 & B_0 & 0 \end{bmatrix} = uB_0\bar{k}$$

From which

$$J \times B = \begin{bmatrix} i & j & k \\ 0 & 0 & \sigma uB_o \\ 0 & B_0 & 0 \end{bmatrix} = -\sigma B_0^2 u$$

Substituting the divergence of stress tensor of the problem 1 in the momentum equation 2 we have:

$$\rho[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}] = \mu u_{yy} + \alpha_1[u_{yty} + uu_{yxy} + vu_{yyy} + u_x u_{yy} + u_y u_{yx}] - \sigma\beta_0^2 u$$

Divided the above equation by ρ we have:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon u_{yy} + \frac{\alpha_1}{\rho}[u_{yty} + uu_{yxy} + vu_{yyy} + u_x u_{yy} + u_y u_{yx}] - \frac{\sigma\beta_0^2}{\rho}u \quad .. \quad (3)$$

Where $\upsilon = \dfrac{\mu}{\rho}$

And the corresponds conditions are:

$u = U$, $v = 0$ at $y = 0$,

$\dfrac{\partial u}{\partial y} = 0, v = \dfrac{dh}{dt}$ at $y = h$

in the last equation, unsteady state, also if there is no magnetic field, we obtain the corresponding equation as given [17].

We can write down the momentum equation in non-dimensional form through using scaling and order of magnitude analysis.

Following Williams and Rhyne [5], we use the following new similarity transformations:

$$u = \frac{bx}{1-at}f'(\eta) \ , v = -\sqrt{\frac{b\upsilon}{1-at}}f(\eta) \ ,$$

$$\eta = \sqrt{\frac{b}{\upsilon}}(1-at)^{-\frac{1}{2}}y$$

The substituting of these quantities into the momentum equation (3) gives:

$$\frac{abx}{2(1-at)^2}\eta f'' + \frac{abx}{(1-at)^2}f' + \frac{b^2x}{(1-at)^2}f'^2 - \frac{b^2x}{(1-at)^2}ff'' = \frac{b^2x}{(1-at)^2}f''' + \frac{\alpha_1}{\rho}[\frac{2ab^2x}{\upsilon(1-at)^3}f''' + \frac{ab^2x}{2\upsilon(1-at)^3}\eta f'''' + \frac{b^3x}{\upsilon(1-at)^3}ff''' + \frac{b^3x}{\upsilon(1-at)^3}ff''' + \frac{b^3x}{\upsilon(1-at)^3}f''^2 - \frac{b^3x}{\upsilon(1-at)^3}ff''''] - \frac{\sigma B_0^2}{\rho}\frac{bx}{1-at}f'$$

The Multiplication of the above equation by $\dfrac{(1-at)^2}{b^2x}$ gives:

$$\frac{a}{2b}\eta f'' + \frac{a}{b}f' + f'^2 - ff'' = f''' + \frac{\alpha_1}{\rho}[\frac{2a}{\upsilon(1-at)}f''' + \frac{a}{2\upsilon(1-at)}\eta f'''' + \frac{2b}{\upsilon(1-at)}ff''' + \frac{b}{\upsilon(1-at)}f''^2 - \frac{b}{\upsilon(1-at)}ff''''] - \frac{\sigma B_0^2}{\rho}\frac{1-at}{b}f'$$

We select:

$$s = \frac{a}{b} \ , \ \alpha = \frac{b\alpha_1}{\mu(1-at)} \ , \ M = \frac{\sigma B_0^2}{\rho}\frac{1-at}{b}$$

Where *M* is the magnetic number (or *M* is the MHD parameter), *s* is unsteadiness parameter and α is material constant.

Then the last equation becomes:

$$f''' - f'^2 + ff'' - s(f' + \frac{1}{2}\eta f'') + \alpha[2ff''' +$$

$$s(2f''' + \frac{1}{2}\eta f''') + f''^2 - ff''''] - Mf' = 0 \quad .. \quad (4)$$

$$f = 0 \ , \quad f' = 1 \quad \text{at } \eta = 0,$$

$$f = \frac{1}{2}\beta S \ , \ f'' = 0 \quad \text{at } \eta = \beta .$$

We will solve Eqs. (4) analytically using HAM in the next section.

3. Analytical Solution

In order to solve the Eq. (4) by (HAM), we select:

$$f_0(\eta) = \eta - \frac{2 - S}{4\beta^2}(3\beta - \eta)\eta^2 \quad .. (5)$$

as initial approximation of f, which satisfy the linear operator and corresponding boundary condition. We use the method of higher order differential mapping [1] to choose the auxiliary linear operator $\int$ which is defined by:

$$\int (f) = \frac{d^4 f}{d\eta^4} \quad \text{, such that}$$

$$\int (c_1\eta^3 + c_2\eta^2 + c_3\eta + c_4) = 0$$

where c_1, c_2, c_3 and c_4 are arbitrary constants.

Let p ∈ [0, 1] denote an embedding parameter and h a non-zero auxiliary parameter. We construct the following zeroth-order deformation problem

(1-p) $\int$ [f*(η; p) – f_0(η; p)] = ph $\mathcal{N}$1 [f*(η; p)] ..(6)

f*(0; p) = 0, $f'^*(0; p) = 1$,

$$f^*(\beta; p) = \frac{S\beta}{2}, f''^*(\beta; p) = 0 .$$

Where we define a nonlinear operator

$$N_1[f^*(\eta; p)] = \frac{\partial^3 f^*(\eta; p)}{\partial\eta^3} - \left(\frac{\partial f^*(\eta; p)}{\partial\eta}\right)^2 +$$

$$f^*(\eta; p)\frac{\partial^2 f^*(\eta; p)}{\partial\eta^2} - s\left(\frac{\partial f^*(\eta; p)}{\partial\eta} + \frac{1}{2}\eta\frac{\partial^2 f^*(\eta; p)}{\partial\eta^2}\right)$$

$$+\alpha[2\frac{\partial f^*(\eta; p)}{\partial\eta}\frac{\partial^3 f^*(\eta; p)}{\partial\eta^3} + s\left(2\frac{\partial^3 f^*(\eta; p)}{\partial\eta^3} + \frac{1}{2}\eta\frac{\partial^4 f^*(\eta; p)}{\partial\eta^4}\right)$$

$$+\left(\frac{\partial^2 f^*(\eta; p)}{\partial\eta^2}\right)^2 - f^*(\eta; p)\frac{\partial^4 f^*(\eta; p)}{\partial\eta^4}] - M\frac{\partial f^*(\eta; p)}{\partial\eta} \quad (7)$$

For p = 0 and p = 1, we respectively have:

f*(η; 0) = f_0(η),f*(η; 1) = f(η; p) .. (8) when p increases form 0 to 1, f*(η; p) vary form f_0(η) to f(η; p).

Using Taylor's theorem and Eqs. (8), we can write:

$$f^*(\eta; p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m \quad .. (9)$$

Where $f_m(\eta) = \frac{1}{m!}\frac{\partial^m f^*(\eta; p)}{\partial p^m}\Big|_{p=0}$

The convergence of the series (9) depends upon h. We choose h in such a way that the series (9) is convergence at p = 1; then due to equation. 8 we have:

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)$$

Differentiating m times the zero-order deformation equation.6 with respect to p and then dividing it by m! and finally setting p = 0, we get the following high-order deformation equation:

$\int$ [f_m(η) – x_m f_{m-1}(η)] = h R_m (f_{m-1}(η)]

$$f_m(0) = f_m'(0) = f_m(\beta) = f_m''(\beta) = 0$$

where $x_m = \begin{cases} 0, m \le 1 \\ 1, m > 1 \end{cases}$ and

$$R_m(f_{m-1}, \eta) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N_1[f^*(\eta; p)]}{\partial p^{m-1}}\Bigg|_{p=0} \quad (10)$$

Substituting the equation. 7 in to equation. 10 we have:

$$R_m[f_{m-1}(\eta)] = \frac{\partial^3 f_{m-1}}{\partial \eta^3} - S\left(\frac{\partial f_{m-1}}{\partial \eta} + \frac{1}{2}\eta \frac{\partial^2 f_{m-1}}{\partial \eta^2}\right)$$

$$+\alpha S\left(2\frac{\partial^3 f_{m-1}}{\partial \eta^3} + \frac{1}{2}\eta \frac{\partial^4 f_{m-1}}{\partial \eta^4}\right) - M\frac{\partial f_{m-1}}{\partial \eta} +$$

$$\sum_{k=0}^{m-1}\left[f_{m-1-k}\frac{\partial^2 f_k}{\partial \eta^2} - \frac{\partial f_{m-1-k}}{\partial \eta}\frac{\partial f_k}{\partial \eta} +\right.$$

$$\alpha\left(2f_{m-1-k}\frac{\partial^3 f_k}{\partial \eta^3} + \frac{\partial^2 f_{m-1-k}}{\partial \eta^2}\frac{\partial^2 f_k}{\partial \eta^2} -\right.$$

$$\left.\left. f_{m-1-k}\frac{\partial^4 f_k}{\partial \eta^4}\right)\right]$$

For the solution of the high-order problem, we use the symbolic computation software MATHEMATICA up to first few order of approximation.

It is found that the general solution of momentum equation is given by:

$$f_m(\eta) = \sum_{n=0}^{5m+3} \eta^n a_{m,n} \quad .. (11)$$

Where $a_{m,n}$ are the coefficient of $f_m(\eta)$ for $m \ge 1$.

Now, we try to find the above coefficient:

1. The initial approximation $f_0(\eta)$ defined by equation. 5 has the same structure as equation. 11.

If we assume that the first (m-1) solutions $f_k(\eta)$ (k = 0, 1, 2,…, m-1) have the same structure as (11), then we want to prove that $f_m(\eta)$ has the same structure as (11) to prove this, we have form equation. 11:

$$\frac{\partial f_m}{\partial \eta} = \sum_{n=1}^{5m+3} n a_{m,n} \eta^{n-1} = \sum_{n=0}^{5m+3} (n+1) a_{m,n+1} \eta^n$$

$$= \sum_{n=0}^{5m+3} b_{m,n} \eta^n \quad .. (12)$$

By the same way, we have:

$$\frac{\partial^2 f_m}{\partial \eta^2} = \sum_{n=0}^{5m+3} c_{m,n} \eta^n \quad .. (13)$$

$$\frac{\partial^3 f_m}{\partial \eta^3} = \sum_{n=0}^{5m+3} d_{m,n} \eta^n \quad .. (14)$$

$$\frac{\partial^4 f_m}{\partial \eta^4} = \sum_{n=0}^{5m+3} e_{m,n} \eta^n \quad .. (15)$$

Where

$$b_{m,n} = (n+1) a_{m,n+1}$$

$$c_{m,n} = (n+1) b_{m,n+1}$$

$$d_{m,n} = (n+1) c_{m,n+1}$$

$$e_{m,n} = (n+1) d_{m,n+1}$$

Now, from equations. 11 and 13, we have:

$$\sum_{k=0}^{m-1} f_{m-1-k}\frac{\partial^2 f_k}{\partial \eta^2} = \sum_{k=0}^{m-1} \sum_{n=0}^{5(m-1-k)+3} a_{m-1-k,n} \eta^n \sum_{n=0}^{5k+3} c_{k,n} \eta^n$$

$$= \sum_{k=0}^{m-1} \sum_{j=\max\{0, n-5m+5k+2\}}^{\min\{n, 5k+3\}} c_{k,j} a_{m-1-k,n-j} \eta^{2n}$$

From above equation, we have:

$$\sum_{k=0}^{m-1} f_{m-1-k}\frac{\partial^2 f_k}{\partial \eta^2} = \alpha_{m,n} \eta^{2n} \quad .. (16)$$

Where

$$\alpha_{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max\{0,n-5m+5k+2\}}^{\min\{n,5k+3\}} c_{k,j}\, a_{m-1-k,n-j}$$

$$\sum_{k=0}^{m-1} \frac{\partial f_{m-1-k}}{\partial \eta} \frac{\partial f_k}{\partial \eta} = \beta_{m,n} \eta^{2n} \quad .. (17)$$

$$\sum_{k=0}^{m-1} f_{m-1-k} \frac{\partial^3 f_k}{\partial \eta^3} = \delta_{m,n} \eta^{2n} \quad .. (18)$$

$$\sum_{k=0}^{m-1} \frac{\partial^2 f_{m-1-k}}{\partial \eta^2} \frac{\partial^2 f_k}{\partial \eta^2} = \gamma_{m,n} \eta^{2n} \quad .. (19)$$

$$\sum_{k=0}^{m-1} f_{m-1-k} \frac{\partial^4 f_k}{\partial \eta^4} = \omega_{m,n} \eta^{2n} \quad .. (20)$$

Where

$$\beta_{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max\{0,n-5m+5k+2\}}^{\min\{n,5k+3\}} b_{k,j}\, b_{m-1-k,n-j}$$

$$\delta_{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max\{0,n-5m+5k+2\}}^{\min\{n,5k+3\}} d_{k,j}\, a_{m-1-k,n-j}$$

$$\gamma_{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max\{0,n-5m+5k+2\}}^{\min\{n,5k+3\}} c_{k,j}\, c_{m-1-k,n-j}$$

$$\omega_{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max\{0,n-5m+5k+2\}}^{\min\{n,5k+3\}} e_{k,j}\, a_{m-1-k,n-j}$$

Substituting equations.12, 13, 14, 15,16,17, 18 and 19 into equation. 20, we have:

$$R_m = \sum_{n=0}^{5m-2} d_{m-1,n} \eta^n - s\{\sum_{n=0}^{5m-2} b_{m-1,n} \eta^n + \frac{1}{2} \sum_{n=0}^{5m-2} c_{m-1,n} \eta^n\} + \alpha s\{2 \sum_{n=0}^{5m-2} d_{m-1,n} \eta^n + \frac{1}{2} \sum_{n=0}^{5m-2} e_{m-1,n} \eta^n\} - M \sum_{n=0}^{5m-2} b_{m-1,n} \eta^n + \{\alpha_{m,n} \eta^{2n} - \beta_{m,n} \eta^{2n} + \alpha(2\delta_{m,n} \eta^{2n} + \gamma_{m,n} \eta^{2n} - \omega_{m,n} \eta^{2n})\}$$

The above equation is the right hand side of the following equation

$$\int [f_m(\eta) - x_m f_{m-1}(\eta)] = h R_m(\eta)$$

By solving the last equation, we found that the general solution is given by:

$$f_m(\eta) = \chi_m \sum_{n=0}^{5m-2} a_{m-1,n} \eta^n + h[\sum_{n=0}^{5m-2} \frac{d_{m-1,n}}{(n+1)(n+2)(n+3)(n+4)} \eta^{n+4} - s\{\sum_{n=0}^{5m-2} \frac{b_{m-1,n}}{(n+1)(n+2)(n+3)(n+4)} \eta^{n+4} + \frac{1}{2} \sum_{n=0}^{5m-2} \frac{c_{m-1,n}}{(n+2)(n+3)(n+4)(n+5)} \eta^{n+5}\} + \alpha s\{2 \sum_{n=0}^{5m-2} \frac{d_{m-1,n}}{(n+1)(n+2)(n+3)(n+4)} \eta^{n+4} + \frac{1}{2} \sum_{n=0}^{5m-2} \frac{e_{m-1,n}}{(n+2)(n+3)(n+4)(n+5)} \eta^{n+5}\} + \frac{\beta^{2n+2}(\alpha_{m,n} - \beta_{m,n} + \alpha(\gamma_{m,n} + \delta_{m,n} - \omega_{m,n}))}{(2n+1)(2n+2)}] - s(\sum_{n=0}^{5m-2} \frac{\beta^{n+2} a_{m-1,n+1}}{n+2} + \frac{1}{2} \sum_{n=0}^{5m-2} \frac{(n+1)\beta^{n+3} a_{m-1,n+2}}{n+3}) + \sum_{n=0}^{5m-2} (n+3)\beta^{n+2} a_{m-1,n+3} + \alpha s(\sum_{n=0}^{5m-2} (n+3)\beta^{n+2} a_{m-1,n+3} + \frac{1}{2} \sum_{n=0}^{5m-2} (n+1)(n+4)\beta^{n+3} a_{m-1,n+4})]] + \frac{1}{12\beta^4}(-\beta^3(\chi_m \sum_{n=0}^{5m-2} n(n-1)\beta^{n+2} a_{m-1,n} - M \quad . \quad (21)$$

Where

$$c_1 = \frac{1}{6\beta}[\chi_m \sum_{n=0}^{5m-2} n(n-1)\beta^{n-2} a_{m-1,n} - M \sum_{n=0}^{5m-2} \frac{\beta^{n+2} a_{m-1,n+1}}{n+2}$$

$$\sum_{n=0}^{5m-2}\frac{\beta^{n+3}a_{m-1,n+1}}{n+2}+h(\frac{\beta^{2n+2}(\alpha_{m,n}-\beta_{m,n}+\alpha(\gamma_{m,n}+\delta_{m,n}-\omega_{m,n}))}{(2n+1)(2n+2)}$$

$$-s(\sum_{n=0}^{5m-2}\frac{\beta^{n+2}a_{m-1,n+2}}{n+2}+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+3}a_{m-1,n+2}}{n+3})+\sum_{n=0}^{5m-2}(n+3)\beta^{n+2}a_{m-1,n+3}+s\alpha(2\sum_{n=0}^{5m-2}(n+3)\beta^{n+2}\alpha_{m-1,n+3}$$

$$+\frac{1}{2}\sum_{n=0}^{5m-2}(n+1)(n+4)\beta^{n+3}\alpha_{m-1,n+4})))+$$

$$6\beta(\chi_m\sum_{n=0}^{5m-2}\beta^n a_{m-1,n}-M\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+1}}{(n+2)(n+3)(n+4)}+$$

$$h(\frac{\beta^{2n+4}(\alpha_{m,n}-\beta_{m,n}+\alpha(\gamma_{m,n}+\delta_{m,n}-\omega_{m,n}))}{(2n+1)(2n+2)(2n+3)(2n+4)}-s(\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+1}}{(n+2)(n+3)(n+4)}+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+5}a_{m-1,n+2}}{(n+3)(n+4)(n+5)})$$

$$+\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+3}}{(n+4)}+s\alpha(2\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+3}}{(n+4)}$$

$$+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+5}a_{m-1,n+4}}{(n+5)})))).$$

$$c_2=-\frac{1}{4\beta^3}(-\beta^3(\chi_m\sum_{n=0}^{5m-2}n(n-1)\beta^{n-2}a_{m-1,n}-M\sum_{n=0}^{5m-2}\frac{\beta^{n+2}a_{m-1,n+1}}{n+2}$$

$$+h(\frac{\beta^{2n+2}(\alpha_{m,n}-\beta_{m,n}+\alpha(\gamma_{m,n}+\delta_{m,n}-\omega_{m,n}))}{(2n+1)(2n+2)}$$

$$-s(\sum_{n=0}^{5m-2}\frac{\beta^{n+2}a_{m-1,n+1}}{n+2}+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+3}a_{m-1,n+2}}{n+3})+$$

$$\sum_{n=0}^{5m-2}(n+3)\beta^{n+2}a_{m-1,n+3}+s\alpha(2\sum_{n=0}^{5m-2}(n+3)\beta^{n+2}a_{m-1,n+3}+$$

$$\frac{1}{2}\sum_{n=0}^{5m-2}(n+1)(n+4)\beta^{n+3}a_{m-1,n+4})))+6\beta(\chi_m\sum_{n=0}^{5m-2}\beta^n a_{m-1,n}-M\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+1}}{(n+2)(n+3)(n+4)}+h(\frac{\beta^{2n+4}(\alpha_{m,n}-\beta_{m,n}+\alpha(\gamma_{m,n}+\delta_{m,n}-\omega_{m,n}))}{(2n+1)(2n+2)(2n+3)(2n+4)}-$$

$$s(\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+1}}{(n+2)(n+3)(n+4)}+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+5}a_{m-1,n+2}}{(n+3)(n+4)(n+5)})+$$

$$\sum_{n=0}^{5m-2}\frac{\beta^{n+4}a_{m-1,n+3}}{(n+4)}+$$

$$s\alpha(2\sum_{n=0}^{5m-2}\frac{\beta^{n+4}\alpha_{m-1,n+3}}{n+4}+\frac{1}{2}\sum_{n=0}^{5m-2}\frac{(n+1)\beta^{n+5}\alpha_{m-1,n+4}}{n+5})))).$$

$c_3 = 0$ for all $m > 0$ and $c_4 = 0$ for all $m \geq 0$.

Finally, to obtain the coefficient $a_{m,n}$ of the function $f_m(\eta)$, substitute the c_1 and c_2 values into equation (21) and equating the equal power of η.

We obtain in fact the following explicit, totally analytic solution of the present flow.

$$f(\eta)=\sum_{m=0}^{\infty}f_m(\eta)$$

$$= \lim_{M \to \infty} \left[\sum_{n=1}^{5M+3} \left(\sum_{m=n-1}^{5M} a_{m,n} \eta^n \right) \right] \quad (22)$$

4. Convergence of the solution

It is noticed that the explicit, analytical expression (11) contains auxiliary parameter h. As pointed out by Liao [12], the convergence region and rate of approximations given by homotopy analysis method are strongly dependent upon h. figure 1 portray the h-curve of the velocity profile. The range for admissible value of h for the velocity is $-1.3 \le h \le -0.2$. We see that series given by equations. (22) converges in the whole region of η when $h = -0.8$. This value of h lie in the admissible range of hf '"(0)

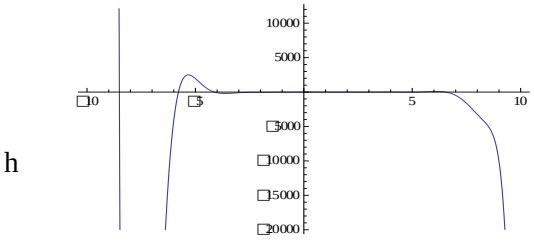

Figure 1- H-curve for velocity at second-order approximation

5. Results and Discussion

we have studied the effects of MHD pressure "M", material constant "α", dimensionless film thickness "β" and unsteadiness parameter "S".

As MHD parameter increases, there is small decreasing in the velocity range. See figure 2.

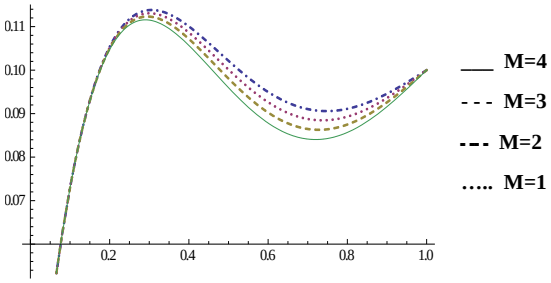

Figure 2- Together, S=0.2, β=1, α=0.2, h=0,8.

As material constant "α" increases, there is a decreasing in the velocity range. See figure 3.

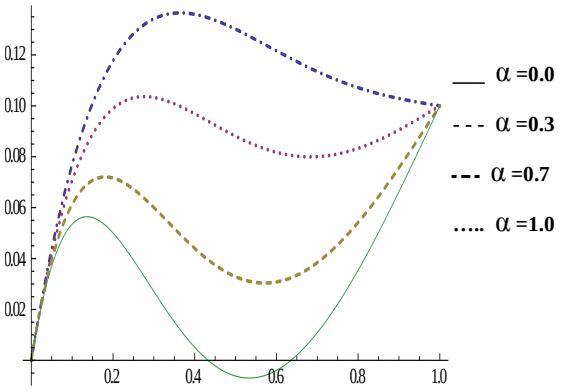

Figure 3- Together, M=1,S=0.2,β=1,h=-0,8.

As dimensionless film thickness "β" increases, there is an increasing in the velocity range. See figure 4.

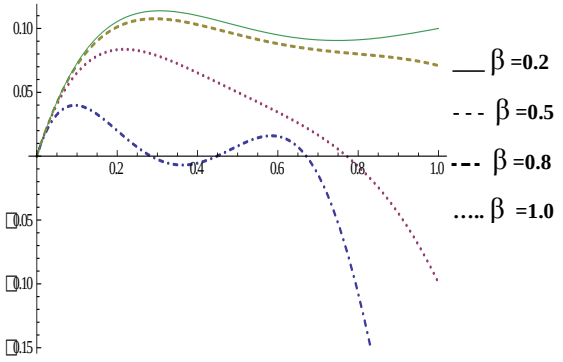

Figure 4

As unsteadiness parameter "s" increases, there is an increasing in the velocity range. See figure 5.

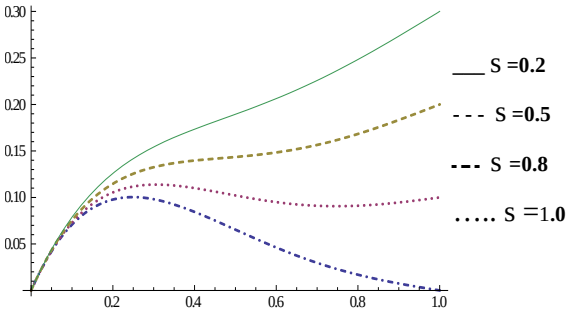

Figure 5

The comparison will be given in following table 1.

Table 1-Velocity Comparison between unsteady flow and unsteady flow under the influence of the magnetic field for $\alpha = 0.2$, $\beta = 0.2$, $S = 0.2$, $h = -0.8$, $\eta = 0.5$.

M	Unsteady flow with out magnetic field	unsteady flow under the influence of the magnetic field
1	0.104122	0.1021400
3	0.104122	0.1001180
7	0.104122	0.0980559
10	0.104122	0.0959555

From table 1 we can see that, as expected, the velocity in unsteady flow is greater than the velocity in unsteady flow under the influence of the magnetic field.

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