On β –I– Continuous Mappings in Ideal Bitopological Spaces

[1] Audie Sabrie and [2] Zahir D. Al–Nafie

[1] Collage of Basic Education, University of Babylon.

[2] Collage of Education Ibn– Hayan, University of Baghdad

1.introduction

The notion of ideal topological introduced and studied by Kuratowski[9],in 1992,Jankovic and Hamlett[8] introduced the concept of I–open sets in topological spaces ,Abd EL–Monsef et al.[1] further investigated I–open sets and I–continuity in 1999, Abd EL–Monsef et al.[2]introduced the notion of almost–I–open sets and almost–I–continuity, Hatir and Noiri[6] have introduced the notion of $\beta - I - open$ sets Recently, M. Galdas, S. Jafari and N. Rajesh[4] introduced the notion of pre–I–open sets in ideal bitopological space, Hatir and Noiri[6] have introduced the notion of these functions .

2. Preliminaries

Throughout the present paper, spaces always mean a bitopological spaces, the closure and the interior of any sub set A of X will be denoted by $\tau_i - cl(A)$,and $\tau_i - \text{int}(A)$ respectively $i, j \in \{1,2\}$, An ideal is defined as a non–empty collection I of sub sets of X satisfying (i) if A$\in$I, and B$\subset$ A, then B$\in$I; (ii) A$\in$I and B$\in$I then A$\cup$B$\in$I.

Let (X,τ) be a topological space and I an ideal of sub sets of X ,An ideal topological space is the triple (X,τ, I),For a sub set A of X, A* (I) ={x$\in$ X : U$\cap$A$\notin$I for each neighborhood U of x }is called the local function [7] of A with respect to τ and for easy will written by A*. it is well known that Cl* (A)= (A$\cup$ A*) .

Definition 2.1

For $i, j \in \{1,2\}$, $i \neq j$, a sub set of an ideal bitopological space (X, τ_1, τ_2, I) is said to be

i) $ij - I - openn$ [4] if. A$\subset$ τ_i – (A $_j$ *)

ii) $ij - almost - I - open$,if A$\subset$ τ_i –Cl (τ_j – Int (A*),

iii) $ij-\beta-I-open$,if A$\subset$ τ_i –Cl (τ_j – Int (τ_i – Cl* (A))).

iv) $ij-pre-I-open$ [4] if A$\subset$ τ_i – Int (τ_j – Cl* (A))).

v) $ij-\beta-open$,if A$\subset$ τ_i –Cl (τ_j – Int (τ_i – Cl (A))).

Remark 2.2

The family of all $ij-I-openn$ (resp. $ij-almost-I-open$, $ij-\beta-I-open$, $ij-pre-I-open$) sets are defined by $IO(X)$ (resp. $AIO(X), \beta IO(X), PIO(X)$) .

Proposition 2.3

Every $ij-almost-I-open$ is $ij-\beta-I-open$.

Proof:

let (X, τ_1, τ_2, I) be an ideal bitopological space and A an $ij-almost-I-open$ set of X ,then A$\subset$ τ_i –Cl (τ_j – Int (A*)$\subset$ τ_i –Cl (τ_j – Int (A*$\cup$A)) = τ_i –Cl (τ_j – Int (τ_i – Cl* (A))),there for A is $ij-\beta-I-open$.

The converse of this proposition is is not true as the following example Example:

Example 2.4

let X={1,2,3} and τ_1 = τ_2 ={X, ϕ ,{1} } and I={ ϕ ,{1} },then {1} is $ij-\beta-I-open$ but not $ij-almost-I-open$ set.

Proposition 2.5

i) Every $ij-\beta-I-open$ set is $ij-\beta-open$,and the converse is not true.

ii) Every open set is $ij-\beta-I-open$,and the converse is not true.

Example 2.6

let X={a,b,c}and τ_1 ={X, ϕ ,{a},{b},{a,b} }, τ_2 ={X, ϕ } , I={ ϕ ,{c} }

then {b,c} is $ij-\beta-I-open$ but not open set and {a,c} is $ij-\beta-open$ but it is not $ij-\beta-I-open$.

Proposition 2.7

i) Every $ij-I-openn$ set is $ij-pre-I-open$. and the converse is not true[4].

ii) Every $ij-pre-I-open$ set is $ij-\beta-I-open$

Theorem 2.8

A sub set A of an ideal bitoplogical space (X,τ_1,τ_2,I) is $ij-\beta-I-open$ if, and only if τ_i – Cl (A)= τ_i –Cl (τ_j – Int (τ_i – Cl* (A))).

Proof 2.9

Let A be a $ij-\beta-I-open$ set ,then A$\subset$ τ_i –Cl (τ_j – Int (τ_i – Cl* (A))),hence τ_i – Cl (A)$\subset$ τ_i –Cl (τ_j – Int (τ_i – Cl* (A)))$\subset$ τ_i –Cl (τ_j – Int (τ_i – Cl (A))) $\subset$ τ_i – Cl (A),therefore τ_i – Cl (A)= τ_i –Cl (τ_j – Int (τ_i – Cl* (A))).

Proposition 2.10 [4]

Let A be a sub set of an ideal bitoplogical space (X,τ_1,τ_2,I) and A is an $ij-pre-I-open$ set then τ_i – Cl (A)= τ_i –Cl (τ_j – Int (τ_i – Cl* (A))).

Definition 2.11

A sub set F of an ideal bitoplogical space (X,τ_1,τ_2,I) is said to be $ij-\beta-I-closed$ if it is complement is $ij-\beta-I-open$.

Theorem 2.12

A sub set of an ideal bitoplogical space (X,τ_1,τ_2,I) is said to be $ij-\beta-I-closed$ if, and only if τ_i –Int (τ_j – Cl (τ_i – Int* (A))) $\subset$ A.

Proof:

let F be a $ij-\beta-I-closed$ in (X,τ_1,τ_2,I) ,then X/F is $ij-\beta-I-open$ and hence X/F $\subset\tau_i$ –Cl (τ_j – Int (τ_i – Cl* (X/F))) = $X/$ τ_i –Int (τ_j – Cl (τ_i – Int* (F)).

Therefore we have τ_i –Int (τ_j – Cl (τ_i – Int* (F)))$\subset F$.

Conversely:

Let τ_i –Int (τ_j – Cl (τ_i – Int* (F)))$\subset F$,then $X/F\subset\tau_i$ –Cl (τ_j – Int (τ_i – Cl* (X/F))) and hence X/F is $ij-\beta-I-open$ therefore F is $ij-\beta-I-closed$.

Theorem 2.13

let F be any $ij-\beta-I-closed$ set in (X,τ_1,τ_2,I) then τ_i –Int (τ_j – Cl* (τ_i – Int (F)))$\subset F$

Proof:

let F be any $ij-\beta-I-closed$ set in (X,τ_1,τ_2,I) then

τ_i –Int (τ_j – Cl* (τ_i – Int (F)))$\subset\tau_i$ –Int (τ_j – Cl* (τ_i – Int* (F)))$\subset\tau_i$ –Int (τ_j – Cl (τ_i – Int* (F)))and by theorem () above we obtain τ_i –Int (τ_j – Cl* (τ_i – Int (F)))$\subset F$.

3. Pair wise $-ij-\beta-I$ and pair wise $ij-pre-I-$ continuous

Definition 3.1

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2)$ is said to be $ij-\beta-I-$ continuous (resp. $ij-almost-I-$continuous, $ij-I-$continuous, $ij-\beta-$continuous)if the inverse image of every ρ_i-open set in (Y,ρ_1,ρ_2) is $ij-\beta-I-open$ (resp. $ij-almost-I-open$, $ij-I-openn$, $ij-\beta-open$) in (X,τ_1,τ_2,I) where $i\neq j$ and $i,j=1,2$.

Definition 3.2 [4]

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2)$ is said to be $ij-pre-I-$ continuous

if the inverse image of every ρ_i-open in (Y,ρ_1,ρ_2) is $ij-pre-I-openn$ set in (X,τ_1,τ_2,I) where $i\neq j$ and $i,j=1,2$.

Proposition3.3 [4]

Every $ij-I-$continuous function is $ij-pre-I-$ continuous.

Proof: see[4].

Remark: the converse of proposition[ ] above need not be true by the following example

Example 3.4

Let X={1,2,3} and $\tau_1=\tau_2=\{\phi,X,\{2\}\}$ and $I=\{\phi,\{2\}\}$,then the identity function $f:(X,\tau_1,\tau_2,I)\to(X,\tau_1,\tau_2)$ is $ij-pre-I-$ continuous but not $ij-I-$continuous.

Theorem 3.5

For a function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2)$,the following statement are equivalent:

i) f is $ij-\beta-I-$ continuous ,

ii) for each $x\in X$ and each $V\in\rho_i$ containing f (x),there exist $U\in\beta IO(X)$ containing x such that $f(U)\subset V$,and $i=1$ or 2,

iii) $f^{-1}(V)$ is $ij-\beta-I-closed$,for each closed set V in Y .

Proposition 3.6

every $ij-almost-I-$continuous function is $ij-\beta-I-$continuous.

Proof:

let $f:(X,\tau_1,\tau_2,I)\rightarrow(Y,\rho_1,\rho_2)$ be $ij-almost-I-$continuous and let V be ρ_i-open set in (Y,ρ_1,ρ_2) then $f^{-1}(V)$ is $ij-almost-I-open$ in (X,τ_1,τ_2,I) where $i\neq j$ and $i,j=1,2$, since every $ij-almost-I-open$ is $ij-\beta-I-open$ set then f is $ij-\beta-I-$continuous.

Proposition 3.7

every $ij-\beta-I-$ continuous function is $ij-\beta-$continuous.

Proof:

let $f:(X,\tau_1,\tau_2,I)\rightarrow(Y,\rho_1,\rho_2)$ be $ij-\beta-I-$continuous and let V be ρ_i-open set in (Y,ρ_1,ρ_2) then $f^{-1}(V)$ is $ij-\beta-I-open$ in (X,τ_1,τ_2,I) where $i\neq j$ and

$i,j=1,2$, since every $ij-\beta-I-open$ is $ij-\beta-open$ set then f is $ij-\beta-$continuous.

Proposition 3.8

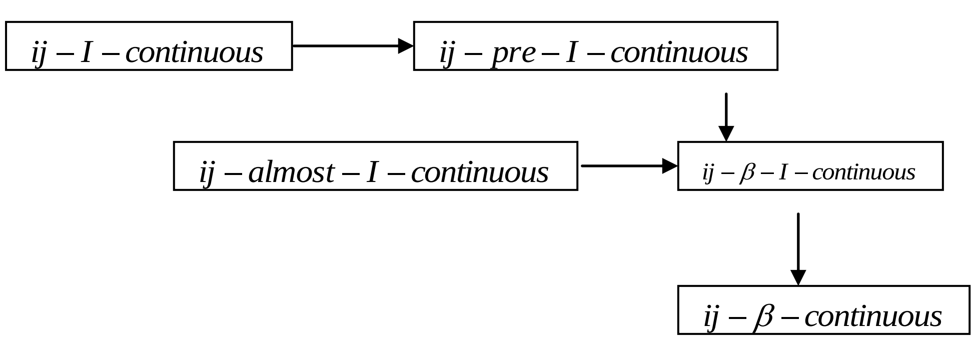
every $ij-pre-I-$ continuous function is $ij-\beta-I-$ continuous

proof:

let $f:(X,\tau_1,\tau_2,I)\rightarrow(Y,\rho_1,\rho_2)$ be $ij-pre-I-$continuous and let V be ρ_i-open set in (Y,ρ_1,ρ_2) then $f^{-1}(V)$ is $ij-pre-I-open$ in (X,τ_1,τ_2,I) where $i\neq j$ and

$i,j=1,2$, since every $ij-pre-I-open$ is $ij-\beta-I-open$ set then f is $ij-\beta-I-$continuous.

The following diagram show us the relation ships between these functions:



4.pair wise $ij-\beta-I$ and pair wise $ij-pre-I$ irresolute functions

Definition 4.1

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is said to be $ij-\beta-I-$ irresolute if $f^{-1}(V)$ is $ij-\beta-I-open$ for every $ij-\beta-J-$set V of (Y,ρ_1,ρ_2,J) .and J is an ideal on (Y,ρ_1,ρ_2)

Definition 4.2

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is said to be $ij-pre-I-$ irresolute if $f^{-1}(V)$ is $ij-pre-I-open$ for every $ij-pre-J-$set V of (Y,ρ_1,ρ_2,J) .and J is an ideal on (Y,ρ_1,ρ_2)

Definition 4.3

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is said to be $ij-I-$ irresolute if $f^{-1}(V)$ is $ij-I-open$ for every $ij-J-$set V of (Y,ρ_1,ρ_2,J).

Definition 4.4

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is said to be $ij-\beta-$ irresolute if $f^{-1}(V)$ is $ij-\beta-open$ for every $ij-\beta-$set V of (Y,ρ_1,ρ_2,J).

Definition 4.5

A function $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is said to be $ij-almost-I-$ irresolute if $f^{-1}(V)$ is $ij-almost-I-open$ for every $ij-almost-J-$set V of (Y,ρ_1,ρ_2,J).

Theorem 4.6

If $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is $ij-\beta-I-$ irresolute and $g:(Y,\rho_1,\rho_2,J)\to(Z,\mu_1,\mu_2)$ be $ij-\beta-J-$continuous then gof is $ij-\beta-I-$ irresolute.

Proof:

let $g:(Y,\rho_1,\rho_2,J)\to(Z,\mu_1,\mu_2)$ be $ij-\beta-J-$continuous and let V be an μ_i-open in (Z,μ_1,μ_2) then $g^{-1}(V)$ is $ij-\beta-J-$open in (Y,ρ_1,ρ_2,J) ,since $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is $ij-\beta-I-$ irresolute, then $f^{-1}(g^{-1}(V))$ is $ij-\beta-I-$open set in (X,τ_1,τ_2,I) but $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$,hence gof is $ij-\beta-I-$ irresolute function.

Theorem 4.7

i) $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is $ij-pre-I-$ irresolute and $g:(Y,\rho_1,\rho_2,J)\to(Z,\mu_1,\mu_2)$ be $ij-pre-J-$continuous then gof is $ij-pre-I-$ irresolute.

ii) $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is $ij-I-$ irresolute and $g:(Y,\rho_1,\rho_2,J)\to(Z,\mu_1,\mu_2)$ be $ij-J-$continuous then gof is $ij-I-$ irresolute.

iii) $f:(X,\tau_1,\tau_2,I)\to(Y,\rho_1,\rho_2,J)$ is $ij-almost-I-$ irresolute and $g:(Y,\rho_1,\rho_2,J)\to(Z,\mu_1,\mu_2)$ be $ij-almost-J-$continuous then gof is $ij-almost-I-$ irresolute.

Proof:

same proof of theorem (4.7) above and definition of each one.

References:

[1] M.E. Abd El-Monsef , E.F. Lashien and A.A. Nasef, On I-open sets and I-continuous functions, kyungpook Math. J. 32 (1) (1992),21-30.

[2] M.E. Abd El-Monsef, R.A Mahmoud and A.A. Nasef , Almost I-openness and almost I-continuity. Egyptian Math. Soc.7 (2) (1999) ,191-200.

[3] M. Caldas ,S.Jafari and N.Rajesh ,On I-open sets and I-continuous functions in ideal bitopological spaces (submitted).

[4] M.Caldas ,S.Jafari and N.Rajesh , Pre open sets in ideal bitopological spaces ,Bol.Soc. Paran.Math. 29 (2) (2011),61-68.

[5] E.Hatir. and T.Noiri , On decomposition of continuity via idealization ,Acta Math. Hunger 96 (4) (2002) ,341-349.

[6] E.Hatir. and T.Noiri, on β-I-open sets and a decomposition of almost –I-continuity, Bull. Malays. Math. Sci. Soc. (2)29 (1) (2006),119-124.

[7] E. Hayashi, topologies defined by local properties Math. Ann. 156 (1964),205-215.

[8] D.Jankovic. and T.R. Hamlet, Compatible extensions of ideals, Boll Un. Mat. Ital. B (7)6 (3) (1992),453-465.

[9] K.Kuratowski, Topology, Academic Press, New York, (1966).