Monte Carlo Observations of Atomic Number Dependence Of Saturation Thickness For Multiple Back-Scattering Photons From Thick Samples

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Abstract

Multiple scattering of 662 keV photons in the backward direction from elemental scatterers, (6C, 13Al, 26Fe, 50Sn, 65Tb and 82Pb), is simulated by Monte Carlo method with a view to estimate its atomic number Z-dependence. The thickness of scatterers was varied up to 20 cm (5.718 mean free paths). The relative contribution of multiple scattering events in the backward direction is found to increase with the sample (scatterers) thickness and to saturate around particular value called saturation thickness (S.T.) that satisfies the fitted relation, that of type rational functions, as:

\[ S.T. = \frac{(a + bZ)}{(1 + cZ + dZ^2)} \]

which shows that the number decreases with increasing atomic number of material sample and these results show good agreement with the reported experimental results.

1. Introduction:

The scattering technique has some attractive superiorities. First, we can have more freedom in selecting the relative positions of a radiation source and detectors and can arrange the source and detectors on the same side of the examined sample, which is very important for a superficial testing or for objects embedded underground [1]. Second, this technique has greater sensitivity in density variations for low density materials [2] and for superficial measurement. Furthermore, this technique can have larger relative contrast and is more efficient for thin objects [3].

The interaction processes of gamma rays in various media result in complete absorption, elastic scattering and inelastic scattering of incident photons [4]. Compton backscattering is a most dominant process in the intermediate energy range and in low Z-elements at near backward angles. In the collision between photons and electrons while
dealing with thick targets, some higher order processes occur due to a large number of secondary radiations produced in the first encounter, also known as multiply backscattered radiations. These multiply backscattered photons have a noticeable effect in the lower energy region of Compton continuum because these radiations also reach the detector and get counted [5]. Therefore, multiple backscattering in finite volumes has been a major drawback in the extraction of information from scattered photon flux because during the interaction of gamma photons with material, these photons continue to decrease in energy as the number of scatterings increases in the target. These low energy gamma photons get registered in the spectrum along with the singly scattered events. So, the energy spectrum of such photons is broad and never completely separate from the singly scattered distribution. This makes rather difficult to judge the exact contribution of multiply scattered photons in the lower energy region near the backscattered peak. For this reason, it acts as noise when one wants to measure the differential Compton cross section. Radiation shielding calculations take into account this process. It is, therefore, important to estimate accurately the intensity and spectral distribution of multiply scattered photons.

The present simulation were carried out with a view to estimate the atomic number $Z$-dependence of the multiple scattering in the backscattering of 662 keV photons from $^{6}$C, $^{13}$Al, $^{26}$Fe, $^{50}$Sn, $^{69}$Tb and $^{82}$Pb scatterers.

2. Simulation procedure:

For a good statistical distribution, $10^6$ photons with 662 keV energy have been followed. The scattering medium was considered as a rectangular surface with semi-finite dimensions. The thickness of scattering medium was changed from (0.1 – 5) cm. The system simulated is schematically shown in Fig.1.
In this model, it was assumed that every photon is emitted from the source with a 45° toward the target. The predicted history of photon in our simulation is defined during the following steps:

\textbf{a.} For simplicity, it is assumed that the emitted photons from the source undergo only one scattering. Also it ignored the small attenuation by the air between the sample and the source-detector system.

\textbf{b.} The position of the interaction within the sample is determined by random sampling of the exponential distribution:

\begin{equation}
    x = -\frac{1}{\mu} \ln(1 - R_n) \quad \cdots \cdots \cdots \cdots (1)
\end{equation}

Where, \( R_n \) is a uniform random number on the interval \([0, 1]\), and \( \mu \) is the linear attenuation coefficient of \( \gamma \)-radiation (with the energy \( E_{\gamma} \)) in the material.

XCOM program \cite{6} has been used to calculate the mass attenuation coefficients \( \left( \mu_m = \mu / \rho \right) \), where, \( \rho \) is the bulk density.

\textbf{c.} According to the value of distance \( x \) and incident angle (\( \theta_{\text{inc.}} \)), if the location of interaction is outside the dimensions of the sample, the history of photon will end. So the energy absorbed into the material is considered zero, counter concerned this process will increase (\( N_{\text{out}} \)), and the program starts from the beginning for another photon.

\textbf{d.} If the location of an interaction is inside the material, counter concerned this process will increase (\( N_{\text{int.}} \)), and the type of interaction is sampled (i.e. forward- or back-scattering) using Kahn’s method \cite{7} (a non-uniform rejection method). The final result of this step is the scattered photon energy.

\textbf{e.} In the Compton scattering process, by which an incoming photon is deflected by an angle \( \theta \) with respect to its origin direction after transferring a part of its energy to an electron, the energy of scattered photon \( E'_{\gamma} \) is given by:\cite{4}

\begin{equation}
    \cos \theta_{\text{scat.}} = 1 + \left( \frac{1}{E_{\gamma}} - \frac{1}{E'_{\gamma}} \right) m_o c^2 \quad \cdots \cdots \cdots (2)
\end{equation}

where, \( m_o c^2 = 511 \text{ keV} \) is the rest mass of the electron, and \( E_{\gamma} \) is the energy of the incident photon (the electron is assumed initially at rest).

In order to use this relation, our virtual set up allows a clear determination of this single scattered process. So, both the radiation source and the detector are strongly collimated.

\textbf{f.} According to the scattering angle, if the type of scattering is in the forward direction, a counter concerned this process will increase (\( N_{\text{fsc.}} \)), and the history of photon will end.
g. Whereas, if the type is backscattering, a counter concerned this process will increase ($N_{bsc}$), and the new scattered photon trajectory is determined.

h. The path length of this trajectory determines whether the scattered photon is still within the dimension of sample or escapes from it toward the detector.

i. The escaped scattered photon trajectories lying within a solid angle, covered by the detector collimator, are to be counted ($N_{bsc\_counted}$).

The graph of the number of scattered events has been determined for scattered photons in the backward direction ($N_{bsc\_counted}$) and theoretical saturation thickness has been obtained using these results.

A program has been written for this purpose in FORTRAN-95 language.

3. Results and Discussion

In Compton scattering experiments involving thin targets, the generation of multiply backscattered photons is less as compared to singly backscattered events, and so such photons are not heavily registered in the observed pulse-height distribution. The procedure of present simulation is in agreement with above experimental result as shown in Fig.2. On the other hand, with increase in target thickness, the flux of multiply backscattered photons gets enhanced and more backscattered photons are detected. So, along with the singly backscattered events, these multiply backscattered events also get registered. The numbers of multiply backscattered photons also depend
upon the atomic number of the target used in the experiment. So, in present work, calculations of the backscattered photons are carried out both as a function of thickness and atomic number of the target.

The plots of observed number of multiply backscattered events, having same energy as singly scattered ones, for different atomic numbers as a function of target thickness, shown in Fig. 3. The solid curves provide best-fitted curves to the present theoretical data (filled geometrical markers).

The registered numbers of the multiply scattered photons as a function of thickness are shown in Fig.3 for $^{6}$C, $^{13}$Al, $^{26}$Fe, $^{50}$Sn, $^{65}$Tb and $^{82}$Pb samples. The solid curves provide best-fitted curves to the present calculated data (filled geometrical markers). It can be seen that multiple scattering in the backward direction increases with the sample thickness and saturates after a particular value depending upon the material, called the saturation thickness (depth).

The saturation of multiply backscattered photons is due to the fact that as the thickness of target increases, the probability of the number of scattered events also increases, but on the other hand enhanced self-absorption results in decrease of the number of photons coming out of the target. Thus, a stage is reached when the thickness of the target becomes sufficient to compensate the above increase and decrease of the number of photons and, hence, the number of multiply backscattered photons coming out of the target saturates. Therefore, we can say that the smallest value of target thickness beyond which the number of multiply backscattered events, having energy the same as in singly scattered Compton distribution, emerging from the target remains constant (within one unit of statistical uncertainty) is known as saturation thickness.
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Fig.-3.a&b: Variation of numbers of multiply scattered events for 662 keV gamma photons for different target thicknesses and at a scattering angle of 140° for various elements.

The measured saturation thickness values, in targets of different elements for 662 keV incident gamma photon energy, are given in 4th column (with cm unit) and in 5th column (with mean free path unit) of Table_1. The 3rd column in the table provides the mean distance a photon of a given energy travels in a given element, known as the mean free path (mfp) (λ) given by [4]:

\[ \lambda = \frac{1}{\mu} \quad \ldots \quad (3) \]

Table-1: Simulated and reported experimental values of saturation thickness for various materials.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Z</th>
<th>1 m.f.p.</th>
<th>M.C. (cm)</th>
<th>M.C. (m.f.p.)</th>
<th>Eq.3 (cm)</th>
<th>Exp. (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6</td>
<td>5.718</td>
<td>12.5</td>
<td>2.186</td>
<td>12.2772</td>
<td>12</td>
</tr>
<tr>
<td>Al</td>
<td>13</td>
<td>4.963</td>
<td>7</td>
<td>1.410</td>
<td>7.39631</td>
<td>7.2</td>
</tr>
<tr>
<td>Fe</td>
<td>26</td>
<td>1.728</td>
<td>3.5</td>
<td>2.025</td>
<td>3.27344</td>
<td>3.2</td>
</tr>
<tr>
<td>Sn</td>
<td>50</td>
<td>1.815</td>
<td>0.9</td>
<td>0.496</td>
<td>0.9482</td>
<td>0.833</td>
</tr>
<tr>
<td>Tb</td>
<td>65</td>
<td>1.392</td>
<td>0.5</td>
<td>0.359</td>
<td>0.4896</td>
<td>0.57</td>
</tr>
<tr>
<td>Pb</td>
<td>82</td>
<td>0.800</td>
<td>0.25</td>
<td>0.313</td>
<td>0.24964</td>
<td>0.26</td>
</tr>
</tbody>
</table>
As shown in Fig.4, the saturation thickness decreases with increasing atomic number (Z) of material sample, and these results agree with the experimental results (7th column of Table_1 and filled cross in Fig.4) reported by G. Singh and et al. [8]. The solid curve is the best-fit curve through the simulated data (hollow triangle) of saturation thickness in various elements, and indicates that saturation thickness (depth) varies with atomic number as it satisfies the fitted relation of rational function type:

$$S.T. = \frac{(a + bZ)}{(1 + c + dZ^2)} \quad \ldots \ldots (4)$$

where:  
$$a = 1.2397506, \quad b = -0.73402695, \quad c = -0.57623571, \quad d = 0.10283287$$

with standard error 0.03221134 and correlation coefficient 0.99954377

The saturation thickness that could be used in the radiation shielding calculation can easily be obtained from Eq.(4).

Fig.-4: A log-log scale of the saturation thickness vs. atomic number Z of material sample.

The deviation of some of the simulated points from the experimental values may be caused by non-inclusion of multiply scattered contribution from the moving electrons and polarization effects of scattered photons, which are not taken into account in the present simulation.

4. Conclusions:

Our simulated results, for 0.662 MeV incident gamma photons, confirm that, for thick targets, there is significant contribution of multiply backscattered radiation emerging from the target, having energy equal to that of a singly scattered Compton process. The counted events of multiply backscattering increase with increase in target thickness and saturates beyond a particular value, called the saturation thickness. It has also been concluded that the saturation thickness decreases for increase in the atomic number of the target. The simulated and experimental data show similar behavior.
Here, it is also important to note that attempts on this objective have been very rare. So our present findings will serve very good reference for further comparison with experimental data of these processes. The work on multiple scattering is in progress under different geometrical conditions for better understanding of the multiple scattering processes. It is further planned to simulate the future multiple scattering experiments with more commonly used EGS4 (electron gamma ray showers) [1] and Geant4 codes [9].

5. REFERENCES: