

Experimental Study For Punching Shear Behavior In RC Flat Plate With Hybrid High Strength Concrete

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Abstract :

This Paper is conducted to investigate experimentally the punching shear behavior of hybrid flat plate. Five slab specimens with (1000x1000x70) mm dimensions have been tested, two of which have been made fully with High Strength Concrete and Steel Fiber Reinforced Concrete , while, the others have been made by mixing two types of concrete (Steel Fiber Reinforced Concrete in the critical zone and High Strength Concrete in the other part of slab specimen). The slab dimensions, main reinforcement have been kept constant in all slab specimens. The variables are concrete type and dimensions of critical area. Experimental results show that the use of SFRC improves the punching shear resistance and allows higher forces to be transferred through the slab-column connection. For slab specimen which made fully with SFRC, the ultimate shear capacity increased by (25%) in comparison with the normal slab. While, for slab specimens which made with hybrid concrete the ultimate shear capacity increased by (5%-13%) in comparison with the normal slab.

KeyWords: Punching Shear, RC Slabs, Hybrid Concrete, High Strength Concrete, Steel Fiber Reinforced Concrete.

دراسة عملية لمقاومة القص الثاقب للسقوف الخرسانية المستوية المصنوعة من الخرسانة الهجينية عالية الانضغاط

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الخلاصة :

تأثير استخدام أنواع مختلفة من الخرسانة (الخرسانة الهجينية) في المنطقة الحرجة لالتقاء العمود مع البلاطة على مقاومة القص الثاقب في البلاطات الخرسانية الصفانحية تمت دراستها في هذا البحث. تم فحص خمسة بلاطات مربعة الشكل بأبعاد (1000*1000*70) ملم بالإضافة الى سلسلة من الفحوصات التي تم إجراءها على نماذج السيطرة لدراسة الخواص الميكانيكية لأنواع الخرسانة المستخدمة. المتغيرات في هذه الدراسة هي نوع الخرسانة في المنطقة الحرجة وأبعاد هذه المنطقة مع الإبقاء على أبعاد البلاطات و حديد التسليح دون تغيير. تعرضت كلا من النماذج المرجعية و بلاطات الفحص الى نفس الظروف المختبرية. أظهرت النتائج المختبرية أن الزيادة في مقاومة انضغاط الخرسانة حسن مقاومة القص الثاقب وهذا يسمح بانتقال قوة اكبر من خلال المفصل الرابط بين العمود والبلاطة. كذلك، بينت النتائج أن مقاومة القص الثاقب قد ازدادت بمقدار (25%) بالنسبة للبلاطات المسلحة بالكامل بألياف الحديد عند مقارنتها مع البلاطة العادية. أظهرت النتائج، كذلك، حصول زيادة في المقاومة القصوى للقص الثاقب بمقدار (5%- 13%) بالنسبة للبلاطات المسلحة جزئياً بألياف الحديد عند مقارنتها مع البلاطة العادية.

1. Introduction

In general, slabs are classified as being one-way or two-way. Two-way slabs may be strengthened by the addition of beams between the columns, by thickening the slab around the columns (drop panels), by flaring the columns under the slabs (column capitals)^[1], or by adding shear reinforcement near the columns.

Reinforced concrete slabs may be carried directly by the columns without using beams or girders. Such slabs are described as flat plates. Since the thickness of a typical slab is relatively small, its capacity to transfer load into the columns by shear is often low. As a result, most failure of flat plates is initiated by overstress in shear at the columns area. These failures are termed Punching-Shear failures.

Punching shear failure of the slab is usually sudden and leads to a progressive collapse of the flat plates structures; therefore, caution is needed in the design of slabs and attention should be given to avoid the sudden failure condition^[2].

There are two main ways to develop resistance to punching shear stresses, one deals with reinforcement pattern and thickness of slabs and another deals with the materials used to improving the resistance to punching shear. The way that deals with reinforcement and

thickness is used as drop panels and columns capitals^[1,3], conventional shear stirrups^[4], shear head reinforcement^[5,6] and shear studs^[5].

The other way that deals with the material is increased compressive strength of concrete [**Gardener**^[7] and **Tuan**^[8]]. As well as the use of steel fibers reinforced polymers is another type of improved mechanical properties as discussed by **Al-karkhy**^[9].

2. Concepts of Hybrid Sections

To increase the load carrying requirement of steel sections, a hybrid section is used. The concept of hybrid section in steel structures is not a new idea. **Salmon** and **Johnson**^[10] defined a hybrid girder as one that has either the tension flange or both flanges of steel section made with a higher strength grade of steel than used for the web.

For steel fiber reinforced concrete sections, **Kawamata et. al.**^[11] defined a hybrid fiber reinforced concrete as a composite material which contains two different types of fiber together. The hybrid matrix (concrete) containing the steel fibers becomes more ductile and the tensile strength due to crack arrest mechanisms of steel fibers is much improved.

In order to repair or strengthen structural elements, layers of new concrete are often applied to an old structure. Hence, **Bernard. et. al.**^[12] defined hybrid concrete structures as structural elements consisting of new and old concrete layers.

When extending the hybrid concept to composite concrete members and due to advances in concrete technology, it is relatively easy to produce composite sections which possess high compressive strength, high ductility, high energy absorption and high tensile strength at the same time.

These characteristics can be achieved by placing two or more different types or strengths of concrete layers together so that each layer is used to its best advantage and as a result, the concrete section becomes a "hybrid" section. For flat plate slabs under punching shear stress, the critical area zone (near the injection point between slab and column) represents the main effective part to resist these stresses; therefore, it is more adequate to employ steel fiber reinforced concrete in this part alone to increase the shear strength.

In the present study, the hybrid slabs is defined as one that made with different concrete type (or concrete strength).

Plain, unreinforced concrete is a brittle material, with a low tensile strength and a low strain capacity; therefore, steel fibers are added to enhance some mechanical properties of plain concrete.

Fiber reinforced concrete (FRC) may be defined as a composite materials made with Portland cement, aggregate and incorporating discrete discontinuous fibers.

The role of randomly distributes discontinuous fibers is to bridge across the cracks that develop provides some post- cracking "ductility". If the fibers are sufficiently strong, sufficiently bonded to material and permit the FRC to carry significant stresses over a relatively large strain capacity in the post-cracking stage.

3. Objective of Present Study

The present experimental study is deals with the strength and deformation characteristics of flat plate made with different concrete types by using the concept of hybrid sections (using two types of concrete in same specimen).

In the critical area zone (near the column area) and to enhanced the mechanical properties of concrete, steel fiber reinforced high strength concrete with ($f'_c=70$ MPa) has been used, while in the area around the critical area zone part, the high strength concrete with ($f'_c=60$ MPa) has been used.

4. Experimental Work

4.1. Experimental Program

Five slab specimens with (1000x1000x70) mm dimensions have been tested, two of which were made fully with HSC and SFRC, while, the others were made by mixing two types of concrete (Steel Fiber Reinforced Concrete in the critical zone and High Strength Concrete in the other part of slab specimen). The slab dimensions, reinforcement in both directions have been kept constant in all slab specimens. The variables are concrete type and dimensions of critical area.

All slabs are simply supported along the all edges and subjected to single point load applied at the center of slab. The applied load is transformed from testing machine through a central column of dimensions (150x150) mm.

4.2. Description of Experimental Work

As mentioned previously, the test slab specimens have been divided into five groups according to the concrete type and dimensions of critical zone. Table (1) show the slabs designation. **Figures (1) and (2)** shows the details, dimensions and slab reinforcement.

Table (1) Slabs designation

Slab Designation	Concrete Type		Critical zone designation
	Critical Area	Rest of Panel	
S-1*	HSC		-
S-2**	SFRC		-
S-3	SFRC	HSC	3d+150
S-4	SFRC	HSC	4.5 d+150
S-5	SFRC	HSC	6 d+150

*Reference Slab (Entire Slab made with HSC)

** Entire Slab made with SFRC

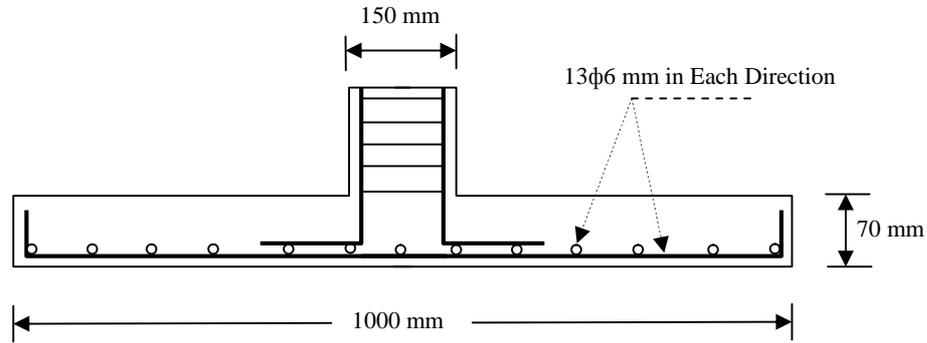


Fig. (1) Details, Dimensions and Reinforcement of slab specimen

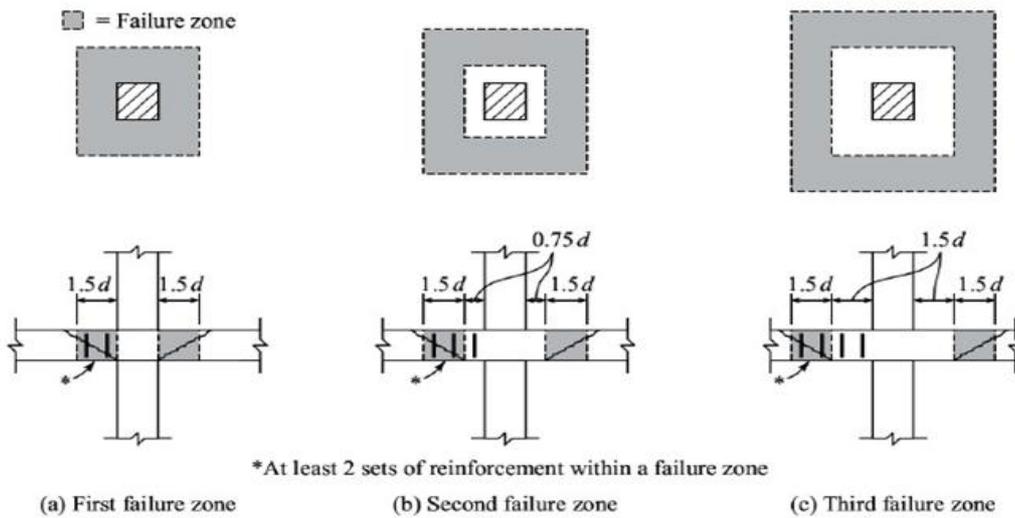


Fig. (2) Details and Dimensions, Reinforcement in Slab

4.3. Materials

4.3.1. Concrete

In manufacturing test specimens, the following materials have been used: ordinary Portland cement (Type I) complying with the Iraqi standard specification No. 5/1984⁽¹³⁾; crushed gravel with maximum size of (12.5mm); natural sand from Al-Ukhaider region (Karbala-Iraq) with fineness modulus (2.72) complying with the Iraqi standard specification No. 45/ 1984⁽¹⁴⁾; Lime stone powder (L. S. P.) with fineness (3100 cm²/gm);

high water reducer super plasticizer (SP) complies with ASTM C-494/C 494M –01⁽¹⁵⁾; crimped mild carbon steel fibers with average length of (25mm), nominal diameter of (0.5mm), aspect ratio of (50) and yield strength of (1130MPa) have been used with volume fraction of (V_f) equal to (0.5%). Clean tap water has been used for both, mixing and curing.

The concrete mix proportions are reported and presented in Table (2). It may be noted that the control specimens (cylinders and prisms) are cast and cured with each slab in a water bath at about) 25⁰C) for (28) days and then tested at the same time of slab testing.

4.3.2. Steel Bar Reinforcement

Plain wires of (6mm) diameter have been used as flexural reinforcement placed in the tension face of the slab (at bottom). Each wire have an average yield strength (f_y) of (382 N/mm²) determined from tensile test. The wires were cut to the desired length of (1050mm), and bent with (90-degree) to formed hooks at the ends of each bar (to prevent bond failure of bars) dimensioned according to the ACI-318 Code^[16].

The wires is uniformly spaced and placed in two perpendicular directions at (70 mm c/c) spacing in each way to obtain the desired steel ratios of (0.5%). A clear cover of (15mm) has been provided below the mesh. It may be noted that, the steel reinforcement have been designed to ensure the tested specimens to fail either by punching shear or flexural.

4.3.3. Specimen Preparation and Testing

A wooden mould with clear dimensions of (1000x1000x70) mm has been used, **Figure (3)**. The specimens have been cast using two types of concrete, in the critical zone area (near the column part) steel fiber reinforced high strength concrete ($f'_c=70$ MPa) has been used, while in the area around the critical zone area part, high strength concrete with ($f'_c=60$ MPa) has been used, to enhanced (improved) strength capacity of concrete in critical zone.

After (28days) of curing (in water bath), the specimens dried in the laboratory temperature and painted with white color to monitoring the specimens during test. The slab specimens have been placed on the testing machine and adjusted so that the centerline, supports, point load and dial gauge were in their correct or best locations.



Fig. (3) Specimen Preparation

4.3.4. Test Measurements and Instrumentation

Hydraulic universal testing machine (MFL system) has been used to test the slabs specimens as well as control specimens. Central deflection has been measured in each loading stage by means of (0.01mm) accuracy dial gauge (ELE type) and (30mm) capacity. The dial gauges have been placed underneath the bottom face at the center, load arrangement with loading frame is shown in **Figure (4)**.



Fig. (4) Slab Specimen Setup

4.3.5. Mix Design and Proportions for HSC

In manufacturing the control and tested specimens, two concrete mixes has been used. These mixes have given an average compressive cylinder strength of (60MPa) and (70MPa) at (28 days). It may be noted that, both mixes were identical in raw materials except that the second mix content steel fibers with volume fraction of (V_f) equal to 0.5%. Mixes proportions and details for all mixes are reported and presented in **Table (2)** below.

Table (2) Concrete Mixes

Material	Mix Designation	
	Mix-1	Mix-2
Cement (kg/m ³)	535	535
Limestone Powder (kg/m ³)	64	64
Sand (kg/m ³)	784	784
Gravel (kg/m ³)	863	863
Water (L/m ³)	155	155
S. P (By weight of cement)	64	64
Steel fiber (V _f %)	-	0.5

5. Results and Discussion

5.1. Crack Pattern and Failure Mode

The initial cracking of all the tested slabs has been observed first in the tension zone of the slab near the column stub in the form of flexural cracks towards the slab edges, with increasing loading, cracks appeared around the region of steel fiber concrete until failure by punching shear (at ultimate load the punching shear failure occurs suddenly). **Figure (5)** show the crack patterns of specimens after punching shear failure on the bottom surface, the cracks patterns of all specimens are similar, the difference appear in the punching shear zone where the distance between the face of column and the punching shear crack become larger.

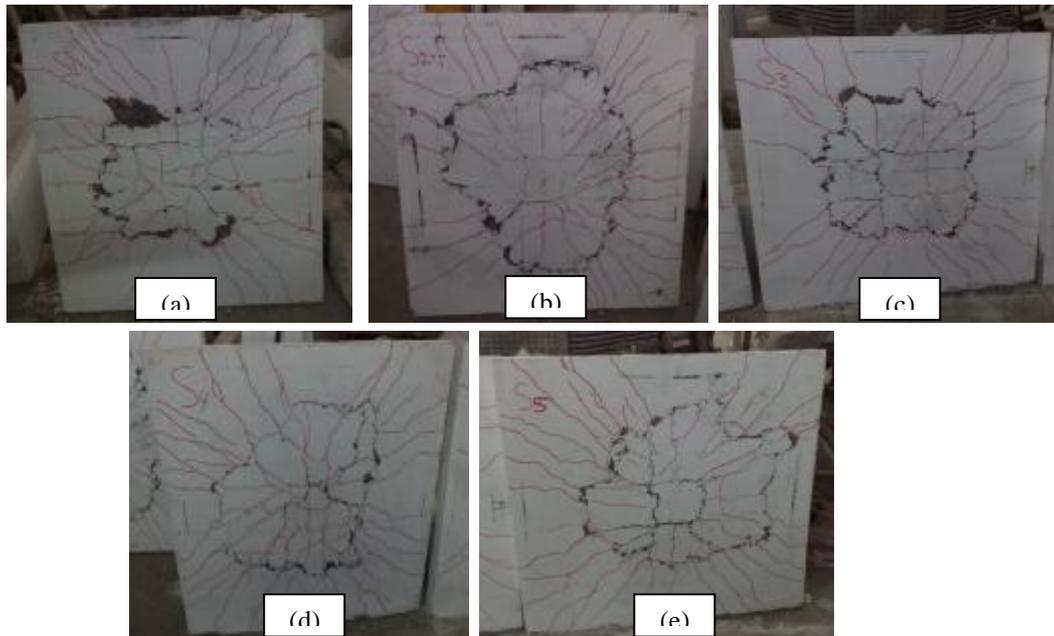


Fig. (5) Cracking and Failure Modes of the Tested Slabs

(a) S-1 (b) S-2 (c) S-3 (d) S-4 (e) S-5

In general , the first cracking load appeared at (21-51)% of the ultimate load, and the fiber reinforced concrete slab (S2) had first cracking load more the slab with high strength concrete only (S1) by about (26%) , and had increasing in first cracking load about (31%), (39%) and (51%) in comparison with hybrid slabs (S3), (S4) and (S5) respectively. This improvement is due to the steel fiber reduce or relive internal faces by blocking microscopic cracks from forming within the concrete and the fibers work as a bridges to hold the concrete to gather. In addition, the cracking load increase with increasing the fiber concrete zone around column for the same reasons we mentioned about, (**Table (3)**).

5.2. Ultimate Load Capacity

The main goal of this section is to find a difference in ultimate load capacity when using FRC in punching shear zone. The observed ultimate load at failure is listed in Table (3). It can be seen that there is a good improvement in ultimate load capacity when increasing the area of steel fiber concrete around column. According to these results the ultimate load capacity of fiber reinforced concrete slab (S2) had increasing in ultimate load by about (25%) in comparison with reference slab (S1).

For the slab specimens (S3, S4 and S5), there is an increasing in ultimate load capacity about (5%), (10%) and (13%) respectively with respect to reference slab (S1). This increasing in ultimate load capacity can be explain to two main reason, first the compression strength for samples (S2, S3, S4 and S5), slab containing SFRC, in punching shear zone has been increased in comparison with reference slab (S1) and the second reason that is the steel fiber work as obstruction elements to delay shear crack from continue to other face of specimen which cause delay failure and as a result increasing in ultimate load capacity.

Table (3) Experimental Results of the Tested Slabs

Slab Designation	P_c (kN)	P_u (kN)	Δ_c (mm)	Δ_u (mm)	$(P_u)_i/(P_u)_R$ %	P_c/P_u %	(Perimeter) ** (mm)	(θ) *** deg
S1*	43	206	0.38	28.4	-	21	2100	60
S2	120	256	0.45	18.5	1.25	47	2650	26
S3	68	216	0.35	24.5	1.05	31	2030	43
S4	88	225	0.28	23	1.10	39	2180	51
S5	118	232	0.45	21	1.13	51	2420	29

*Reference Slab

**Failure Perimeter which Corresponding to maximum punching failure zone.

***Crack angle of punching shear.

5.3. Cracking Loads

Results presented in **Table (3)** show that the cracking loading has been decreased with increasing the strength of concrete. For slab specimens (S1, S2, S3, S4 and S5), the crack has been occurred at shear force of approximately (21%), (47%), (31%), (39%) and (51%) of ultimate load respectively.

This means the cracking load (initial stage of loading) depends essentially on concrete strength.

5.4. Deflections

Load-deflection curves for all specimens are shown in **Figures (6) to (8)**. In general, all load deflection curves of all tested slabs have similar behavior and there are three stages can be seen in load-deflection curve. First, linear relationship between load and deflection this zone of load-deflection curve is called elastic zone and the first crack occurs at the end of this zone. Second, also linear relationship between load and deflection and the yielding of steel reinforcement occurs at the end of this zone, the third stage will start and the deflection continues to increase without any increasing in load. It is apparent that for specimens reinforced with steel fibers (S2, S3, S4 and S5), the deflection (during loading stage) were less than the slab without steel fiber (S1), (at the same loading stage) also, the area under the load-deflection curves were greater in slab(S2, S3, S4 and S5) than in reference slab (S1). This behavior can be justified that the compressive strength has been increased due to adding the steel fibers and as a result deflection has been reduced. Also, this means the specimens reinforced with steel fibers exhibits more ductile than the non-fibrous ones.

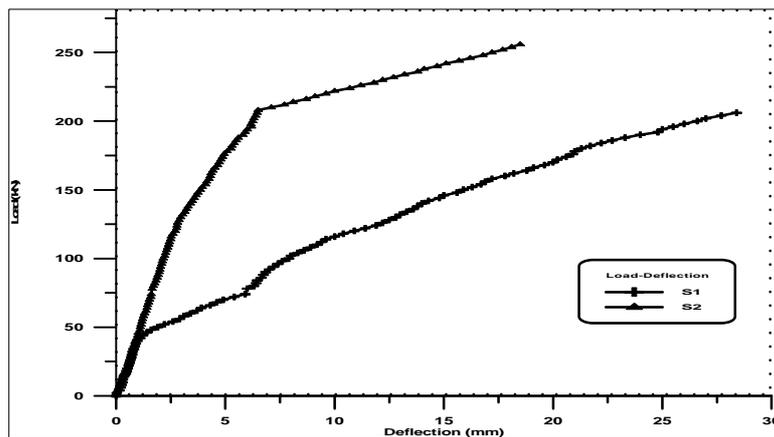


Fig. (6) Load-Deflection Curves for (S1 & S2)

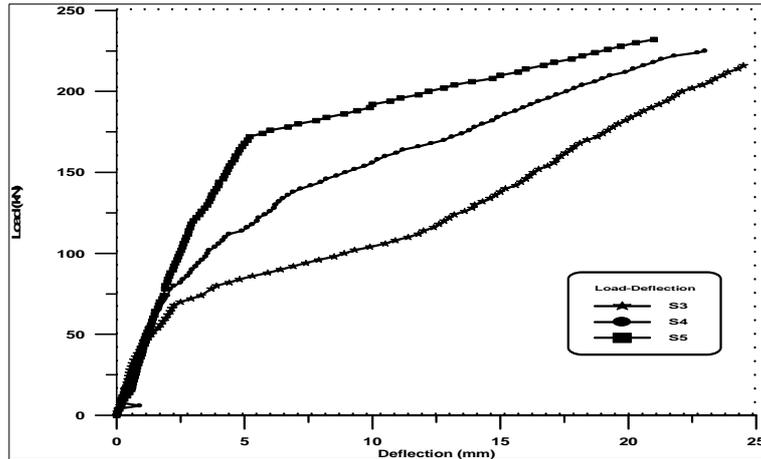


Fig. (7) Load-Deflection Curves for (S3, S4 & S5)

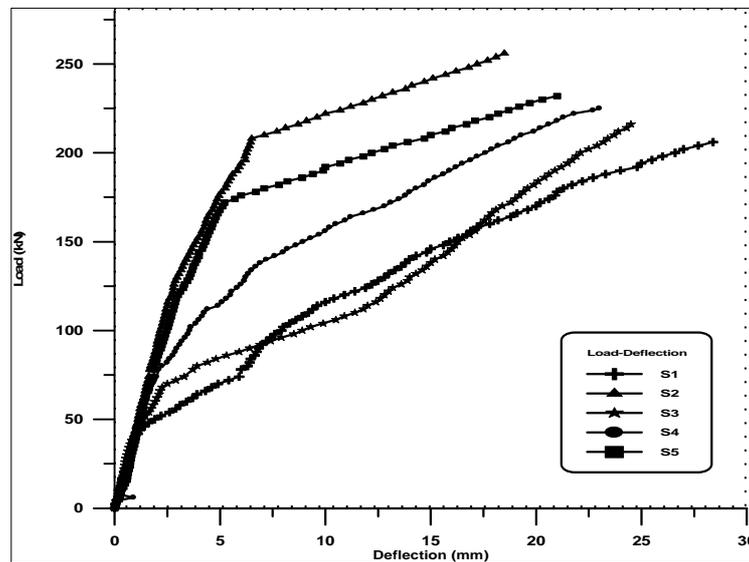


Fig. (8) Load-Deflection Curves for (S1, S2, S3, S4 & S5)

5.5. Area of the Failure Zone

The failure perimeters and corresponding maximum periphery punching failure zones are measured and presented in **Table (3)**. For the tested slab specimen, with increasing the strength of concrete, the failure perimeter increased significantly.

The crack angle of punching shear was found to be approximately between (26) to (60) degrees. It may be noted that, for specimens which made with steel fiber high strength concrete, the crack angle was less inclined (crack angle of approximately 26 degrees). This means, the failure cracks extend in more distance and cross more number of reinforced bars.

5.6. Effect of Steel fibers in Ultimate Capacity

According to the experimental results, the ultimate load capacity of fiber reinforced concrete slab (S2) increased by about (25%) in comparison with reference slab (S1). For the slab specimens (S3, S4 and S5), there is an increasing in ultimate load capacity about (5%), (10%) and (13%) respectively with respect to reference slab (S1). Presence of SFRC improves the punching shear resistance and allows higher forces to be transferred through the slab-column connection.

5.7. Punching Shear Strength Analysis based on ACI-318

ACI-318-08 state in articles (11.11.1.2) and (11.11.2.1), for two-way action, each of the critical sections to be investigated shall be located so that its perimeter (b_o) is a minimum but need not approach closer than ($d/2$) to the face of column, and for non-prestressed slabs, (V_c) shall be the smallest of:-

$$V_c=0.17(1+\frac{2}{b})\sqrt{f'_c} b_o.d \dots\dots\dots(1)$$

$$V_c=0.083(\alpha_s \frac{d}{b_o} +2)\sqrt{f'_c} b_o.d \dots\dots\dots(2)$$

$$V_c=0.33\sqrt{f'_c} b_o.d \dots\dots\dots(3)$$

The corresponding punching shear force, based on ACI-318-08, are calculated and presented in **Table (4)**. While, the comparison between measured and calculated punching shear forces for tested slab specimens are presented in **Table (5)**. As shown in **Table (5)**, the ACI-318 formulas give underestimated shear strength by (79, 106, 73, 81 and 86) % for slab specimens (S1, S2, S3, S4 and S5) respectively. This may be due to neglected the contribution of the steel fibers and high strength concrete. Therefore, it is recommended that the all formulas related to the punching shear (ACI formulas) is required to be modified to take into consideration the effect of presence of the steel fibers and high strength concrete.

Table (4) Punching Shear Force Based on ACI-318-08

f'_c (MPa)	V_c (kN)			Note
	Eq. (1)	Eq. (2)	Eq. (3)	
60	178.2	135.8	115.3	Eq.(3) governed
70	192.4	146.7	124.5	Eq.(3) governed

$d=55\text{mm}$, $b_o=4(150+55)=820\text{mm}$, $\beta=1$, $\alpha_s=40$

Table (5) Comparison between Measured and Calculated Punching Shear Force

Slab Designation	Concrete Type at Critical Section	f'_c (MPa)	V_n (kN)		$(V)_{EXP./} (V_n)_{ACI}$
			EXP.	ACI	
S1	HSC	60	206	115.3	1.79
S2	SFRC	70	256	124.5	2.06
S1	SFRC	70	216	124.5	1.73
S2	SFRC	70	225	124.5	1.81
S3	SFRC	70	232	124.5	1.86

6. Conclusions

From the previous discussions, it can be seen the following points:

- 1- Based on the experimental results, the ultimate load capacity of fiber reinforced concrete slab has been increased about (25%) in comparison with non fibrous concrete.
- 2- For the hybrid slab specimens, there is an increasing in ultimate load capacity was about (5-13) % in comparison with reference slab. This increasing in ultimate load capacity can be explain to two main reason, first the compression strength for samples (slab containing SFRC) in punching shear zone was increased in comparison with reference slab and the second reason that is the steel fiber work as obstruction elements to delay shear crack from continue to other face of specimen which cause delay failure and as a result increasing in ultimate load capacity.
- 3- For specimens which made with steel fiber high strength concrete, the crack angle was less inclined (crack angle of approximately (26) degrees). As a result, the failure cracks extend more distance and cross more number of reinforced bars.
- 4- For the tested slab specimen, with increasing the strength of concrete, the failure perimeter increased significantly.
- 5- The specimens reinforced with steel fibers exhibits more ductile than the non-fibrous ones.
- 6- The ACI-318 formulas give underestimated shear strength by (79-106) % because the adopted equations of ACI-318 neglect the contribution of the steel fibers and high strength concrete. Therefore, it is recommended that the all formulas related to the punching shear (ACI formulas) is required to be modified to take into consideration the effect of presence of the steel fibers and high strength concrete.

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9. Notation

b_o =Critical section perimeter;

d =Effective depth of concrete section;

f'_c = Ultimate cylinder compressive strength;

P_u = Ultimate load;

P_c = Cracking load;

$(P_u)_i$ = Ultimate load of considered slab;

$(P_u)_R$ =Ultimate load of reference slab;

f_y = Yield tensile strength;

ϕ = Diameter of reinforced bars;

Δ_c =Deflection corresponding to cracking load;

Δ_u = Deflection corresponding to ultimate load;

Θ = Crack angle of punching shear;

β = The ratio of long side to short side of the column;

α_s =Column location parameter (40 for interior columns, 30 for edge columns, 20 for corner columns)

FRC=Fiber reinforced concrete;

HSC=High strength concrete;

SFRC=Steel fiber reinforced concrete.