CONTROLLR MODELLING AND DESIGN OF ROTATIONAL SPEED FOR INTERNAL COMBUSTION ENGINE

Dr. Raoof M. Radhi

Lecher, shawajin@yahoo.com

Dr. Emad Q. Hussien

Lecher, emadalslamy@yahoo.com

ABSTRACT

This paper proposes a controller design of a nonlinear model of an internal combustion engine based on linear quadratic regulator (LQR) technique. The design takes into consideration the effect of external disturbances that occurs during the operation of system. The equations of motion are linearized according to the perturbation theory in order to inspect the dynamic stability of the engine motion, and the subsequent design of appropriate feedback controllers for the system, which can be obtained by solving the LQR problem. The results show that the proposed controller has good performance and stability than PID controller. The simulations have been carried out in Matlab/Simulink environment.

1 INTRODUCTION

There are many kinds of systems in common industrial, transportation, and domestic use that need to be controlled in some manner, and there are many ways in which that can be done. In all types of heat engine, the expansion of the high-temperature and high pressure gases produced by combustion applies direct force to some component of the engine, such as pistons, turbine blades, or a nozzle. This force moves the component over a distance, generating useful mechanical energy [1].
An automotive engine is a typical multiple-input multiple output system that has to satisfy a number of performance criteria under different operating conditions. Controller development for an internal combustion engine is a challenging engineering problem that requires background of different disciplines such as thermal engineering, dynamics, and control theory [2].

Most engine systems have idle speed control built into the electronic control unit (ECU). The engine rotation (rpm) is monitored by the crankshaft position sensor which plays a primary role in the engine timing function for fuel injection, spark events, and valve timing. The idle speed control (ISC) system regulates engine, by adjusting the volume of air that is allowed to bypass the closed throttle valve.

The main source of performance deterioration of the speed control system is disturbances such as rapid external load changes and slow varying changes in operating conditions. External load changes are the result of loading due to changes in operation requirements, such as an attempt for sudden stop which subject the engine to a very high deceleration level, or may be fast acceleration for sudden high speed demand, as well as city driving during busy traffic. All of these operating conditions lead to variable power demand and thereby engine speed fluctuation. However, the engine may be equipped with means to satisfy such requirements, but sacrifices have to be accepted.

Power demand variation and so, engine speed fluctuation causes instantaneous air/fuel mixture changes between lean and rich values, which both have adverse effects on engine [3], i.e:

1. Rich mixture burns faster causing maximum pressure concentration near top dead center (TDC) and so, resulting in rough operation, as well as reduces time for heat transfer to occur from cylinder, thus raising overall gas temperature which increases NOx formation rate.
2. Lean mixture slows flame speed and thus combustion lasts well past TDC. This keeps the high pressure well into power stroke, again raising gas temperature. High temperature together with unused oxygen of lean mixture oxidizes exhaust valves and seats, as well as increasing NOx formation rate.

During transient period, the engine speed settling time may be significant, giving thereby a fair instant of time for the rotational speed to bounce between acceleration/deceleration modes. This will cause, as mentioned above, severe deterioration in both engine power and
operation economy. The scope of this work is therefore set to overcome all of these disturbances problems.

Many different closed loop designs have been proposed in the literature including $H_\infty$ control [4], $H_2$ control [5], sliding mode control [6], $\ell_1$ optimization [7], feedback linearization [8], proportional-integral-derivative (PID) control [9], linear quadratic control (LQ) [10], and adaptive control [11,12]. A comparison between different control algorithms methods can be found in [13].

The objective of this paper is to maintain engine speed at a prescribed set-point in the presence of random disturbances. It’s proposed LQR technique as a method for controlling the engine rotational speeds.

2-SYSTEM DESCRIPTION

The engine model contains inlet and exhaust manifolds, torque generation, internal friction and crank shaft dynamic. This creates a torque and a rotational motion on the crankshaft which depend on load, pressure in the cylinder, mass of all parts in motion and the geometry of the engine as shown in Fig.(1)[14].

![Fig. (1) Schematic of engine speed system](image)

3- DYNAMIC OF ENGINE NON-LINEAR MODEL

The nonlinear model is based on the work of Powell and Cook [15]. It’s block diagram is shown in Fig.(2). The engine speed can be controlled by the throttle angle and ignition timing. The manipulated variable (input) is computed by the error between the engine speed, which can be measured by a crank sensor, and the desired one. An application of conventional PID control is invalid to such nonlinear engine control system with variation of the desired speed and system parameters.
Fig. (2) Non-linear engine model

The differential equations describing the overall engine dynamics are given by [14, 16]:

\[
\begin{align*}
\dot{m}_a(t) &= C_d(P_m)A_{th}(\alpha) \frac{P_a}{\sqrt{R T_a}} \psi\left(\frac{P_a}{P_m}\right) \\
\dot{m}_\beta(t) &= \frac{P_m}{RT_m} \xi_{vol}(P_m, \omega)V_d \frac{\omega}{2\pi} \\
\dot{p}_m(t) &= \frac{R T_m}{V_m} \left(\dot{m}_a - \dot{m}_\beta\right) \\
T_e(t) &= \xi_{ind} H_0 \frac{Z}{4\pi} \dot{m}_\beta \\
\dot{\omega}(t) &= \frac{1}{J_e} (T_e - T_d)
\end{align*}
\]

Where, \( C_d(P_m) \) is the discharge coefficient, this coefficient compensates for flow losses and variations of the effective throttle area \( A_{th}(\alpha) \), and \( \psi\left(\frac{P_a}{P_m}\right) \) is the pressure ratio. Can be expressed the rate of change of manifold pressure \( \dot{p}_m(t) \) as a nonlinear function of \( \alpha(t), P_m(t) \) and \( \omega(t) \) as follows [14, 17]:

\[
\dot{p}_m(t) = f_1(P_m(t), \omega(t), \alpha(t))
\]
The load torque on the engine is not known a priori. It is due to aerodynamic resistance, road, and the internal engine friction. In general, the generated torque \( T_p \) is a nonlinear function of engine speed, mass flow rate into the engine cylinders, equivalence ratio \( \phi \) and spark advance \( \sigma \). The time delay \( \tau \) in the engine model equals approximately 180 degrees of crank angle advance, and Thus is a speed dependent parameter. Additionally, most engine control activities are event driven and synchronized with position. Therefore, the rate of change of engine speeds can be expressed in the following form:

\[
\dot{\omega}(t) = f_2(P_m(t), \omega(t), T_d(t))
\]  

(3)

4- FEEDBACK LINEARIZATION

The basic idea with feedback linearization is to transform the nonlinear systems dynamics into a linear system. Conventional control techniques like pole placement and linear quadratic optimal control theory can then be applied to the linear system. Feedback linearization allows us to design the controller directly based on a nonlinear dynamic model that better describes a shape maneuvering behavior.

The equations of motion are linearized according to the perturbation theory in order to inspect the dynamic stability of the engine motion [18]. The linearized state-space representation around an operating point is developed from the dynamic equations:

\[
\dot{x} = Ax + Bu + B_d d
\]  

(4)

\[
y = Cx + uD
\]  

(5)

State vector are defined by \( x^T = [P_m \quad \omega] \), \( u = [\alpha] \) and state matrices A, C and D are given by;

\[
A = \begin{bmatrix}
\frac{\partial f_1}{\partial P_m} & \frac{\partial f_1}{\partial \dot{\omega}} \\
\frac{\partial f_2}{\partial P_m} & \frac{\partial f_2}{\partial \dot{\omega}}
\end{bmatrix}, \quad C = [0 \quad 1], \quad D = [0]
\]

Setting
In this case, this model is used to design a controller for the system.

5-LQR DESIGN

Linear Quadratic Regulator (LQR) is a widely used control technique. It is preferred because of its easy implementation and its optimality for linear time invariant systems. It is an optimal and robust technique for Multi-Input Multi-Output (MIMO) control using this method [10, 8].

From Eqns.(4) and (5) above, the derivation which is given a linear time invariant system in state variable with disturbance. For LQR control the following cost function is defined

\[
J = \frac{1}{2} \int_{t_0}^{t_1} (X'Qx + u'Ru)dt
\]

(6)

The object of LQR control is to find a state feedback gain matrix, \( K \) such that the cost function is minimized. The matrices \( Q \) and \( R \) are weighting matrices, which determine the closed loop response of the system. The solution to this problem starts with finding a control law in the form of the following;

\[
u = -R^{-1}B'P(t)x(t) - R^{-1}B'\xi(t)
\]

(7)

Where, \( P \) and \( \xi \) are obtained from the following equations

\[
\dot{P} + PA + A'P - PBR^{-1}BP + C'QC = 0
\]

(8)

\[
\dot{\xi} + (A' - PBR^{-1}B')\xi - PB_d d = 0
\]

(9)

If \( t_1 \equiv \infty \), \( P \) and \( \xi \) are steady state solutions;

\[
A'P + PA - PBR^{-1}BP + C'QC = 0
\]

(10)
\[ \dot{x} = -(A' - PBR^{-1}B')x - PB_d d \]  

(11)

In other words, the control law contains both feedback states and feed forward disturbance

\[ u = -Kx(t) + K_d d \]  

(12)

With:

\[ K = R^{-1}B'P \]

\[ K_d = R^{-1}B'(A' - PBR^{-1}B')PB_d \]

6-SIMULATION RESULTS

A linear engine model was established in Matlab/Simulink to evaluate the performance of LQR controller. The numerical simulations have been performed using engine having characteristics shown in Table 1, and are taken from reference [19].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>1.013*10^5 Pa</td>
</tr>
<tr>
<td>( V_m )</td>
<td>6.8*10^-3 mm^3</td>
</tr>
<tr>
<td>( V_d )</td>
<td>3.77 mm^3</td>
</tr>
<tr>
<td>( T_m )</td>
<td>350 k</td>
</tr>
<tr>
<td>( R )</td>
<td>281 J/kg.k</td>
</tr>
<tr>
<td>( T_a )</td>
<td>296 k</td>
</tr>
<tr>
<td>( Z )</td>
<td>4</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>45.6*10^6 J/kg</td>
</tr>
<tr>
<td>( J_e )</td>
<td>0.15 kg.m^2</td>
</tr>
<tr>
<td>( T_d )</td>
<td>2 N.m</td>
</tr>
</tbody>
</table>

The space matrices \( A, B, C \) and \( D \) are obtained after substituting the operating point state values, as shown in Fig.(3). The state weight matrix, \( Q \) and the control weighting matrix, \( R \) are chosen by Bryson’s rule. These choices are used as just the starting point for a trial and error iterative design procedure aimed for the desirable properties in the close-loop system.
The PID controller is used for controlling the engine speed. The PID controller parameter are tuned to give the optimal performance; $K_p = 0.0001$, $K_i = 0.01$, and $K_d = 0.0001$.

Such controller is illustrated in Fig.(4), where $\Delta \omega$ represents the input engine speed (error signal) and $\Delta \alpha_c$ represents the output throttle angle change.

Assuming the measurable output is rotational speed of the engine and the control input is the angle of the throttle plate. The linear model was given a step change in throttle angle from 5.5° deg. to 6.5° deg. at 15 seconds, as shown in Fig.(5). The load disturbance of the engine speed is taken randomly at any time, as sinusoidal in cross-section with amplitude of 2 N.m, as shown in Fig.(6).

The effect of load disturbance (torque) on the rotational speed of the engine at the time of operation $t = (5 - 10) sec$ and $t = (25 - 30) sec$ is shown in Fig.(7). For the open-loop system is first tested on the simulator before applying the proposed controllers.

The simulation results of the engine speed shown in Figs.(8) and (9), to verify the effectiveness of the LQR controller comparing to both the open-loop system and PID.
controller. The effectiveness of each controller is tested and verified using Matlab/Simulink environment. The PID controller has a percentage of overshoot and consequently takes some time to stabilize the system. It also has the settling time that can reach more than 5 sec, which will affect the effectiveness of the system.

Therefore, it's concluded that the LQR controller gives the best performance in terms of lower amplitude and faster settling time compared to the PID controller.

7-CONCLUSIONS

Rotational speed control of an internal combustion engine is an important issue, so this paper presents a design method for engine speed control using LQR method. The objective of the research was to evaluate the existing methods for estimating the engine speed variations. An application of LQR law to such nonlinear engine control system is valid for the variation of the desired speed and system parameters. The obtained results showed that the presented controller has shorter settling time and smaller overshoot compared to both PID controller and open-loop system.

Fig.(5) Variation of air throttle angle with time of operation system

Fig.(6) Disturbance load variation with time of operation system
Fig.(6) Random disturbance load variation with time of operation system

Fig.(7) Variation of engine speed with time of operation for open loop system

Fig.(8) Time response of engine speed for closed loop system at different system
Fig.(9) Time response of engine speed for closed loop system at different method

REFERENCES


LIST OF SYMBOLS

\( d \)  Disturbance input, N. m

\( H_0 \)  Calorific value of fuel, J/Kg

\( J_e \)  Mass moment of inertia for crankshaft, Kg m²

\( \dot{m}_\alpha \)  Air mass flow rate, Kg/s

\( \dot{m}_\beta \)  Mass flow rate of air inducted into cylinder, Kg/s

\( P_a \)  Atmospheric pressure, N/m²

\( P_m \)  Intake manifold pressure, N/m²

\( R \)  Universal gas constant

\( t \)  Time, sec

\( T_a \)  Air temperature, K

\( T_d \)  Disturbance torque, (N.m)

\( T_m \)  Intake manifold temperature, K

\( u \)  Control unit torque, N.m

\( V_c \)  Clearance volume, m³

\( V_d \)  Volume of cylinder, m³

\( V_m \)  Intake manifold volume, m³

\( Z \)  Number of cylinders

\( \alpha \)  Throttle angle, deg.

\( \xi_{vol} \)  Volumetric efficiency

\( \xi_{ind} \)  Indicated efficiency

\( \omega \)  Rotational speed, rad/s

\( \omega_{ref} \)  Reference rotational speed, rad/s
New Base and Burner Design for Utilizing the Compound Gases with Spilled Oil at Using Power Chimney Techniques

Dr. Rafid M. Hannun
College of Engineering, Thi-Qar University

eng_rafid005@yahoo.com

Abstract:

The renewable or non-renewable modern energy kinds have many shapes to utilize power generation. The power chimney tower is one of them developed by low losses, simple and has high facilities that used in spilling oil and/or oil refinery stations and to save clean environment.

In this paper, new base design of power chimney used to increase the utilized energy by increasing the number of burners in addition to introducing new burner design by adding directional vanes. Many parameters influencing the system operation were exactly studied to predict new operation phenomenon. This fundamentally depends the combustion of gases compound the spilled oil. Velocity distribution is the important parameter which gives the first prediction to put the position of erection of power turbine, made or not. The numerical analysis was presented by using GAMBIT and FLUENT 6.3 to predict the high velocity at the expansion of chimney near the Centre of burned gases cover collector. This position is very suitable for promoting and building the power turbine since the velocity was high when the compounded gases is combusted (that there components are the methane gas with other friendly gases and waste). Also, it is concluded that there are high temperature increased by using the new burner design reach (1400-1800) K in comparison with old design without swirlers. It is easy to erect steam or gaseous boiler in contact with furnace for utilizes the heat generated in electrical power generation. So, the other factors, temperature and pressure were studied to coincide with previous papers in this field. The validity of this study was done by comparison to same flow rate of air/fuel ratio of previous study findings to give similar results of parameters.

Keywords: Power, Chimney, Burner, Oil.
1- Introduction:

Power chimney technology is a promising large scale of power generation. This technology was first described by Günter in 1931 and tested with the 50 kW Manzanares prototype plant since 1980 [1]. There are four components for the combination of this prototype: combustion chamber, chimney, turbine connected with electrical generator.

The installation of power chimney at locations near the oil spilling stations for utilizing of compound fuels which burned in this suggested power plant.


Sislian et.al. (1988) [5] measured experimentally many mechanical parameters in combustor and concluded that turbulence in the jet diffusion flame was appreciably more anisotropic than in the corresponding cold jet in all regions of the flow. Gaseous fuels are usually characterized by clean combustion, with low rates of soot and nitric oxides. The main problem is that of achieving the optimal level of mixing in the combustion zone.
A mixing rate that is too high produces narrow stability limits, but a mixing rate that is too low may make the system prone to combustion-induced pressure oscillations. Many different methods have been used to inject gas into conventional combustion chambers, including plain orifices, slot, swirlers, and venture nozzles [6]. Backstrom and Fluri [7] developed two analyses for finding the optimal ratio of turbine pressure drop to available pressure drop in a chimney power plant to be 2/3 for maximum fluid power and using the power law model for this prediction.

Pretorius and Kröger [8] evaluated the influence of a recently developed convective heat transfer equation, more accurate turbine inlet loss coefficient, the performance of a large scale chimney power plant. This simulation of study concluded that the new heat transfer equation reduce the annual plant power output by 11.7%, but, the more realistic turbine inlet loss coefficient only accounts for a 0.66˚rise in annual power production.

Ninic and Nizetic [9] developed and used the availability of warm, humid air via the formation of up draft “gravitational vortex column” situated over turbine with numerical solution for chimney power plant.

The compound gases pass through burner vents which circumstances with air vent to create high mixing rate to combust inside the cub of combustion chamber. The air near the ground absorbs the heat to decrease its density. The hot air particles move up to hit the cub ceiling continuously and go to chimney vent. This series heating generates continuous movement of air, then, produces electrical energy by installing turbine connected to electrical generator. The height of chimney causes high pressure difference between the upper and lower points. This pushes increasingly the movement of air particles between the lower points of chimney to up. In addition, the power may be generated by connecting gas or steam turbine to use the resulting hot gases before chimney inlet vent [10].

Love et.al. (2009) [11] developed an experimental method for the rapid characterization of combustion properties, and measured the amounts of NOx and combustion products. Azazi (2001)[12] presented a study to Hartha power plant furnace in Iraq, used a two dimensional aerodynamics and thermal aspects by using FORTRAN computer programme. He concluded that 1500 °C inside temperature of furnace and the tangential velocity played a great role for keeping the stability of the fire ball. Alhabbubi (2002) [13] presented a prediction of temperature distribution and heat flux along the walls of Al-Mussaib thermal power plant furnace in Iraq by using zonal method to analyze the radiative heat transfer. He found that the temperature range from 1450K to 2100 K. Sobolev et.al. (2008)[14] presented a numerical calculation results of methane turbulent diffusion jet flames of rectilinear-swirl
burner in the furnace of high capacity boiler by using CFD AnsysCFX10.0 programme. Hannun (2009) [15] studied the combustion of liquid and gaseous fuel in Nassiriya power plant furnace, analyzed numerically the mechanical properties by using FLUENT code. Hannun et.al. (2011) [16] presented a prediction for mechanical parameters influenced the operation of solar power chimney at Nassiriya city. Hannun (2011) [10] predicted the parameters inside the combustion chamber of power chimney by using similar eight burners consist of vents for air and fuel without swirlers.

The aim of study is to redesigning a burner and number of burners of furnace base to reach high efficient combustion during utilizing the compound gases at oil refinery stations.

2-Numerical model

2-1 Physical model

In this study, practical prototype logically depended on as shown in Fig. 1 is selected as a physical model for simulation. The chimney height is 50 m and radius 1 m, the frustum (cup) of 15 m radius at the base and 10m height. There are four burners arranged around the chimney Centre to ensure continuous, efficient flame stability. Each burner consists of five central cylindrical vents (each 0.2m radius) for entering gaseous fuel, the outer vents (the inner radius 1m and the outer 2m) designed as separated inclined swirler vents for flowing of air with high turbulent mixing rate with fuel to ensure perfect combustion inside the chamber. The vanes of one burner are designed to swirl air in a direction reversed to that of neighbor burner to prevent the friction among mixture (fuel and air) particles [15, 19].
There are many prototypes in the world have large different heights of chimney up to 500 m, with different radii. The designs of combustion chamber have many shapes and different dimensions. The design mentioned in previous paragraph (2-1) is considered as model to be analyzed.

The continuity, Navier – Stokes, energy equations and k-ε equation are shown below [5]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0$$  \hspace{1cm} (1)

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho uu)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} = \rho g \beta (T - T_e) + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$  \hspace{1cm} (2)

$$\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho vv)}{\partial y} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$  \hspace{1cm} (3)

$$\frac{\partial (\rho cT)}{\partial t} + \frac{\partial (\rho cu T)}{\partial x} + \frac{\partial (\rho cv T)}{\partial y} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$  \hspace{1cm} (4)

$$\frac{\partial}{\partial t} (\rho K) + \frac{\partial}{\partial x_j} (\rho K u_j) = \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial^2 K}{\partial x_j^2} \right) + \frac{\partial K}{\partial x_j} \right) + G_k + G_e - \rho \varepsilon + S_k$$  \hspace{1cm} (5)

**2-2 Mathematical model**

There are many prototypes in the world have large different heights of chimney up to 500 m, with different radii. The designs of combustion chamber have many shapes and different dimensions. The design mentioned in previous paragraph (2-1) is considered as model to be analyzed.

The continuity, Navier – Stokes, energy equations and k-ε equation are shown below [5]:

![Fig.2 Base and Burners of Prototype](image-url)
\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\mu_i + \frac{\mu_i}{\sigma _K} \frac{\partial \varepsilon}{\partial x_j}\right) + C_{ie} (G_k + C_{3e} G_b) - C_{2e} \rho \frac{\varepsilon^2}{K} + S_e \tag{6}
\]

Where:

<table>
<thead>
<tr>
<th>C_1</th>
<th>C_2</th>
<th>C_µ</th>
<th>(\sigma_{k,t})</th>
<th>(\sigma_{e,t})</th>
<th>(\varepsilon)</th>
<th>k</th>
<th>(\sigma_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>1.92</td>
<td>0.09</td>
<td>1</td>
<td>1.3</td>
<td>0.7</td>
<td>0.4</td>
<td>0.9</td>
</tr>
</tbody>
</table>

2-3 Boundary conditions:

1. For chimney

\[
\frac{\partial T}{\partial x} = 0 , u = 0 , v = 0 \tag{7}
\]

For the base inside the combustion chamber

\[
\frac{\partial T}{\partial y} = 0 , u_d = 0 , v_d = 0 \tag{8}
\]

For outlet conditions:

At \(x = \pm \infty\), \(T = \text{constant} = 300K\), \(P = P_{\text{atmosphere}}\), \(u = 0\), \(v = 0\) \(\tag{9}\)

2. Symmetrical axis at chimney centre axis, i.e:

\[
\begin{align*}
&u_{(x=+x)} = u_{(x=-x)}, \quad v_{(x=+x)} = v_{(x=-x)}, \quad p_{(x=+x)} = p_{(x=-x)} \tag{10} \\
&\mu_m \left( \frac{\partial u_d}{\partial y} + \frac{\partial v_d}{\partial x} \right)_{(x=+x)} = \mu_m \left( \frac{\partial u_d}{\partial y} + \frac{\partial v_d}{\partial x} \right)_{(x=-x)} \tag{11}
\end{align*}
\]

The gaseous fuel was natural gas with high ratio of methane which burns according to the following chemical reaction (complete combustion):

\[
\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \tag{12}
\]
The molecular weight of carbon (C), hydrogen (H) and oxygen (O) are 12, 2 and 32 respectively. Therefore, the combustion of one mole of methane with two moles of oxygen produce one mole of carbon dioxide with two moles of water, that means one kg of methane with four kg of oxygen to produce the following:

\[ M_{H_2O} = m_{CH_4} \times \frac{W_{H_2O}}{W_{CH_4}} \]  \hspace{1cm} (13) \\
\[ M_{CO_2} = m_{CH_4} \times \frac{W_{CO_2}}{W_{CH_4}} \]  \hspace{1cm} (14)

Therefore, the fuel burning inside the chimney chamber follows the equation:

\[ Q = m_a c_p (T_f - T_a) + \dot{\varepsilon} A_p (T_f^a - T_w^a) \]  \hspace{1cm} (15)

Where \( Q \) (kW) = mass flow rate × calorific value of fuel

**2-4 Numerical analysis:**

The turbulent flow of mixture (air and fuel) before and after the combustion process inside the system would be analyzed by standard k-\( \varepsilon \) model. The SIMPLE algorithm with QUICK Scheme method used to solve the pressure –velocity coupling, momentum and energy equations respectively. These methods were explained by many references such as Ref.[17]. So, the Gambit and Fluent Codes are used to describe the results of this paper. Fig.3, 4 and 6 the study case of new burner design and chimney system as designed by GAMBIT.

Fig. 3 Front and side view of burner as designed by GAMBIT
3- Results and Discussion

The heat energy transferred from the flame inside the combustion chamber to circumferential wall which may be built as heat exchanger with another fluid (air or distilled water). This fluid absorbs heat from the wall by convective heat transfer to rotate gas turbine or steam turbine. So, it might be use turbine at chimney depend on the flue gases motion outside the system.
In this paper, three cases of gaseous fuel velocity input depending on the chamber capacity and/or iterate that to capacity of chamber. The cases are 0.05, 0.1, 0.14 m/s of gaseous velocity input through anyone of the four velocity input vents at the base (0.052, 0.105, 0.157kg/s mass flow rate). The central section plane which cut the system into two parts and cut two burners only is taken at this paper. The combustion process is taken for gaseous fuel (methane) density (CH4 fuel) is 0.6679kg/m$^3$.

Fig. 6 shows the temperature distribution inside the system. Fig.6A is an indication of heat absorbed or transferred to different trends of parts chose at 0.105kg/s of fuel input flow rate. This shape of temperature range (2200 K to 300 K) which is at ambient temperature of 300 K.

**Fig. 6 Static temperature at A (0.105kg/s), B (0.052kg/s) and C (0.157kg/s) of fuel input flow rate**

FLUENT Code demonstrates wide ranges of measurements near the effective domain. The dominated temperature of air inside the chimney is 1200K as observed by limited range contour but the maximum temperature value is at the bottom side distance of chimney and inside upper ceiling of the chamber. It is notably high hot place than other system space.
because the combustion of fuel and air was happen under this place. The air speed increases in the direction of chimney Centre.

The other shapes of Fig.6B and 6C denote to temperature difference at fuel flow rate of 0.052 and 0.157kg/s respectively. It is normally, there are gradually higher temperature ranges than 0.105kg/s of fuel flow rate. In 0.052kg/s fuel velocity, it is normally lower than 0.2m/s because of lower mass flow rate. But in 0.157kg/s observe lower value too since the combustion process takes place near the chimney base which gone outside the system as shown in fig.6C with yellow colour. This heat forces the velocity magnitude in the direction of chimney Centre as a result to natural convection of air and forced convection of fuel input [10].

Therefore, fig.7 shows the circumferential average of static temperature for three cases of paper study. It is found that the higher value of temperature at the chimney is for 0.157kg/s of fuel as described previously but the lower temperature lies between 0.052kg/s and 0.105kg/s. There is an increasing in the temperature at the left hand side of figure for mass flow rate of fuel 0.052kg/s because of swirling and friction of mixture particles inside the combustion chamber [18].

To ensure the validity of this study, the circumferential average of static temperature compared with previous paper [10] in Fig.7-B. The mass flow rates of combusted fuel of this study (0.052, 0.105 and 0.157 kg/s) are similar to that of velocity magnitude (0.1, 0.2 and 0.3m/s) respectively for [10]. The findings of comparison of Fig.7-A with Fig.7-B give that higher temperature at present study (1400-1800)K as a result of new burner design and swirling flow which leads to high turbulent mixing rate for air and fuel particles and high combustion efficiency.

Also, lower temperature is at 10m radially in Fig.7-A because of non-premixed combustion with high mixing rate of burners at this position. The chamber of chimney is closed for the end edge side of base therefore no air input, so, the temperature is high.

The velocity vectors of the system are shown in Fig.8 which indicates that high velocity magnitude at inlet fuel flow rate of 0.105kg/s reaches 5.6m/s at the position of turbine at the lower part of chimney. So, there is a low velocity gradient at the outer end of the base part of system. The velocity notably increases when directed to the chimney Centre due to the heat accumulation increase, narrow area of chimney cover and low pressure gradient. The velocity range increases directly with increasing the heat flux of combustion. These ranges are suitable for using big turbines at lower position of chimney or using multi-stage turbine to have high gain energy.
Fig. 7 Circumferential average of static temperature A: for three present cases and B: for Reference [10]

The circumferential average of velocity magnitude distribution on the system domain with its position in the central section for whole system with axial coordinates is presented by Fig. 9A. It is denoted that high velocity values lie at the chimney but low values at the end edges of chamber since the reasons mentioned in this paper before. The upper black curve of Fig. 9A denotes the velocity magnitudes in the axial coordinate calculated as average values of the central section of domain for flow rate of fuel input 0.157 kg/s which interpritates high velocity of flue gases of combustion with increase the energy of fuel. It is observed that higher velocity values trend from the outer edge to the central part of domain (chimney) as mentioned before. The validity of these values shown by fig. 9B for Hannun [2] which predict the same trend of curves for different values of power.
Fig. 8 Velocity vector distribution (m/s) at 0.105kg/s fuel flow rate

So, the validity of this work that the programme code works properly to get logic results. Fig. 10 shows the concentration of species in combustion which is calculated as mass fraction. The first one is the gaseous fuel (methane CH4) which reasonable prediction for incoming fuel from burners to combustion chamber before the combustion process has high mass fraction at this place then circumferentially disappeared after the mixing outside the burner, but the second one for Hannun [10] the fuel concentration gradually disappeared without swirling and mixing process inside the furnace as a result to combustion. The forth is carbon dioxide concentration where it has maximum value at the end of combustion after the reaction complete with the same manner of water particles concentration (Fig. 10). It is vice versa to concentration of oxygen (fifth slip) because it inters the reaction before burning of fuel. The sixth slip is for nitrogen, its concentration is very high in large space of domain except in the combustion space because it is not interring in the combustion process but it has high portion of air constituents.

Fig. 9 Circumferential average of velocity magnitude for different velocity input of fuel

A for present study, and B for [10]
Fig. 10 The concentration of species (mass fraction) at 0.105kg/s flow rate of fuel input and [10]
4- Conclusions:

It is noted from the numerical solution and the analysis of chimney tower system for the predicted case study, there are some conclusions presented about the circumferential parameters such as:

1- The present design could be depended to erect in oil stations mentioned in the 1 above or may be arranged to be favorable for anyone.

2- The present design of burners and their situations were enhanced by using swirlers to get complete combustion process with high efficiency.

3- It is reasonable to build a new burner design of chimney tower at the oil spilling units or refinery units to use the compound gases outgoing the petroleum wells.

4- The pressure gradient is approached to atmosphere at the inlet vent of the system but it is increased while reach the top of chimney since high difference in relative pressure due to the heat. Also, the height of chimney which work as vacuum pressure to increase the air velocity.

5- Due to the high velocity recorded at the expansion of chimney, the power turbine is preferred to build in.

References


### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>( a )</td>
<td>Thermal diffusivity (m(^2)/s).</td>
</tr>
<tr>
<td>( A_p )</td>
<td>Area of the inside faces of combustion chamber (m(^2)).</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational body force (m/s(^2)).</td>
</tr>
<tr>
<td>( k )</td>
<td>Kinetic energy (m(^2)/s(^2)).</td>
</tr>
<tr>
<td>( L )</td>
<td>Combustion chamber canopy height (m).</td>
</tr>
<tr>
<td>( m_a )</td>
<td>Air mass flow rate (kg/s).</td>
</tr>
<tr>
<td>( M )</td>
<td>Air mass flow rate (kg/s).</td>
</tr>
<tr>
<td>( c_p )</td>
<td>Molecular weight (kg/kmol).</td>
</tr>
<tr>
<td>( p )</td>
<td>Specific heat at constant pressure (kJ/kg. K).</td>
</tr>
<tr>
<td>( R_a )</td>
<td>Rayleigh number.</td>
</tr>
<tr>
<td>( T_a )</td>
<td>Air temperature (K).</td>
</tr>
<tr>
<td>( T_f )</td>
<td>Fuel temperature (K).</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Wall temperature (K).</td>
</tr>
<tr>
<td>( u, v )</td>
<td>Average velocity at x, y directions respectively (m/s).</td>
</tr>
<tr>
<td>( W )</td>
<td>Weight (kg).</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Emissivity factor.</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Dissipation rate (m(^2)/s(^3)).</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Coefficient of Thermal expansion (1/K).</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>The heat conduction coefficient (W/m. K).</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity (kg/m.s).</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity (m(^2)/s).</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (kg/m(^3)).</td>
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</tbody>
</table>
A Study on a Developed Electro-Mechanical Epicyclic Power-Drive (EMEC) for Hybrid Electric Vehicles (HEV)

Abdul Baki K. Ali
PhD in Mechanical Engineering / University of Basrah, Basrah, Iraq

westofbas@yahoo.com, tel 009647801012423

Abstract:

Hybrid electric vehicles (HEV) draw much attention during the last years due to the need for alternative power resources with reduced emissions. The major disadvantages of HEV are the heavy weight, high cost and complicated control strategies. This paper focuses on a developed design for electro-mechanical differential unit. Most designer uses differential block as a mechanical linkage between two power sources. In this paper the differential unit is modified to do more than linkage. It acts as a continuously-varying-transmission (CVT), electric motivation unit and power regeneration unit. This of course leads to more compact and light weight vehicles. The developed model uses three main units: Synchronous block, Differential block, and Induction block the three blocks are combined in a compact design. Mechanical-electrical power transformation is carried out through the synchronous block which acts also as regeneration unit during braking. On the other hand electrical-mechanical transformation is carried out through the induction unit. The differential unit plays the role of smart linkage between the synchronous and the induction units. Simulation through Matlab-simulink is carried to show the validity of the model. The developed model proved to satisfy the properties of CVT and driving demands torques.

دراست تطوير وحدة حركة كهروميكانيكية تعمل بشكل تفاضلي لنظام القدرة في السيارات الكهربائية الهجينة.

لقد جذبت السيارات الكهربائية الهجينة اهتماماً متزايداً خلال السنوات الأخيرة بسبب الحاجة إلى مصادر طاقة دبلة أقل ثقلًا. لكن من ابرز عيوبها هو ثقل وزنها وكثافتها العالية مع أنظمة السيطرة المعقدة. استخدم بعض المصممون النموذج التفاضلي كوسيلة ربط بين الحمل من جهة وبين مصادر الطاقة من جهة أخرى. في هذا البحث تم تطوير النموذج التفاضلي ليكون أكثر من وسيلة ربط. انة يعمل كغير سرعة مستمر CVT ووحدة تحريك كهربائي ووحدة استرجاع للطاقة مما يؤدي إلى سيارة أقل حجما ووزناً. يكون النموذج المقترح من ثلاث وحدات هي الوحدة الترانزيتية والوحدة التفاضلية والوحدة الحثية. يتم التحويل الميكانيكي-الكهربائي من خلال الوحدة الترانزيتية والتي تؤدي أيضاً دور استرجاع الطاقة أثناء الفرملة. بينما يتم التحويل الكهربائي-الميكانيكي من خلال الوحدة الحثية. تعمل الوحدة التفاضلية
Keywords

Hybrid electric vehicle (HEV), Power-Drive, Continuously Varying transmission (CVT), Powertrain, Control

1. Introduction

The studies for hybrid electrical vehicle (HEV) have attracted considerable attention because of the necessity of developing alternative methods to generate energy for vehicles due to limited fuel energy, global warming and exhaust emission limits in the last century [1]. HEV incorporates internal composition engine, electric machines and power electronic equipments. Many researchers classify HEV according to the power flow into serial, parallel and complex HEV’s [2]. Recent papers focus on new configurations such as planetary [3] and differential [4] configurations. In planetary configuration (PC), the power-drive uses a planetary gear mechanism to connect an internal combustion engine, an electric motor and a generator. A highly efficient engine can simultaneously charge the battery through the generator and propel the vehicle (Fig.1). It is important to be able to set the engine operating point to the highest efficiency possible and at sufficiently low emission levels of undesirable exhaust gases such as hydrocarbons, nitrogen oxides and carbon monoxide. The motor is physically attached to the ring gear. It can move the vehicle through the fixed gear ratio and either assist the engine or propel the vehicle on its own for low speeds. The motor can also return some energy to the battery by working as another generator in the regenerative braking mode.

![Fig.1](image) [3] the prius car PC
Another innovative configuration is called differential configuration (DIfC) [4]; The (DIfC) is simple, cost-effective and easy to implement. The differential unit, which acts as mechanical torque-overflow keeps engine torque within a predefined value. This is achieved by the aid of torque loop attached to one terminal of the differential. The (DIfC) configuration proved to act as a CVT (continuously varying transmission) and also can perform efficient control strategies. Minimum emissions can be assured by running the IC-engine within best engine performance zone which is characterized by engine torque and engine speed. Fig.2 shows the main components of DIFC power-drive. IC engine is connected at shaft (B), DC generator connected to shaft (F1) and DC motor connected to shaft (F2). The DC motor is connected to the vehicle wheels either directly or by means of gear reducer. The other important thing to notice in this new configuration is the torque loop. Many researchers interested in torque control for hybrid electric vehicles [5]. The torque signal is generated by the load cell attached to the engine supports. The signal is conditioned then through an electronic circuit to be compared with a predefined value namely the reference torque. The result of comparison is fed to a well tuned PID controller to produce a suitable field voltage for the DC generator. The field voltage of the DC motor is considered to be fixed to the rated value. The driver has only one variable to control the vehicle with. It is the fuel pedal. As he pushes the pedal, engine speed increases. Generator speed is also increased the torque loop pushes an amount of field voltage to the generator. This amount is exactly the amount that produces a certain torque. Regulating the generator torque which is connected to on terminal of the differential leads to regulate the torque of IC-engine because all the terminal of the differential has constant torque relation. This model succeeds to run the engine at its rated torque.
Developed Electro-Mechanical Epicyclic Power-Drive (EMEC)

The Differential configuration (DIFC) invented by Ali et al [4] has the advantage of mixed serial and parallel configuration. At vehicle speedup the (DIFC) acts serial from one side of view and parallel from another point. The mechanical power at one terminal of the differential unit is transformed to electrical power through the generator attached to this terminal; the generator passes this power to the motor. This type of power flow is a serial flow. On the other hand the other terminal of the differential receives the mechanical power from the ICE and passes it directly to the wheels. When the vehicle approaches the top-gear speed most of the power of the ICE flows mechanically to the wheels, while during the speedup period only a small mechanical power is passing directly to the wheels. The differential block allows for flexible mixing between mechanical and electrical power. In this paper a new modification is added to the configuration (DIFC). This modification implies a self gear shifting possibility without the need for torque loop regulation. Fig. 3 shows the new developed configuration (EMEC).
The (EMEC) power drive consists of three major blocks:

1.1 Epicyclic block: The epicyclic block is the mechanical part of the power-drive. It contains sun, planet, and ring gears. The planet gear(s) are mounted on an arm which is attached to the rotor of the synchronous block, the second block in the drive. The sun gear is linked to the vehicle engine (ICE). It receives the power from the ICE and delivers it either to the arm or to the ring in a differential manner. The ring gear is attached to output shaft which transmits power for
wheels. Sometimes, speed reducer is needed before wheels. The output shaft passes through the rotor of the induction block and keyed with it. The ring gear also carries the permanent magnets (PM) of the exciter. The ring gear assembly can be made of many parts to be assembled together. It also represents oil casing for all gears assembly. Seals are used when necessary as shown in Fig.3. The torque and speed relationship of the epicyclic block can be shown through Fig.4:

\[
R_A = \frac{R_R + R_S}{2}
\]

(1)

Speed relationships are [6]:

\[
\frac{\omega_S - \omega_A}{\omega_R - \omega_A} = -\frac{R_R}{R_S}
\]

(2)

Where: speed, R is radius.

Torque relationship:

\[
T_R = F \times R_R
\]

\[
T_S = F \times R_S
\]

\[
T_A = 2F \times R_A = F \times (R_R + R_S)
\]

One can write the torque relationship as:

\[
T_S = \alpha T_R, \quad \alpha = \frac{R_S}{R_R}
\]

(3)

\[
T_A = \beta T_R, \quad \beta = \frac{R_R + R_S}{R_R}
\]

(4)
1.2 Synchronous block

The synchronous block represents the impulsive hart of the power-drive. Permanent magnet synchronous machines are known as a good candidate for hybrid electric vehicles due to their unique merits [7]. Many researchers recommend using synchronous machines for HEV [8, 9, 10, and 11]. Fig.5 shows the synchronous block. It consists of rotor, which is the arm carrying the planet gear, and stator, which is procreator of electric power. The rotor itself consists of two parts exciter coil and salient pole rotor. The exciter coil cuts the magnetic field of the permanent magnet of the ring gear assembly. The cutting speed depends on the relative movement or relative speed between the ring and the arm assembly. The generated power rectified into DC power and passed to the salient poles of the Synchronous generator. The stator of the generator is attached firmly to the frame of the drive. The generated AC power has a frequency equal to the arm speed and a voltage proportion to the relative speed between the arm and the ring.

![Fig.5 the Synchronous Block](image)

The principle of operation can be explained as follows: When the vehicle is just start to move the ICE moves the sun gear which in turns moves the planet gear, the planet gear tries to move the ring gear which is attached finally to vehicle wheels. The starting torque need to be high at start-up, So when it is difficult to propel the ring gear, then the planet begins to roll inside the ring causing the arm to spin around its centre. The arm is attached to the synchronous block, so it moves exciter winding inside the magnetic flux of the permanent magnet of the ring gear, which is now either stationary or rotates slowly. The generated power is rectified with diodes before be fed to the salient poles rotor which is also attached to the arm assembly. The rotating flux of the salient poles generates AC power in the stator winding.
of the generator. According to the theory of synchronous generator [12], the generated frequency is equal to the speed of rotation of the rotor. On the other hand the voltage amplitude depends on two factors: 1- the excitation current which in turns depends on the rate of cutting the magnetic flux of the permanent magnet ($\omega_A - \omega_R$), and 2- the rate of cutting the flux of the salient poles by the stator winding ($\omega_A$). Ring gear rotates at $\omega_R$, the arm rotates at $\omega_A$, so the generated voltage is:

$$E_1 = C_1 \omega_A (\omega_A - \omega_R) \sin(2\pi \omega_A t)$$  

(5)

Where:

$C_1$ is a constant depends on magnetic properties of the synchronous block

### 1.3 The induction block

The induction block assembly is shown in Fig.6. It consists of a squirrel-cage induction motor attached to the ring gear. The stator of the induction motor is attached firmly to the frame of the drive. It receives AC power from the synchronous block. The torque generated from the induction motor, $T_m$, is added to the mechanical torque of the epicyclic, $T_R$ to form the total propulsion torque directed to the wheels, $T_w$. It is gained, and then by a constant depends on axel gears.

$$T_w = T_R + T_m$$  

(6)
2. Electrical analyses of the new (EMEC) power drive:

Electrical analysis of the drive assumes a two AC machines: synchronous generator and induction motor, connected end to end (or by using drive circuits). The generator generates AC power due to arm movement and flows it to the motor which is firmly attached to the ring gear. The simplified equivalent circuit of the two machines is shown in Fig. 7.

For the generator:
The equivalent impedance $Z_{eq,g}$ is the vector sum of the stator resistance and reactance:

$$Z_{eq,g} = R_g + jX_g$$  \hspace{1cm} (7)

Where: $R_g$ and $X_g$ are resistance and reactance of generator stator.

For motor block:

$$Z_{eq,m} = R_{eq,m} + jX_{eq,m}$$  \hspace{1cm} (8)

Where $R_{equationm}$ and $X_{equationm}$ are the equivalent resistance and reactance of motor seen from stator.

The term $X_{equationm}$ is constant and depends on winding and magnetizing properties. The term $R_{equationm}$ is not constant but it can be divided into two parts [12]: continuous ($R_o$) and load equivalent resistance ($R_L$) which is equal to:

$$R_L = \hat{R}_2 \left( \frac{1}{s} - 1 \right)$$  \hspace{1cm} (9)

Where $\hat{R}_2$ is the resistance of the rotor seen from the stator; $s$, is the slip and given by:

$$s = \frac{\omega_A - \omega_R}{\omega_A}$$  \hspace{1cm} (10)

The synchronous speed here is $n_A$ since it is the speed of the synchronous generator.

The circuit current, $I$, is:
The equivalent resistance and reactance of the circuit seen from the terminals E1.

The total circuit power delivered by the generator is given by:

\[ P_g = 3E_1I \cos \phi \]  

Where \( \phi \) is the power factor of the circuit:

\[ \phi = \tan^{-1} \left( \frac{X_{eq}}{R_{eq}} \right) \]  

If one neglect windage and frictional losses then the generator torque is:

\[ T_g = \frac{3E_1I \cos \phi}{\omega_A} \]  

On the other hand, the mechanical power of the induction motor is given by [12]:

\[ P_m = 3I^2 \dot{R}_2 \left( \frac{1 - s}{s} \right) = T_m \omega_R \]  

But, \( \omega_R = (1 - s) \omega_A \)

\[ T_m = 3I^2 \frac{\dot{R}_2/s}{\omega_A} \]  

Substituting the value of the current \( I \) in equation (16) yields to:

\[ T_m = 3 \frac{E_1^2 \dot{R}_2/s}{(R_{eq}^2 + X_{eq}^2) \omega_A} \]  

Substituting the value of \( E_1 \) (equation 5) and \( s \) (equation 10) in equation (17) gives:

\[ T_m = 3C_1^2 \frac{\omega_A^2(\omega_A - \omega_R)\dot{R}_2}{(R_{eq}^2 + X_{eq}^2)} \]  

\[ T_m = 3C_2^2 \frac{\omega_A^2(\omega_A - \omega_R)R_2}{(R_{eq}^2 + X_{eq}^2)} \]  

Where the ratio \( \dot{R}_2/R_2 \) is included in the new constant, \( C \).

If we let \( m \) to be the ratio:

\[ m = \frac{\text{Number of motor poles}}{\text{number of generator poles}} \]  

Then

\[ T_m = 3mC_2^2 \frac{\omega_A^2(\omega_A - \omega_R)R_2}{(R_{eq}^2 + X_{eq}^2)} \]
3. The Simulink model:

The Simulink model of the new proposed power-drive (EMEC) is shown in Fig. 8. The model is constructed in two subsystems: electrical and mechanical. Connections between these systems are shown in Fig. 8. The type of analysis is continuous and a powergui block is necessary for power drive system analysis.

![Simulink model](image)

**Fig. 8 Simulink model**

3.1 The Electrical Subsystem

The electrical subsystem implies the mathematical modelling of the electrical part of the power-drive. It contains the synchronous generator block connected directly to the induction motor block. Torque and speed relation derived in sec. 3 are implemented in this block. The inputs are speed of arm \(W_A\) and speed of ring \(W_R\) (the letter W used instead of the symbol \(\omega\)). \(W_A\) input is connected to the synchronous generator and hence it determines the electrical synchronous speed of the system. \(W_R\) input is connected to the rotor of induction motor and hence it represents the shaft speed of the motor. Electrical slip \(s\) is calculated according to the two inputs \((W_A\) and \(W_R\)) as in equation (10). The electrical subsystem exports two signals: motor torque, \(T_m\) and generator torque, \(T_G\). It contain also all the necessary measuring scopes and devices.

3.2 The Mechanical Subsystem

The mechanical subsystem is shown in Fig. 9. This unit contain mechanical parts of the power-drive, Tyre block, vehicle block, ICE block and driving cycle block. The epicyclic unit is connected to the ICE engine as the primary source of power in one terminal. It delivers power to the arm (=generator) through the second terminal and receives power through third
terminal, the ring (=motor). Inertia blocks are used at each terminal of the differential block. Motion sensor block senses the motion of a driveline axis. The block can output the motions angular velocity (\(\omega\)), in radians/second. The Torque Actuator block actuates the connected driveline axis with a torque. You specify this torque as a Simulink input signal in Newton-meters. The generator torque represents a load on the mechanical unit so it multiplied with (-1) while motor torque is multiplied with (+1) because it encourages movement. The two inputs Tm and Tg are Simulink signals, so a conversion block is needed to convert them to power-drive signals, the block is the torque actuator. Inertia blocks are necessary to make the model simulate real systems. Other blocks such as vehicle and tyre blocks are standard blocks in Simulink program and can be customised according to real systems. Shaft sensors are necessary to convert power drive signal into Simulink type. Vehicle model used in this simulation assumes a mass of 1200kg. Tyre block receives mechanical signal and coverts it to driving forces.

Fig.9 Mechanical subsystem
4. Tests and Results:

Many tests were carried on the model (EMEC) to show model validity. All tests are performed in the environment of Matlab-simulink. The first test was to show the response to reference speed. Fig. 10 shows the references speed, throttle and vehicle speed. In driving cycle block, a virtual reference speed is considered. The profile assumed to be a ramp signal. The throttle opening seems to response well to the reference speed demand. It is saturated at a value of unity (full throttle). The developed power drive show accepted response. The vehicle moves smoothly from stationary to the desired speed. The time needed to pick the desired speed depends on the initial design parameters such as engine horsepower, motor size and vehicle properties.

![Fig.10 Response to reference speed](image-url)
The other test was to show the automatic gear shifting capability. Fig. 11 shows the motor and generator torque verses vehicle speed (Vx). Motor poles number is 4 times that of the generator, this led to the electrical torque gain shown. The automatic gear shifting is clear from the descending value of the torque against the vehicle speed which is exactly the situation in automatic transmission. This tests show the (CVT) capability of the EMEC power drive. For a vehicle speed of 9 m/s for example, the value of $T_m$ is about 700 N.m while the value of $T_G$ is 220 N.m. The ratio is about 3.2. When the vehicle speed increased it is no further need for high torques and one can see that the values of $T_m$ and $T_G$ begin to decrease. It is the desired behavior in real Vehicles, where high torqueses are needed for speedup and less torques are needed for steady high-speeds. At a speed of 24 m/s the values of $T_m$ and $T_G$ are 300 and 75 Nm respectively. The ratio is about 4 which is the pole ratio (m). It is clear also from the figure that although the torques $T_m$ and $T_G$ are decrease with vehicle speedup, the ratio $T_m/T_G$ is seemed to be fixed.

![Fig.11 Automatic gear shifting test](image)

The second test was to show the in/out efficiency of the power-drive. Fig. 12 shows the drive efficiency against vehicle speed. The efficiency is inversely related to copper losses of the electrical machines. At start up, where low rotating speeds and hence low terminal voltage the losses are small and hence the efficiency appears to be near 100%, as the vehicle grew faster, the electrical machines are also faster and hence high terminal voltage and high power losses. When the vehicle about to pick its top-gear speed, the need for electrical torque became less and hence less electrical power is needed, this means that the power drive behave electro-mechanic at low speeds and became approximately 100% mechanically at near
top-gear speed. This is the major advantage of the EMEC power drive. The estimated efficiency represents the power drive efficiency. It is calculated by dividing the motion power (force x speed) by the total mechanical and electrical powers.

![Power-drive efficiency against vehicle speed](image1.png)

**Fig.12 Power-drive efficiency against vehicle speed**

The automatic mixing behaviour of the proposed power drive between mechanical and electrical behaviour has a major role in governing power losses in conductor's resistance. The electrical behaviour is activated at low speeds and high torques and it is eliminated at high speeds.

Fig.13 shows the effect of selecting a certain pole ratio \( m \), equation (20), on torque gain \( \frac{T_m}{T_G} \). The result is expected according to equation (21). This can also eliminate the use of post gear reducer to adjust the propulsion torque. It is important to note that the selection of \( m \) can be carried on the phase of design. High values are suggested for heavy vehicles and light values can be used for light vehicles. Now days, some electric machine can shade some poles and hence variable pole numbers can be assumed. This can add extra flexibility for speed ratios during operation as well as during initial design.

![Electrical torque gain \( \frac{T_m}{T_G} \) verses pole ratio \( m \)](image2.png)

**Fig.13 Electrical torque gain \( \frac{T_m}{T_G} \) verses pole ratio \( m \)**
5. Conclusions

The developed model (EMEC) proved to drive the vehicle according the driving cycle demand. It gives high electrical torque at start up and reduces that torque when vehicle begin to pick its speed. The Generator-motor set is connected directly and hence no complicated control circuits are needed. This also allows for simple control strategies for Hybrid electric vehicle which is the main goal of this paper.
The EMEC power-drive behaves electro-mechanically at low speeds, where high torques is needed by electrical torque gain between generator and motor. At speeds near the Top-Gear speed it behaves approximately 100% mechanically; this will eliminate the electrical losses. This behaviour gave the model advantages over the serial hybrid configuration where the electrical losses still acts even near high vehicle speeds.
Properly selecting the motor/generator pole ratio results in more flexibility in power-drive design. It allows for properly selecting electrical torque-gain.

6. Recommendations

Adding driving circuits (inverters) between Generator and motor could affect the torque ratio of the two machines and allow for battery power to be delivered to the motor. The regeneration process could be tested and validated in the future works.

7. References


## 8- Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
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<tr>
<td>$\alpha, \beta$</td>
<td>Constants</td>
<td>$R_G$</td>
<td>Generator resistance (Ohm)</td>
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<td>Motor resistance (Ohm)</td>
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<td>Continuously varying transmission</td>
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<td>Slip</td>
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<td>Sun torque (N.m)</td>
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<td>Internal combustion</td>
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<td>Arm speed (rad/sec)</td>
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<td>Equivalent reactance (Ohm)</td>
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<td>Equivalent impedance/motor</td>
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Use of Neural Networks to Predict Ultimate Strength of Circular Concrete Filled Steel Tube Beam-Columns

Ahmed Sagban Saadoon
Department of Civil Engineering, University of Basrah, Iraq, e-mail:asagban@yahoo.com

Kadhim Zuboon Nasser
Department of Civil Engineering, University of Basrah, Iraq, e-mail:kadhimzuboon@yahoo.com

Abstract

Artificial neural networks (ANNs) are useful computing system which can be trained to learn complex relationship between two or more variables. It learns from examples and storage the knowledge for future use. In this study, a model for predicting the ultimate strength of circular concrete filled steel tube (CCFST) beam-columns under eccentric axial loads has been developed in ANN. The available experimental results for 181 specimens obtained from previous studies were used to build the proposed model. The predicted strengths obtained from the proposed ANN model were compared with the experimental values and current design provision for CCFST beam-columns (AISC and Eurocode4). Results showed that the predicted values by the proposed ANN model were very close to the experimental values and were more accurate than the AISC and Eurocode4 values. As a result, ANN provided an efficient alternative method in predicting the ultimate strength of CCFST beam-columns.

Keywords: beam-columns, artificial neural networks, concrete filled steel tube.

استخدام الشبكات العصبية في تقدير المقاومة القصوى للعنبات - الأعمدة ذات مقطع أنبوب حديدي دائري مملوء بالخرسانة

المستخلص

إن الشبكات العصبية نظام مفيد ممكن تدريبه لتعليم العلاقات المعقدة بين عدة متغيرات من خلال إدخال مجموعة من الامثلة الحقيقية. إن الهدف الرئيسي من الدراسة الحالية هو بناء شبكة عصبية لتقدير مقاومة العنبات - الأعمدة ذات مقطع أنبوب حديدي دائري مملوء بالخرسانة والمعرضة إلى أحمال ضغط لامركزية. وقد استعملت النتائج المختبرية لـ(181) عينة (مستخلصة من بحوث سابقة) في بناء الشبكة المترحزة. وقررّت القيم المقترحة من هذه الشبكة مع القيم المختبرية ومع القيم المحسوبة على ضوء شرط التصميم في الكودين العالميين Eurocode4 و AISC. لقد أظهرت النتائج أن القيم المقترحة من الشبكة المترحزة كانت قريبة جداً من القيم المختبرية وكانت أدقّ من القيم المحسوبة حسب مواصفات الكودين المذكورين. وبالتالي فأنه من الممكن استخدام الشبكات العصبية في تقدير مقاومة مثل هذا النوع من العنبات - الأعمدة.
1. Introduction

Beam-columns are members that are subjected simultaneously to axial forces and bending moments. Thus, their behavior falls somewhere between that of a pure axially loaded column and that of a beam with only moments applied. Also, their behavior must include the effects of the axial loads on the flexural stiffness. This is usually referred to as the second-order elastic analysis. To understand the behavior of beam-columns, it is common practice to look at the response as predicted through an interaction equation between axial loads and moments.

Numerous different structural systems are used today to meet performance or functional requirements in structures. Composite construction is widely used in structural systems to achieve long spans, lower story heights, and provide additional lateral stiffness. Composite construction uses the structural and constructional advantages of both concrete and steel. Concrete has low material costs, good fire resistance, and is easy to place. Steel has high ductility and high strength-to-weight and stiffness-to-weight ratios. When properly combined, steel and concrete can produce synergetic savings in initial and life-cycle costs. Currently composite floor systems are widely utilized in steel buildings in the form of composite beams and joists/trusses. There are two basic kinds of composite beams or columns: steel sections encased in concrete (steel-reinforced concrete sections or SRC) and steel sections filled with concrete (concrete filled tubes or CFT). The latter can be either circular (CCFT) or square/rectangular (RCFT) in cross-section. In composite columns additional synergies between concrete and steel are possible: (a) in concrete-filled tubes, the steel increases the strength of the concrete because of its confining effect, the concrete inhibits local buckling of the steel, and the concrete formwork can be omitted; and (b) in encased sections, the concrete delays failure by local buckling and acts as fireproofing while the steel provides substantial residual gravity load-carrying capacity after the concrete fails.

The structural behavior of circular concrete filled steel tube (CCFST) beam-columns has been investigated through many experimental tests [1-5]. The main objective of these tests was to determine the different parameters that influence the beam-columns structural behavior.

For the last two decades, different modeling methods based on artificial neural networks (ANN) have become popular and have been used by many researchers for a variety of civil engineering applications [6-10]. ANNs are natural complementary tools in building intelligent systems and are low level computational structures that perform well when dealing with raw data. The basic strategy for developing ANN systems based models for material behavior is to
train ANN systems on the results of a series of experiments using that material. If the experimental results contain the relevant information about the material behavior, then the trained ANN systems will contain sufficient information about material’s behavior to qualify as a material model. Such trained ANN systems not only would be able to reproduce the experimental results, but also they would be able to approximate the results in other experiments through their generalization capability.

The aim of this study is to propose a model using ANN to predict the ultimate strength of CCFST beam-columns under eccentric axial loads (Fig. (1)).

![Diagram of CCFST beam-column under eccentric axial loads](image)

**Fig. (1) CCFST beam-column under eccentric axial loads**

2. Artificial Neural Networks (ANN)

2.1. General

An Artificial Neural Network (ANN) is a computational tool that attempts to simulate the architecture and internal features of the human brain and nervous system. Comparing ANN with other digital computing techniques, ANNs are advantageous because of their special features such as the possibility of non–linear modeling relationship between input and target specially for problem where the relationships are not very well known and low sensitive to error.

The first structural engineering application of ANN goes back only to the year 1990. Since then a wide range of applications have emerged. These applications include [11]:-
- Mapping of input–output data of non–linear relation for materials and structures.
- Damage identification of structures and structural control against dynamic loads.
- Preliminary design of structure.
- Optimum design and analysis.
ANNs are enormously parallel systems composed of many processing elements connected by links of variable weights. Generally, ANNs are made of an input layer of neurons, sometimes referred to as nodes or processing units, one or several hidden layer of neurons and output layer of neurons. The neighboring layers are fully interconnected by weight. The input layer neurons receive information from the outside environment and transmit them to the neurons of the hidden layer without performing any calculation. Layers between the input and output layers are called hidden layers and may contain a large number of hidden processing units. All problems, which can be solved by a perceptron can be solved with only one hidden layer, but it is sometimes more efficient to use two or three hidden layers. Finally, the output layer neurons produce the network predictions to the outside world.

Figure (2) shows a symbol neuron model with input, sum function, sigmoid activation function and output. The input to a neuron from another neuron is obtained by multiplying the output of the connected neuron by the synaptic strength of the connection between them. The weighted sums of the input components \( (\text{net})_j \) are calculated by using the following equation:

\[
(net)_j = \sum_{i=1}^{n} W_{ij} Y_i + b,
\]  

Where \((\text{net})_j\) is the weighted sum of the \(j\)th neuron for the input received from the preceding layer with \(n\) neurons, \(W_{ij}\) is the weight between the \(j\)th neuron in the preceding layer, \(Y_i\) is the output of the \(i\)th neuron in the preceding layer. The quantity \(b\) is called the bias and is used to model the threshold. The output signal of the neuron, denoted by \(Y_j\) in Fig. (2), is related to the network input \((\text{net})_j\) via a transformation function called the activation function. The most common activation functions are sigmoid and Gaussian function due to their nonlinearity property. The output of the \(j\)th neuron \(Y_j\) is calculated by using Eq. (2) with a sigmoid function as follows:

\[
Y_j = f(\text{net})_j = \frac{1}{1 + e^{-\alpha(\text{net})_j}},
\]  

Where \(\alpha\) is a constant used to control the slope of the semi-linear region. The sigmoid function represented by Eq. (2) gives outputs in the range (0,1).
2.2. Feed-forward Neural Networks

Feed-forward NNs are the most popular and most widely used models in many practical applications. They are known by many different names, such as multilayer feed-forward and multilayer perceptrons. In a feed-forward NN, the artificial neurons are arranged in layers, and all the neurons in each layer have connections to all the neurons in the next layer. However, there is no connection between neurons of the same layer or the neurons which are not in successive layers. In general, the feed-forward NN consists of one input layer, one or two hidden layers and one output layer of neurons. The input layer receives input information and passes it onto the neurons of the hidden layer(s), which in turn pass the information to the output layer. The output from the output layer is the prediction of the net for the corresponding input supplied at the input nodes. Each neuron in the network behaves in the same way as discussed in Eqs. (1) and (2). There is no reliable method for deciding the number of neural units required for a particular problem. This is decided based on experience and a few trials are required to determine the best configuration of the network. In this study, the multilayer feed-forward type of neural networks, as shown in Fig. (3), is considered.

Fig. (2) A simple neuron model.
2.3. Back-propagation Algorithm

Multi-layer perceptrons are trained with supervised learning rules. Hopefully, a network that produces the right output for a particular input will be obtained. The most widely used supervised learning algorithm for neural networks is the back propagation, also known as error back propagation. Training is implemented by adjusting the weights according to the error (the distance between the target and the actual output vector) in the output layer. The learning error for rth example is calculated by the following performance function usually called the mean-square error:

$$E = \frac{1}{2} \sum (T_j - Y_j)^2,$$

(3)

Where $T_j$ is the target output at neuron $j$ and $Y_j$ is the output predicted at neuron $j$. As presented in Eqs. (1) and (2) the output $Y_j$ is a function of synaptic strength and outputs of the previous layer. In the back-propagation phase, the error between the network output and the desired output values is calculated using the so called generalized delta rule, and weights between neurons are updated from the output layer to the input layer. These operations are repeated for each example and for all the neurons until a satisfactory convergence is achieved.
for all the examples present in the training set. The training process is successfully completed, when the iterative process has converged. The connection weights are captured from the trained network, in order to use them in the recall phase. There are several different back propagation training algorithms. They have a variety of different computation and storage requirements and no one algorithm is best suited to all locations. The resilient back propagation (RPROP) is an algorithm for feed forward networks that often provides faster convergence; therefore it is used in this study.

3. ANN for Beam-columns:

The computer program “MATLAB version 7.0 Neural Network Toolbox” is employed for the proposed ANN model in this study. The advantage of using this program is that many types of networks are included in the program and many training algorithms with different properties can be used for a specific network model. An ANN model was developed to predict the ultimate strength of CCFST beam-columns under eccentric axial loads.

3.1. Selection of Training and Testing Data

The experimental data that are used to build the NN model are obtained from a database developed by Kim [12]. The data used to build the NN model should be divided into two subsets: training data and validating or testing data. The testing data contains approximately 20% from total database. The training phase is needed to produce a NN that is both stable and convergent. Therefore, selection of what data to use for training a network is one of most important steps in building a NN model. The total number of (181) test beam-columns were utilized. The training data contained (147) samples and the testing data comprised of (34) samples which were selected randomly. ANNs interpolate data very well. Therefore, patterns chosen for training set must cover upper and lower boundaries [13].

3.2. Input and Output Layer

The nodes in the input layer and output layer are usually determined by the nature of the problem. In this study the parameters which may be introduced as the components of the input vector consist of yield stress of steel tube ($f_y$), cylinder concrete compressive strength ($f'_c$), outside diameter of circular steel tube ($D$), thickness of steel tube ($t$), laterally unbraced length of member ($L$), and eccentricity of applied load ($e$). The output vector is the ultimate axial load ($P$). Table (1) summarizes the ranges of each different variable.
Table (1) Range of input parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
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<tr>
<td>Yield stress of steel tube ($f_y$) (MPa)</td>
<td>185-435</td>
</tr>
<tr>
<td>Cylinder concrete compressive strength ($f'_c$) (MPa)</td>
<td>20-113</td>
</tr>
<tr>
<td>Outside diameter of circular steel tube ($D$) (mm)</td>
<td>95-324</td>
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<tr>
<td>Thickness of steel tube ($t$) (mm)</td>
<td>0.9-12.8</td>
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<tr>
<td>Laterally unbraced length of member ($L$) (mm)</td>
<td>360-4968</td>
</tr>
<tr>
<td>Eccentricity of applied load ($e$) (mm)</td>
<td>0.3-337</td>
</tr>
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</table>

3.3. Proposed ANN Model

A multilayered feed-forward NN with a resilient back-propagation algorithm was employed in the present study. The NN architecture developed has six neurons in the input layer and one neurons in the output layer as demonstrated in Fig. (4). Two hidden layers were used in the architecture of multilayer feed-forward NN due to its minimum absolute percentage error values for training and testing sets. In the first hidden layer eight and in the second hidden layer two neurons were determined. The transfer (activation) functions used are hyperbolic tangent (tansig) function in first hidden layer and linear (purelin) function in both second hidden and output layer.

Fig. (4) Architecture of proposed ANN
4. Results and Discussion

An important aspect of developing ANNs is determining how well the network performs once training is complete. The performance of a trained network is checked by involving two main criteria:

(1) How well the NN recalls the predicted response from data sets used to train the network (called the recall step). A well trained network should be able to produce an output that deviates very little from desired value.

(2) How well the NN predicts responses from data sets that were not used in the training (called the generalization step). A well generalized network should be able to sensible the new input patterns.

The performance of the proposed ANN is tested by the regression analysis between the output of this network (predicted values) $P(\text{ANN})$ and the corresponding targets (experimental values) $P(\text{exp})$ for both training and testing data as shown in Figs. (5) and (6), respectively. The coefficient of correlation ($R^2$) is a measure of how well the variation in the output is explained by the targets. If this number is equal to 1, then there is a perfect correlation between targets and output. In these figures, the coefficient of correlation $R^2 = 0.984$, 0.979 for training and testing data respectively. These values indicate an excellent agreement between the predicted values and the experimental values.

![Figure (5) Regression analysis between predicted and actual values for training data](image5)

![Figure (6) Regression analysis between predicted and actual values for testing data](image6)
Table (2) presents the actual and predicted values for testing data. As seen from this table, the values obtained are very close to the experimental results. The average value of ratios of actual to predicted ultimate loads is 1.028 with a standard deviation of 0.147. This result demonstrates that ANN can be successfully applied to establish accurate and reliable prediction models.

**Table (2) Actual (experimental) and predicted values for testing data**

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<tr>
<th>Column designation</th>
<th>f_y (MPa)</th>
<th>f_c' (MPa)</th>
<th>D (mm)</th>
<th>t (mm)</th>
<th>L (mm)</th>
<th>e (mm)</th>
<th>P(exp) (kN)</th>
<th>P(ANN) (kN)</th>
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<td>1879</td>
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</table>

Average 1.028

Standard Deviation 0.147
Based on these results, the proposed ANN architecture (6-8-2-1) with activation functions (tansig, purelin, purelin) with (RPROP) is used for this study. Table (3) shows the properties of this network.

Table (3) Values of parameters used in the proposed ANN model

<table>
<thead>
<tr>
<th>Number of input layer neurons</th>
<th>Number of hidden layer</th>
<th>Number of first hidden layer neurons</th>
<th>Number of second hidden layer neurons</th>
<th>Number of output layer neuron</th>
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<th>Learning cycle</th>
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<td>1</td>
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</table>

5. Comparison with Design Strengths

The predicted strengths, of the CCFST beam-columns in Table (2), obtained from the proposed ANN model are compared with unfactored design strengths predicted using the design procedure specified in the American Institute of Steel Construction (AISC) [14] and the Eurocode 4 [15] for CCFST beam-columns as calculated by Kim [12]. The predicted strengths of the proposed ANN model P(ANN) are compared with the design strengths calculated using AISC specifications P(AISC) and the design strengths calculated using Eurocode 4 specifications P(Euro) as shown in Table (4). The values of P(exp)/P(ANN), P(exp)/P(AISC) and P(exp)/P(Euro) ratios with the corresponding averages and standard deviations are shown also in this table.

Fig. (7) Regression analysis between predicted and actual values
It can be seen, from Table (4), that the average ratio of actual to predicted loads is 1.028 for ANN, 1.168 for AISC, and 1.249 for Eurocode4, and that the standard deviation is 0.147 for ANN, 0.236 for AISC, and 0.179 for Eurocode4. Therefore the design strengths calculated using AISC and Eurocode4 specifications are generally conservative.

**Table (4) Comparison between actual (experimental) and predicted values**

<table>
<thead>
<tr>
<th>Column designation</th>
<th>( P(\text{exp}) ) (kN)</th>
<th>( P(\text{ANN}) ) (kN)</th>
<th>( P(\text{AISC}) ) (kN)</th>
<th>( P(\text{Eurocode}) ) (kN)</th>
<th>( \frac{P(\text{exp})}{P(\text{ANN})} )</th>
<th>( \frac{P(\text{exp})}{P(\text{AISC})} )</th>
<th>( \frac{P(\text{exp})}{P(\text{Euro})} )</th>
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In Fig. (7), the predicted strengths $P(\text{ANN})$ and the design strengths $P(\text{AISC})$ and $P(\text{Euro})$ are plotted against the experimental strengths. As shown in this figure, the coefficient of correlation $R^2 = 0.979$, 0.923 and 0.944 for ANN, AISC, and Eurocode4, respectively. These values indicate that the proposed ANN model can predict more accurate results than AISC and Eurocode4 methods and that ANN provided an efficient alternative method in predicting the ultimate strength of CCFST beam-columns.

6. Conclusions

The most important conclusions that can be drawn from the present study are the followings:

1. The ultimate strength of CCFST beam-columns can be predicted by the proposed ANN model in a quite short period of time with tiny error rates.
2. The predicted ultimate strength values were very close to the experimental results.
3. It was noticed that the design strengths calculated using AISC and Eurocode4 specifications are generally conservative.
4. The predicted strengths obtained from the proposed ANN model were compared with current design provision for CCFST beam-columns (AISC and Eurocode4). It was found that the proposed ANN model can predict more accurate results than AISC and Eurocode4 specifications.
5. The above conclusions have demonstrated that ANN provided an efficient alternative method in predicting the ultimate strength of CCFST beam-columns.

References


Notation

The following symbols are used in this paper:

\( \alpha \) = constant;
\( b \) = bias (used to model the threshold);
\( D \) = outside diameter of circular steel tube (mm);
\( E \) = mean-square error;
\( e \) = eccentricity of applied load (mm);
\( f'c \) = cylinder concrete compressive strength (MPa);
\( f_y \) = yield stress of steel tube (MPa);
\( L \) = laterally unbraced length of member (mm);
\( \text{net}_j \) = weighted sums of the input components;
\( P \) = ultimate eccentric axial load;
\( R^2 \) = coefficient of correlation;
\( T_j \) = target output at neuron \( j \);
\( t \) = thickness of steel tube (mm);
\( W_{ij} \) = weight between the \( j \)th neuron in the preceding layer; and
\( Y_i \) = output predicted at neuron \( i \).
Analysis of High Strength Concrete Circular Columns under Axial and Lateral Loading Combinations

Mohammed A. Mashrei
Civil Engineering Department / College of Engineering / Thi_Qar University
moha74ed@yahoo.com

Abstract

Columns are vertical compression members which carry primarily axial Compression load. The axial load may be associated with bending moments in one or two directions. Analysis of high strength concrete columns with circular cross section and spiral transverse reinforcement was presented in this paper. A computer program was used to do the analysis of high strength circular concrete columns. The variables considered in this article were concrete strength ranging from 55 MPa to 80 MPa, volumetric ratios of steel used to confine the core concrete, amount of main reinforcement and axial load level. The results indicate that as increase in the amount of lateral steel, main steel, axial load and increase in concrete strength resulted in increases in strength capacity of column. The deformability of high-strength concrete columns can be improved significantly through confinement. The results obtained in this research are agreed with results obtained by experimental and analytical study conducted by other authors.

Keywords: High-strength concretes; Concrete columns; Computer analysis; Moment–curvature
1. Introduction

Strength of concrete used in the construction has increased gradually over the years. Concretes of up to 120 MPa compressive strength have been used in structural applications, especially in columns of selected multistory buildings [1]. The main advantages of using high strength concrete in building constructions are: increased strength of structures, reduced cross-sections and more durable material. One advantageous use of high-strength concrete is in the columns of tall structures. For a given load, the high-strength concrete column has a smaller cross-sectional area, thus providing more floor space.

Columns are members that carry loads chiefly in compression. Nevertheless, their contribution in horizontal stiffness of building frames is also of great importance. The main reinforcement in columns is longitudinal, parallel to the direction of the axial load, and consists of bars arranged in a square, rectangular or circular pattern. The design primarily considers the compression and bending moments about one or both axes of the cross section. When strong horizontal shaking, as one during an earthquake, is transmitted, the columns may undergo lateral deflection which in turn affects the horizontal stiffness, therefore the study of high strength column is very important. Due to limited amount of experimental research and to the uncertainty inherent in the prediction of failure of the structural elements under earthquake loading, the use of high strength concrete structures in seismic risk areas needs extra caution in order to ensure adequate ductile behavior [2]. Some widely accepted properties of high-strength concrete, as reported in several studies [3-5], are its higher modulus of elasticity, less ductile mode of failure, and larger strain at maximum stress [6]. As mentioned, high-strength concrete offers advantages in performance and economy of construction, but, the brittle behavior of the material remains a major drawback for seismic applications. Since strength and ductility of concrete are inversely proportional, therefore, concrete confinement becomes a critical issue for high-strength concrete columns in seismically active regions. The use of transverse reinforcement in reinforced concrete columns provides confinement to compressed concrete, prevents premature buckling of compressed longitudinal bars, and acts as shear reinforcement. The quantities of transverse reinforcement present in columns designed for seismic resistance should ensure ductile behavior during severe earthquake loading. In the seismic design of moment resisting frames of buildings it is possible to use a strong column-weak beam approach to reduce the likelihood of plastic hinging in columns during a major earthquake [7].
This paper presents an analysis of high-strength concrete columns subjected to combine axial and lateral loading. The columns have a circular cross section and spiral circular transverse reinforcement. The variables studied in this article are the concrete strength, volumetric ratios of steel used to confine the core concrete, amount of main reinforcement and axial load level. The results were compared with experimental results to verify the accuracy of the analysis method. The material model for concrete used in this analysis is based on a model suggested by Mander et al.[8]. This model takes into account the different stress-strain curves of the concrete cover and the confined core. King et al. model [9] was used for the reinforcing steel.

2. Material models

The material constitutive models used in the program are described in the following sections: These include a concrete model for unconfined and confined concrete, and reinforcing steel models.

2.1 Model Proposed by Mander et al. (1988) [8]

Mander et al. [8] proposed a stress-strain model for steel-confined concrete subjected to uniaxial compressive stress (Figure 1). This model is based on the axial compressive tests of concrete with a quasi-static strain rate and monotonic loading. Their model utilized the equations developed by Popovics [10], originally proposed for stress-strain response of unconfined concrete:

\[ f_c = \frac{f_{cc} \cdot x \cdot r}{r - 1 + x^r} \]  

Where:

\[ x = \frac{\varepsilon_c}{\varepsilon_{cc}} \]

\( \varepsilon_c \): longitudinal compressive concrete strain

\( \varepsilon_{cc} = \varepsilon_{co} \left\{ 1 + 5 \left( \frac{f_{cc}}{f_{co}} - 1 \right) \right\} \]  

\( \varepsilon_{co} \): unconfined concrete strain

\( f_{cc} \): confined concrete strength

\( f_{co} \): unconfined concrete strength

\[ r = \frac{E_c}{E_c - E_{sec}} \]
\[
E_{sec} = \frac{f_{cc}'}{\varepsilon_{cc}}
\]

\[
\varepsilon_{cu} = 1.4 \left( 0.004 + \frac{1.4 \rho_s f_{sh} \varepsilon_{su}}{f_{cc}'} \right)
\]

\[E_c: \text{tangent modulus of elasticity of concrete}\]

\textbf{For circular sections:}

\[
f_{cc}' = f_{co}' \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f_t'}{f_{co}'}} - 2 \frac{f_t'}{f_{co}'} \right)
\]

\[
f_t' = \frac{1}{2} k_e \rho_s f_{yh}
\]

\[f_{yh}: \text{yielding stress of transverse reinforcement}\]

\[\rho_s = \frac{4 A_{sp}}{d_s s}\]

\[A_{sp}: \text{cross section area of spiral}\]

\[s: \text{distance between spirals, center to center}\]

\[d_s: \text{diameter of the core (center to center of spirals)}\]

\[
k_e = \frac{1 - \frac{s'}{2d_s}}{1 - \rho_{cc}}
\]

\[s': \text{clear distance between spirals}\]

\[\rho_{cc}: \text{ratio of area of longitudinal reinforcement to area of core section}\]

And for unconfined concrete use equ. (1) with a lateral confined stress \(f_t' = 0\). The part for strains larger than \(2\varepsilon_{co}\) is assumed to be a straight line which reaches zero at \(\varepsilon_{sp}\) as shown in Fig.1.

The ultimate strain was defined to be the strain at first hoop fracture and is calculated from an energy balance approach. The shaded area in Fig. 1 represents the increase in strain energy at failure resulting from confinement, which is equal to the strain energy capacity of the confining reinforcement. By equating the ultimate strain energy capacity of the confining reinforcement per unit volume of concrete core to the shaded area plus additional energy required to maintain yield in the longitudinal steel in compression, the ultimate strain corresponding to hoop fracture can be calculated as above by equ.(4).
2.2 Models for the Reinforcing steel:

2.2.1 King et al. model [9]

The stress-strain relation for the reinforcing steel Fig.2 is the same used by King et al. [9].

\[
f_s = E_s \varepsilon_s \quad \text{for} \quad \varepsilon_s \leq \varepsilon_y \tag{5}
\]

\[
f_s = f_y \quad \text{for} \quad \varepsilon_y < \varepsilon_s < \varepsilon_{sh} \tag{6}
\]

\[
f_s = f_y \left[ \frac{m(\varepsilon_s - \varepsilon_{sh}) + 2}{60(\varepsilon_s - \varepsilon_{sh}) + 2} + \frac{(60 - m)(\varepsilon_s - \varepsilon_{sh})}{2(30r + 1)^2} \right] \quad \varepsilon_{sh} < \varepsilon_s \leq \varepsilon_{sm} \tag{7}
\]

Where:

\[
m = \frac{(f_{su}/f_y)(30r + 1)^2 - 60r - 1}{15r^2}
\]

\[
r = \varepsilon_{su} - \varepsilon_{sh}
\]

Where:

\(f_s\) = steel stress

\(\varepsilon_s\) = steel strain,

\(\varepsilon_{sh}\) = steel strain at commencement of strain hardening

\(f_{su}\) = ultimate tensile strength of steel

\(f_y\) = yield strength of steel
2.2.2 Raynor et al. Model [11]

As proposed by Raynor et al. (2002).

\[ f_s = E_s \varepsilon_s \quad \text{for} \quad \varepsilon_s \leq \varepsilon_y \]  \hspace{1cm} (8)

\[ f_s = f_y + (\varepsilon_s - \varepsilon_y)E_y \quad \text{for} \quad \varepsilon_y < \varepsilon_s < \varepsilon_{sh} \]  \hspace{1cm} (9)

\[ f_s = f_u - (f_s - f_{sh}) \left( \frac{\varepsilon_{sm} - \varepsilon_s}{\varepsilon_{sm} - \varepsilon_{sh}} \right)^{C1} \quad \varepsilon_{sh} < \varepsilon_s < \varepsilon_{sm} \]  \hspace{1cm} (10)

Where:

\[ \varepsilon_y = \frac{f_y}{E} \]

\[ f_{sh} = f_y + (\varepsilon_{sh} - \varepsilon_y)E_y \]

\( E_y \): is the slope of the yield plateau

\( C1 \): is the parameter that defines the curvature of the strain hardening curve as shown in Fig. 3.
3. Analytical Models

Set of codes have been written with Matlab program depend on CUMBIA codes prepared by Montejo and Kowalsky [12] were developed to carry out calculations for theoretical moment-curvature relations of columns with a circular cross. Effect of concrete strength, volumetric ratios of steel used to confine the core concrete, amount of main reinforcement and axial load level on the behavior of column under axial load and lateral load were investigated. The required input data included cross-sectional dimensions of specimens, position, and amount of longitudinal steel, amount of and spacing of transverse steel, properties of longitudinal and transverse steel, unconfined concrete strength and applied axial load. The column was divided into 40 small slices, each one containing two types of elements, core, and cover. It must be expressed that reasonably small tolerance value (0.001) is chosen to stop the procedure.

The analysis procedure involved by assign an initial value of compressive strain at extreme concrete fiber and then increasing levels of the strain; an iterative procedure is used to find the neutral axis depth to satisfy equilibrium at each level of concrete strain; calculate strain at the middle of each element and in longitudinal steel bars. The program stops when the concrete strain in the core exceeds the maximum concrete strain, the tension strain in the steel bars exceeds the maximum steel strain or there is a suddenly lost of strength. The program may also stops if the maximum number of iteration is reached. If the solve of problem does not achieve, the allowable tolerance can be also changed.
5. Results and Discussion

In the research described in this paper, a series of moment curvature analyses was performed on circular columns with varying strength of concrete, levels of axial load ratio, spacing of lateral reinforcement and longitudinal reinforcement ratio. The axial load was varied from 0.1 to 0.5$f_c'$ $A_g$. The baseline section has the following information: the diameter of column was 300 mm. The length of column was 1000 mm. The concrete compressive strength was 55 MPa, while the yield stress of all reinforcement was 460 MPa. The longitudinal reinforcement ratio was chosen as 1.1%, and the transverse bar diameter was chosen as 10mm @ 120mm. The concrete cover to the main reinforcement was 20 mm.

The effects of different variables have been studied by comparing moment-curvature relations of the sections of those columns in which only one major variable differed significantly. These variables are:

5.1 Effect of Compressive Strength

The concrete compressive strength is important parameter. Three different concrete strength (55MPa, 65 MPa and 80 MPa) were considered to investigate this parameter in this study, all these values within high strength concrete range. Figures 4(a) to 4(e) show the effect of the concrete strength on moment-curvature relationships. The axial load was varied from 0.1 to 0.5$f_c'$ $A_g$ respectively for Fig. 4(a) through Fig. 4(e). All figures indicate that higher ductility is obtained in the lower strength concrete column and at lower axial load ratio. While the overall strength of column is increased with increasing the concrete strength. The results obtained here have the same trend as those obtained experimentally by Sheikh et al. [13], Sungjoong[15].
(a) Axial load ratio is 0.1

(b) Axial load ratio is 0.2

(c) Axial load ratio is 0.3
5.2 Effect of amount lateral steel

The effect of amount of the lateral reinforcement was investigated by considering three different values of spacing of lateral reinforcement (35 mm, 70 mm and 120 mm). All other parameters were kept as a baseline section as mentioned above. Figures 5(a) to 5(e) illustrate the effect of spacing of lateral reinforcement on strength and deformability of high strength concrete column. As shown from Figures a smaller spacing enhancement the strength and ductility of circular column. This may be due to that a smaller spacing increases the confined concrete area, resulting in higher confinement efficiency. Also it can be seen that more sudden drop of load resistance after peak load occurred in the columns with lower volumetric ratio or larger spacing of transverse steel. It may be concluded from these results that reducing
The spacing of lateral reinforcement would result in an increased moment capacity of the section. Ductility would also be improved. The reduction of the spacing, from 120 mm to 35 mm for axial load ratios of 0.1, 0.2, 0.3, 0.4 and 0.5 results in increases of the strength gain (16%, 11%, 11%, 14% and 21%) respectively. Also it can be noted that the ductility decreases with increasing the axial load ratio. The results obtained here have the same trend as those obtained experimentally by Sheikh et al. [13].

(a) Axial load ratio is 0.1

(b) Axial load ratio is 0.2
Fig. 5 Effect of spacing of lateral steel on moment-curvature relationships
5.3 Effect of longitudinal reinforcement ratio

Figures 6(a) - 6(e) compare the strength and the ductility of columns with a lower ratio of longitudinal steel to the strength and the ductility of columns with a higher ratio of longitudinal steel. A larger amount of longitudinal bars, provided by a larger bar diameter. Three different ratios of longitudinal reinforcement were considered (1.1%, 2.4 % and 4.2%). Other parameters are almost same in each comparison. From the comparisons, the strength (moment capacity) increases with increasing the ratio of longitudinal reinforcement. The ductility seem to be slightly larger for columns with higher ratio of longitudinal steel. The larger amount of longitudinal steel may provide more restraining action against an inclined shear failure as dowels compared to small amount of longitudinal steel [13]. Therefore, it can be concluded that the amount of longitudinal steel has effect on the strength while has only small effect on ductility of columns. The results obtained here have the same trend as those obtained experimentally by Sheikh et al. [14] and [15].

![Graph showing the effect of longitudinal reinforcement ratio](image)

(a) Axial load ratio is 0.1

![Graph showing the effect of longitudinal reinforcement ratio](image)

(b) Axial load ratio is 0.2
Fig. 6 Effect of ratio of longitudinal steel on moment-curvature relationships

(c) Axial load ratio is 0.3

(d) Axial load ratio is 0.4

(e) Axial load ratio is 0.5
6. Conclusions

The behavior of high-strength concrete columns subjected to combine axial and lateral loading was investigated in this study. The parameters that significantly affect the behavior of high strength circular concrete column, including compressive concrete strength, amount of longitudinal steel, and spacing of lateral steel and the level of axial load have been studied in this study. The following conclusions can be drawn based on and analytical investigations reported in this study.

1- The result presented in this paper indicates that ductility of column decreases with increasing concrete strength.

2- Strength and ductility of concrete columns are improved significantly for well confined concrete core. In other words a larger number of laterally supported longitudinal bars results in higher flexural strength and ductility. While the amount of longitudinal steel has effect on the strength and has only small effect on ductility of columns.

3- The ductility of concrete columns is decreased with increasing axial load ratio.

4- High-strength concrete columns with sufficient lateral confinement pressure can be used to resist combination of axial and lateral loading such as seismic loading.

References


Dynamic Analysis of non-Prismatic Beam under non-Concentric Axial Force by using The Differential Transformation Method

Talib H. Elaikh

Mech. Eng. Depart., College of Engineering, Thi-Qar University
taleb_ehres@yahoo.com

Abstract:

In this study, the dynamic analysis of non-prismatic simply-supported Euler-Bernoulli beam subjected to non-concentric compressive axial force has been presented. Kinetic and potential energy expressions of the model are derived. Hamilton’s principle is used to obtain the governing equations of motion. The fourth order differential equation with variable coefficient of the beam dynamics and buckling is solved using Differential Transform Method (DTM) to obtain mode shape, natural frequencies and buckling load of the system. The computer package Mathematical is used to write a program to calculate the natural frequencies, buckling load and the mode shapes. The effects of the taper ratio, eccentricity, and bucking load are investigated. The results are compared with the results of the analytical solution where a very good agreement is observed. The results show that the natural frequencies of prismatic and non-prismatic beam decreased when the compressive axial force increased. Also, the results show that when the taper ratio and eccentricity increased, the natural frequency of non-prismatic increases. The buckling load factor decreases when the taper ratio increases.

Keywords: Non-prismatic Euler-Bernoulli Beam, Non-concentric force, Hamilton’s Principle, Differential Transform Method
1. Introduction

Non-prismatic beam are increasingly being used in structures for economic, aesthetic, and other consideration. Design of such structures to resist dynamic forces such as wind and earthquakes, requires knowledge of their natural frequencies and the mode shapes of vibration. The Non-prismatic beam has received great attention from engineers due to their capability in optimizing the strength and weight of the structure.

The vibration problems of non-prismatic beam can be solved by analytic or approximate approaches. The analysis of non-uniform beam vibration using a green function method in the Laplace transformation domain have been investigated by Lee and. Ke [1]. Free vibration of tapered beam with flexible ends using Bessel’s’ function have been studied by Auicello[2]. Naquleswaran [3] obtained a direct solution for the transverse vibration of Euler-Bernoulli wedge and cone beam. Vibration problems of non-uniform rods and beams using the Rayleigh-Ritz scheme was solved by Abrate.[4]. Rutta P. [5] applied Chebychev series to solve vibration problem for a non-prismatic beam resting on a two parameter non-homogenous elastic foundation. Buckling analysis of non-prismatic columns by using modified vibrational mode shape and energy method was presented by Rahai and Kazemi [6]. Free vibration of non-prismatic beam by the displacement based formulation (stiffness method) was presented by Reza Attarnejad [7].

The complexity in analysis of non-prismatic beam lies in the presence of variable coefficient in the governing differential equation introduced by variable cross-section area and second moment of area. Due to presence of these variables coefficient , exact solutions are generally unavailable except for some special cases, many references mentioned that such as [8,9,10], therefore, a semi-analytical technique based on the Taylor series expansion method which called differential transformation method is using to solve the differential equation.
DTM was applied to solve linear and non-linear initial value problems and partial differential equations by many researches. The concept of DTM was first introduced by Zhou [11] and he used DTM to solve both linear and non-linear initial value problems in electric circuit analysis. Bert and Zeng [12] used DTM to investigate the analysis of axial vibration of compound bars. Numerical solution to buckling analysis of Bernoulli–Euler beams and columns were obtained using DTM and harmonic differential quadrature for various support conditions considering the variation of flexural rigidity by Rajasekaran [13]. The application of techniques of differential transformation method (DTM) to analyze the transverse vibration of a uniform Euler-Bernoulli beam under varying axial force was presented by Young-Jae and Jong-Hak [14]. The analysis of axially vibrating variable cross-section isotropic rod by using differential transformation method was studied by Mohammed Rafree and Amir Moradi [15]. Transverse vibration of conical Euler-Bernoulli beam using differential transformation method was presented by Torabi et al. [16].

In the present work, the model suggested in Ref. [5] is extended to include the effect of non-concentric compressive axial force on the vibration of non-prismatic beams. The eccentric compressive force can be resolved into a force and a couple moments at the center of the cross section of the beam. The Differential Transform Method (DTM) was used to solve the fourth order differential equation with variable coefficient of the beam vibration and buckling to obtain mode shape, natural frequencies and buckling load of the system. Also, the effect of this force and the effect of eccentricity on fundamental frequency of the non-prismatic beam are conducted.

2. Differential Transform Method

The differential transform method is a semi-analytic transformation technique based on the Taylor series expansion and is a useful tool to obtain analytical solutions of the differential equations. In this method certain transformation rules are applied to both the governing differential equations of motion and the boundary conditions of the system in order to transform them into a set of algebraic equations. The solution of these algebraic equations gives the desired results of the problem. It is different from high-order Taylor series method because Taylor series method requires symbolic computation of the necessary derivatives of the data functions and is expensive for large orders. The basic definitions and the application procedure of this method can be introduced as follows;
Consider a function $y(x)$ which is analytic in a domain $D$ and let $x = x_0$ represent any point in $D$. Then, the function $y(x)$ can be represented by a power series whose center is located at $x_0$, and the differential transform of the function $y(x)$ is given by

$$Y[k] = \frac{1}{k!} \left( \frac{d^k y(x)}{dx^k} \right)_{x=x_0}$$  \hspace{1cm} (1)$$

where $y(x)$ is the original function and $Y[k]$ is the transformed function, which is called T-function. The inverse transformation of $Y[k]$ is defined as

$$y(x) = \sum_{k=0}^{\infty} (x-x_0)^k Y[k]$$ \hspace{1cm} (2)$$

Combining Eqs. (1) and (2), gives

$$y(x) = \sum_{k=0}^{\infty} \frac{(x-x_0)^k}{k!} \left( \frac{d^k y(x)}{dx^k} \right)_{x=x_0}$$ \hspace{1cm} (3)$$

Considering $f(x)$ by a series of finite terms, Eq. 3 is arranged as follows, with assuming the residual terms to be negligibly small. The increase of convergence is determined by the value $q$.

$$y(x) = \sum_{k=0}^{\infty} \frac{(x-x_0)^k}{k!} \left( \frac{d^k y(x)}{dx^k} \right)_{x=x_0}$$ \hspace{1cm} (4)$$

Table 1 shows lists of the transformation properties that are useful in the analysis that follows.

<table>
<thead>
<tr>
<th>Original Function</th>
<th>Transformed Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = g(x) \pm h(x)$</td>
<td>$F(k) = G(k) \pm H(k)$</td>
</tr>
<tr>
<td>$f(x) = \lambda g(x)$</td>
<td>$F(k) = \lambda G(k)$</td>
</tr>
<tr>
<td>$f(x) = g(x)h(x)$</td>
<td>$F(k) = \sum_{l=0}^{k} G(l)H(k-l)$</td>
</tr>
<tr>
<td>$f(x) = \frac{d^n g(x)}{dx^n}$</td>
<td>$F(k) = \frac{(k+n)!}{k!} G(k+n)$</td>
</tr>
<tr>
<td>$f(x) = x^n$</td>
<td>$F(k) = \delta(k-n) = \begin{cases} 0 &amp; \text{if } k \neq n \ 1 &amp; \text{if } k = n \end{cases}$</td>
</tr>
</tbody>
</table>
3. Mathematical model and formulation:

For a non-prismatic Euler-Bernoulli simply supported beam under non-concentric compressive axial force, the governing differential equations of motion is derived by applying Hamilton’s principle. Figure (1) shows a schematic representation of the problem under consideration.

The potential or strain energy $U$ of the beam due to bending and axial compressive force is given by:

$$U = \frac{1}{2} \int_0^L EI(x)(y'')^2 dx - \frac{1}{2} N \int_0^L y'^2 dx$$

(5)

The kinetic energy $T$ of the beam is given by:

$$T = \frac{1}{2} \int_0^L \rho A(x) \dot{y}^2 dx$$

(6)

Hamilton’s principle, which is expressed as follows, is applied to the energy expressions given above in order to obtain the governing equations of motion and the boundary conditions

$$\int_{t_1}^{t_2} \delta \left( T - U \right) dt = 0$$

(7)

Where $t_1$ and $t_2$ are the time intervals in the dynamic trajectory, and $\delta$ is the usual variation operator.

Substituting for $T$ and $U$ from Eqs. (5) and (6) into Eq.(7), using the $\delta$ operator, integrating each term by parts, and collecting terms gives the following governing differential equation in free vibration for the non-prismatic beam under non-concentric axial force.

$$\frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 y}{\partial x^2} \right) + N \frac{\partial^2 y}{\partial x^2} + \rho A(x) \frac{\partial^2 y}{\partial t^2} = 0$$

(8)

The boundary conditions for simply supported beam whose length is $L$ are given as:
\[ y(x) = 0, \quad EI(x) \frac{\partial^2 y(x)}{\partial x^2} - M = 0 \quad \text{at} \quad x = 0 \]  
\[ (9a) \]

\[ y(x) = 0, \quad EI(x) \frac{\partial^2 y(x)}{\partial x^2} - M = 0 \quad \text{at} \quad x = l \]  
\[ (9b) \]

Where \( M = N.e \)

![Schematic Diagram](image)

**Fig. 1. A schematic diagram**

For free vibration analysis of the own problem, let us assume the solution is in the form of sinusoidal variation of \( y = Y(x,t) \) with circular frequency \( \omega \):

\[ y = Y(x)e^{i\omega t} \]  
\[ (10) \]

Substituting equation (10) into Eqs. (8), and (9), equation of motion and boundary conditions are expressed as follows:

\[ \frac{d^2}{dx^2} \left( EI(x) \frac{d^2 Y(x)}{dx^2} \right) + N \frac{d^2 Y(x)}{dx^2} - \rho A(x) \omega^2 Y(x) = 0 \quad x \in (0,l) \]  
\[ (11) \]

\[ Y(x) = 0, \quad EI(x) \frac{d^2 Y(x)}{dx^2} - M = 0 \quad \text{at} \quad x = 0 \]  
\[ (12a) \]

\[ Y(x) = 0, \quad EI(x) \frac{d^2 Y(x)}{dx^2} - M = 0 \quad \text{at} \quad x = l \]  
\[ (12b) \]

Introduce the following non-dimensional quantities:

\[ \xi = \frac{x}{L}, \quad \bar{y} = \frac{Y}{L}, \quad \bar{N} = \frac{NL^2}{\pi^2 EI}, \quad \mu^2 = \omega^2 \sqrt{\frac{\rho A}{EI}} \]

\[ b(\xi) = \frac{I(x)}{I_e}, \quad q(\xi) = \frac{A(x)}{A_e} \]

\[ m = \frac{N.e.l}{EI_e}, \quad g = \frac{N.e.l}{EI(l)}, \quad C = \pi^2 \bar{N} \]  
\[ (13) \]
The governing differential equation and boundary conditions can be rewritten in the following non-dimensional form:

\[ \frac{d^2}{d\xi^2} \left( b(\xi) \frac{d^2 Y(\xi)}{d\xi^2} \right) + C \frac{d^2 Y(\xi)}{d\xi^2} - \mu^2 q(\xi)Y(\xi) = 0 \]  

(14)

\[ Y(\xi) = 0, \quad \frac{d^2 Y(\xi)}{d\xi^2} - m = 0 \quad \text{at } \xi = 0 \]  

(15a)

\[ Y(\xi) = 0, \quad \frac{d^2 Y(\xi)}{d\xi^2} - g = 0 \quad \text{at } \xi = 1 \]  

(15b)

4. Application of DTM

4.1 Free vibration problem of non-prismatic beam

From the definition and properties of DT transformation given in table 1, the DT of the equation of motion (14) after defining \( \lambda = \mu^2 \) is found as [17],

\[ \sum_{r=0}^{\infty} \overline{B}(k-r)(r+1)(r+2)(r+3)(r+4)\overline{Y}(k+4) + 2\sum_{r=0}^{\infty} (k-r+1)\overline{B}(k-r+1)(r+1)(r+2)(r+3)\overline{Y}(r+3) \]

\[ + \sum_{r=0}^{\infty} (k-r+1)(k-r+2)\overline{B}(k-r+2)(r+1)(r+2)\overline{Y}(r+2) + C(r+1)(r+2)\overline{Y}(r+2) \]

\[ = \sum_{r=0}^{\infty} \lambda \overline{Q}(k-r)\overline{Y}(r) \]  

(16)

Where \( \overline{B}(k), \overline{Q}(k), \text{and} \overline{Y}(k) \) are the T-function of \( b(\xi) \), \( q(\xi) \) and \( y(\xi) \) respectively. Additionally, the differential transform method is applied to Eqs (15a)-(15b) and the following transformed boundary conditions are obtained.

\[ \overline{Y}(0) = 0, \quad 2\overline{Y}(2) - m = 0 \quad \text{at } \xi = 0 \]  

(17a)

\[ \sum_{k=0}^{m} \overline{Y}(k) = 0, \quad \sum_{k=0}^{m} k(k-1)\overline{Y}(k) - g = 0 \quad \text{at } \xi = 1 \]  

(17b)

The boundary conditions given by Eqs (17a), (17b) and the missing boundary conditions that are assumed to be \( \overline{Y}(1) = s \), \( \overline{Y}(3) = z \) where \( s \) and \( z \) are constants, are substituted into Eq. (16). Therefore, the following expression is obtained
\( A_{ij}^{(n)}(\lambda)s + A_{ij}^{(n)}(\lambda)z = 0 \quad j = 1,2,3..n \) \hspace{1cm} (18)

Where \( A_{ij}^{(n)}(\lambda) \), \( A_{ij}^{(n)}(\lambda) \) are polynomials of \( \lambda \) corresponding to \( n \).

By Eq. (18), we have the frequency equations as follows.

\[
\begin{bmatrix}
A_{11}^{(n)}(\lambda) & A_{12}^{(n)}(\lambda) \\
A_{21}^{(n)}(\lambda) & A_{22}^{(n)}(\lambda)
\end{bmatrix} = 0
\] \hspace{1cm} (19)

Solving equation (19), we get \( \lambda = \lambda_j^{(n)} \) where \( j = 1,2,3..n \). Here, \( \lambda_j^{(n)} \) is the \( j^{th} \) estimated eigenvalues corresponding to \( n \). The value of \( n \) is obtained by the following equation:

\[
|\lambda_j^{(n)} - \lambda_j^{(n-1)}| \leq \epsilon
\] \hspace{1cm} (20)

where \( \epsilon \) is the tolerance parameter. If equation (20) is satisfied, then we have \( j^{th} \) eigenvalues \( \lambda_j^{(n)} \).

In general, \( \lambda_j^{(n)} \) are conjugated complex values, and can be written as \( \lambda_j^{(n)} = a_j + ib_j \).

Neglecting the small imaginary part \( b_j \), we have the \( j^{th} \) natural frequency.

**4.2 Buckling problem of non-prismatic beam**

When the natural frequency of the system vanishes under the axial loading, the system begins to buckle. By introduction \( \mu^2 = 0 \) into Eq. (16), one gets the relation

\[
\frac{d^2}{d\xi^2}(b(\xi)\frac{d^2\bar{Y}(\xi)}{d\xi^2}) + C\frac{d^2\bar{Y}(\xi)}{d\xi^2} = 0
\] \hspace{1cm} (21)

By applying the DTM to Eq. (22), and using the transformation operation and after some simplifications, the following recurrence equation can be obtained

\[
\sum_{r=0}^{k-r+1}(k-r+2)(r+2)(r+3)\bar{Y}(k+4) + 2\sum_{r=0}^{k-r+1}(k-r+1)(r+2)(r+3)\bar{Y}(r+3) + \sum_{r=0}^{k-r+1}(k-r+2)(r+2)(r+3)\bar{Y}(r+2) + C(r+1)(r+2)\bar{Y}(r+2) = 0
\] \hspace{1cm} (22)
The differential transformation boundary conditions are the same as that of equations (17a), and (17b) for simply-supported beam.

By assuming \( Y(1) = c_1 \) and \( Y(3) = c_2 \), then Eq. (22) can be calculated up to \( n \) terms, and it will be substituted in Eq. (17), and solving these two equations for non-trivial solutions we get

\[
N_{cr} = N_{cr}^{(n)}. \]

Here \( N_{cr}^{(n)} \) is the estimated buckling load value corresponding to \( n \). The value of \( n \) is obtained by the following equation:

\[
\left| N_{cr}^{(n)} - N_{cr}^{(n-1)} \right| \leq \varepsilon
\]

(23)

Where \( \varepsilon \) is the tolerance parameter and four decimal accuracy considered in the present analysis.

5. Results and discussion

5.1 Free Vibration:

Consider a prismatic beam with modulus of elasticity \( E = 200 \) G Pa, beam length \( L = 3 \) m, rectangular cross-sectional area with 0.1 m height and 0.08 width, subjected to a concentric and non-concentric compressive axial force with different values. The natural frequencies of the beam under concentric force are determined according to Eq. (19), and compared with those of Singiresu S. Rao [18] as tabulated in Table (1). It is seen that the present values show a good agreement with those of Singiresu S. Rao [18]. The dimensionless natural frequency \( \lambda \) corresponding to various dimensionless compressive forces \( N_r \) are shown in Fig. (2), where \( N_r = Nl^2 / \pi^2 EI \). As the compressive axial force increased, the natural frequencies of all modes decreased, at a certain compressive force, the critical buckling load, the lowest natural frequency drops to zero and the beams elastically become unstable.
Table 1: natural frequencies of the present study and the exact result Comparison

<table>
<thead>
<tr>
<th>Axial force (kN.)</th>
<th>$\omega_1$ (Rad/s)</th>
<th>$\omega_2$ (rad/s)</th>
<th>$\omega_3$ (rad/s)</th>
<th>$\omega_4$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 0$</td>
<td>160.3046</td>
<td>641.2173</td>
<td>1442.7434</td>
<td>2564.8719</td>
</tr>
<tr>
<td>$N = 500$</td>
<td>130.0443</td>
<td>613.1987</td>
<td>1415.0751</td>
<td>2537.3112</td>
</tr>
<tr>
<td>$N = 1000$</td>
<td>90.1298</td>
<td>583.8353</td>
<td>1386.8454</td>
<td>2509.4612</td>
</tr>
<tr>
<td>$N = 1000$ Ref. (18 )</td>
<td>90.1293</td>
<td>583.8313</td>
<td>1386.8320</td>
<td>2509.4392</td>
</tr>
</tbody>
</table>

Fig. (2) Dimensionless natural frequency $\omega(\rho AI^4 / EI)^{1/2}$ versus dimensionless compressive force $N_r = NI^2 / \pi^2 EI$.

The first three mode shapes for prismatic Euler-Bernoulli simply supported beam without and with compressive axial force are shown in Figs. (3) and (4).
Fig. (3) The first three mode shapes of simply-simply supported beam without compressive axial force

Fig. (4) The first three mode shapes of simply-simply supported beam with compressive axial force \((N = 1 \times 10^3 \text{ KN})\)

On the other hand, when the beam is subjected to non-concentric compressive axial force with eccentricity \((e)\), the dimensionless natural frequencies are obtained by using (DTM) for different values of axial compressive force and eccentricities. Figure (5), shows the effect of eccentricity on the natural frequencies of prismatic beam. In this figure, it is seen that the critical concentric axial compressive force is higher compared to that for beam subjected to non-concentric force due to the moment which results from the eccentricity of this force.
Now, consider a non-prismatic beam with simply-supported boundary conditions is subjected to a concentric and non-concentric compressive axial force. The dimensionless natural frequencies for taper ratio ($\alpha = 0, 0.5$) for different values of compressive axial force are obtained as tabulated in Table (2). It can be seen that, the natural frequencies decrease when the compressive force increases.

**Table (2), Non-dimensional natural frequency of simply supported non-prismatic beam with different non-dimensional axial compressive force**

<table>
<thead>
<tr>
<th>$N_r$</th>
<th>$\alpha=0$</th>
<th>$\alpha=0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega_1$</td>
<td>$\Omega_2$</td>
</tr>
<tr>
<td>$N_r=0$</td>
<td>9.8696</td>
<td>39.4784</td>
</tr>
<tr>
<td>$N_r=0.25$</td>
<td>8.5473</td>
<td>34.1893</td>
</tr>
<tr>
<td>$N_r=0.345$</td>
<td>7.9876</td>
<td>31.9507</td>
</tr>
<tr>
<td>$N_r=0.5$</td>
<td>6.9788</td>
<td>27.9154</td>
</tr>
</tbody>
</table>
5.2 Buckling load

A non-prismatic beam with different second moment of area and different tapered ratio under concentrated compressive force are studied. As a case study, the non-prismatic beam with constant is of and linearly varying height, \( I(\xi) = (1-\alpha \xi)^3 \). The critical buckling load \( (N_r) \) using differential transform methods are calculated and they are listed in Table (3). These results agree very well with these by using finite element method (FEM).

![Table (3). Critical buckling load \( (N_r) \) for tapered beam with \( I(\xi) = (1-\alpha \xi)^3 \)](tables/5.2_Buckling_load_table.png)

Second case study, the non-prismatic beam with constant height and linearly varying width, which means \( I(\xi) = (1-\alpha \xi) \). The results are also evaluated under the simply-supported boundary conditions and compressive axial force. The variation of the critical buckling load with the taper ratio for simply-supported boundary condition and different second moment of area \( (I) \) is plotted in Fig.(6) which illustrates the decreasing of the critical load as the taper ratio increased. By comparison of these two curves in Fig.(6), we can see that the critical buckling loads of the second case study are greater than the corresponding values of first for the same taper ratio. This implies that the beam with constant height and linearly varying width has a stronger than that with constant width and linearly varying height. In other words, beam under compressive axial force is easy to buckle towards the linearly varying height direction, rather than linearly varying width direction. Such conclusion is easily understood since the bending stiffness of \( I(\xi) = (1-\alpha \xi)^3 \) is less than \( I(\xi) = (1-\alpha \xi) \).
6. Conclusions

The main conclusions of the present work can be summarized as:

1- The application of DTM to both the governing equations of motion, buckling, and the boundary conditions are very easy. Moreover, DTM produces simple algebraic equations that can be solved very quickly using the symbolic computational software, Mathematical.

2- The calculated results using differential transformation method (DTM) give good agreement when compared with reference values.

3- The natural frequencies of prismatic and non-prismatic beam decrease when the compressive axial force increases.

4- The result show that when the taper ratio and eccentricity increases, the natural frequency of non-prismatic decreases.

5- The buckling load factor decreases when the taper ratio of non-prismatic beam increases.

6- Beam under compressive axial force is easy to buckle towards the linearly varying height direction, rather than linearly varying width direction.

Fig.(6) Variation critical buckling force \( (N_r) \) with the taper ratio \( (\alpha) \) \( (\longrightarrow I(\xi) = (1-\alpha\xi)), \)

\[ (\cdots I(\xi) = (1-\alpha\xi)^3) \]
7. References


Nomenclature

A(x)   Cross-sectional area at the position x, (m^2)
A_o    Cross-sectional area at x=0, (m^2)
E      Young’s modulus (N/m^2)
e      Eccentricity (m)
I(x)   Moment of inertia at distance at x, (m^4)
I_o    Moment of inertia at x=0, (m^4)
I(l)   Moment of inertia at distance at x=l, (m^4)
L      Beam length, (m)
M      Bending moment (N.m)
N      Non-concentric compressive axial force, (N)
N_r    Nondimensionalized compressive force
U      Potential Energy (Joule)
T      Kinetic Energy (Joule)
t      Time, (second)
y(x, t) Transverse deflection, (m)
x      Longitudinal coordinate
ρ      Mass density of the beam material (kg/m^3)
μ      Non-dimensional natural frequency
α      Taper ratio
ω      Circular natural frequency (rad/s)
i      \( \sqrt{-1} \)
Some Mechanical Properties of Retempered High Strength Concrete

Lec. Dr. Laith Sh. Rasheed
Karbala University – Engineering College
Civil Engineering Department

Eng. Ruaa Hussain Elewi
Karbala University – Engineering College
Civil Engineering Department

Asst. Lec. Alaa M. Hadi
Karbala University – Engineering College
Civil Engineering Department

Eng. Marwa R. Aziz
Karbala University – Engineering College
Civil Engineering Department

Abstract

Iraq climate is characterized by the temperature increase during most of the months. The ambient temperatures are considered the most important factor that effects the production, properties and durability of the concrete.

This study aims to evaluate the effect of retempering concrete after 60 minutes from cast by three different methods (retempering by water, retempering by Superplasticizer (G51) and retempering by remixing) on some of the mechanical properties of High Strength Concrete (HSC).

In this work, different tests methods are adopted such as compressive strength, splitting tensile strength, ultrasonic pulse velocity and a hammer rebound Schmidt tests. These tests are investigated at different ages (7, 14, 28, and 56 days).

The results show the high strength concrete that retempered by superplasticizer (G51) leads to higher strengths, higher Ultrasonic Pulse Velocity (UPV) and higher hammer rebound than direct cast. The rate of increase in the mechanical properties of the high strength concrete that retempered by G51 relative to direct cast concrete at age 56 days were 28.29%, 16.37%, 2.76%, and 16.02% for compressive strength, splitting strength, Ultrasonic Pulse Velocity (U.P.V.) and hammer rebound respectively.

While the concrete that retempered by water leads to lower strengths, lower Ultra Sonic Pulse Velocity (UPV) and lower hammer rebound than the other mixes.

For the High Strength Concrete (HSC) mixes, the compressive strength and splitting tensile strength are closely related to UPV with a high correlation coefficient, R2.
1. Introduction

With the gradual improvement in concrete technology and concrete practice over the years there has been an increasing use of concrete in higher strengths, in which one or more specific characteristics have been enhanced through the selection and proportioning of its constituents. The high strength concrete (HSC) has a simple definition “concrete with compressive strength above the present existing limits in national code about 40 MPa up to 130 MPa” (ACI 363R, 1997). Most of the time high strength concrete has to be made in a ready-mix plant, and it has to be transported and placed in the forms at the job sites. Thus, in situations like delivery of HSC from central mixing plant, using a normal well-designed concrete mix, should arrive at its destination with sufficient workability to enable it to be properly placed and fully compacted (John and Ban, 2003). In such circumstances, where there is a significant period of time between mixing and placing the concrete, there will be a noticeable reduction in the workability of the fresh concrete. Therefore, retempering of the concrete by water, while normally considered to be bad practice, may be contemplated as a possible course of action (West, 1990). The ‘Retempering’ process is defined as the addition of water and remixing of concrete or mortar which has lost enough workability in order to...
restore concrete or mortar slump back to specified limits (*ACI 116, 2000*). This process, which evolved as a solution to long hauls and placing delays, is adaptable to the production of high strength concrete where it is desirable to retain the workability as long as possible. Laboratory research, as well as field experience, shows that strength reduction and other detrimental effects are proportional to the amount of retempering water added. Therefore, water addition in excess of the proportioned maximum water content or w/cm to compensate for loss of workability should be prohibited, while adding chemical admixtures, particularly high-range water reducing admixtures, may be very effective to maintain workability of concrete in hot weather conditions (*ACI 305R, 1999*). Indeed, Iraq is characterized by a long, dry and hot summers. The average maximum summer temperature is as high as 45 °C accompanied by blazing sunshine, which has a strong impact on concrete’s workability and accelerate the loss of slump with time. Therefore, the addition of water to ready mixed concrete at the jobsites is particularly more serious during these hot weather conditions. From the preceding discussion, it could be inferred that, there is a real need to enlighten practitioners in the field of the construction about the importance and effectiveness of different retempering conditions on durability and mechanical properties of high strength concrete.

The main objective of this study is to evaluate the influence of different retempering methods, including: *retempering by G51, retempering by water and retempering by remixing*, on some hardened properties of high strength concrete such as compressive strength, splitting tensile strength, ultrasonic pulse velocity and rebound hammer test, and finally identifies the most effective retempering method for high strength concrete.

2. Experimental Program

2.1 Materials

2.1.1 Cement

The cement used in this study is ordinary Portland cement type I. This cement is tested and checked according to Iraqi standard specification (*IQS No.5:1984*). The chemical and physical properties of this cement are illustrated in Tables 1 and 2, respectively.
### Table 1: Chemical composition and main compounds of cement*

<table>
<thead>
<tr>
<th>Compound composition</th>
<th>Chemical composition</th>
<th>Percentage by weight</th>
<th>Limits of (IQS NO.5 /1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lime</td>
<td>CaO</td>
<td>58.75</td>
<td>/</td>
</tr>
<tr>
<td>Silica</td>
<td>SiO₂</td>
<td>20.38</td>
<td>/</td>
</tr>
<tr>
<td>Alumina</td>
<td>Al₂O₃</td>
<td>3.52</td>
<td>/</td>
</tr>
<tr>
<td>Iron oxide</td>
<td>Fe₂O₃</td>
<td>4.68</td>
<td>/</td>
</tr>
<tr>
<td>Sulfate</td>
<td>SO₃</td>
<td>1.88</td>
<td>≤ 2.5 %</td>
</tr>
<tr>
<td>Magnesia</td>
<td>MgO</td>
<td>3.21</td>
<td>≤ 5 %</td>
</tr>
<tr>
<td>Loss on ignition</td>
<td>L.O.I.</td>
<td>3.8</td>
<td>≤ 4 %</td>
</tr>
<tr>
<td>Insoluble residue</td>
<td>I.R.</td>
<td>1.2</td>
<td>≤ 1.5 %</td>
</tr>
<tr>
<td>Lime saturation factor</td>
<td>L.S.F.</td>
<td>0.93</td>
<td>0.66 – 1.02</td>
</tr>
<tr>
<td>(Al₂O₃÷Fe₂O₃)</td>
<td>/</td>
<td>1.2</td>
<td>/</td>
</tr>
</tbody>
</table>

**Main compounds (Boque’s equations)**

<table>
<thead>
<tr>
<th>Percent by weight of cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tricalcium silicate (C₃S)</td>
</tr>
<tr>
<td>65.17</td>
</tr>
<tr>
<td>Dicalcium (C₂S)</td>
</tr>
<tr>
<td>13.36</td>
</tr>
<tr>
<td>Tricalcium aluminate (C₃A)</td>
</tr>
<tr>
<td>1.41</td>
</tr>
<tr>
<td>Tetracalciumaluminoferite (C₄AF)</td>
</tr>
</tbody>
</table>

*Chemical tests were conducted by the construction materials laboratory of University of Karbala

### Table 2: Physical properties of cement*

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Test results</th>
<th>Limits of (IQS NO.5 /1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setting time (Vicat’s Method)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial, min</td>
<td>190</td>
<td>≥ 45 min</td>
</tr>
<tr>
<td>Final, min</td>
<td>330</td>
<td>≤ 600 min</td>
</tr>
<tr>
<td>Fineness (Blaine Method), m²/kg</td>
<td>278</td>
<td>≥ 230 m²/kg</td>
</tr>
<tr>
<td>Compressive strength, MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 days</td>
<td>17.28</td>
<td>≥ 15, MPa</td>
</tr>
<tr>
<td>7 days</td>
<td>25.97</td>
<td>≥ 23, MPa</td>
</tr>
</tbody>
</table>

*Physical tests were conducted by the constructional materials laboratory of University of Karbala
2.1.2 Sand

Locally available natural sand with 4.75mm maximum size was used in presented work. Its grading was within the limits of the Iraqi specification (IQS No.45:1984). Tables 3 and 4 show the grading and physical properties of this fine aggregate, respectively.

Table 3: Grading of fine aggregate

<table>
<thead>
<tr>
<th>Sieve size (mm)</th>
<th>Cumulative passing%</th>
<th>Limits of Iraqi specification No.45/1984 /zone (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>90.10</td>
<td>100</td>
</tr>
<tr>
<td>4.75</td>
<td>78.39</td>
<td>90-100</td>
</tr>
<tr>
<td>2.36</td>
<td>71.19</td>
<td>75-100</td>
</tr>
<tr>
<td>1.18</td>
<td>58.08</td>
<td>55-90</td>
</tr>
<tr>
<td>0.6</td>
<td>22.65</td>
<td>35-59</td>
</tr>
<tr>
<td>0.3</td>
<td>1.33</td>
<td>8-30</td>
</tr>
<tr>
<td>0.15</td>
<td>0.0</td>
<td>0-10</td>
</tr>
</tbody>
</table>

Table 4: Physical properties of fine aggregate*

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Test result</th>
<th>Limits of Iraqi specification No.45/1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
<td>2.62</td>
<td>/</td>
</tr>
<tr>
<td>Sulfate content</td>
<td>0.32%</td>
<td>≤ 0.5 %</td>
</tr>
<tr>
<td>Absorption</td>
<td>2.21%</td>
<td>/</td>
</tr>
<tr>
<td>Dry-Loose density</td>
<td>1610</td>
<td>/</td>
</tr>
<tr>
<td>Fineness modulus</td>
<td>2.98</td>
<td>/</td>
</tr>
</tbody>
</table>

*Physical tests were conducted by the constructional materials laboratory of University of Karbala

2.1.3 Gravel

Natural rounded gravel of maximum size 14 mm was used in presented work. Table 5 shows the grading of this aggregate, which conforms to the Iraqi specification (IQS No.45:1984). The specific gravity, sulfate content and absorption of coarse aggregate are illustrated in Table 6.
### 2.1.4 Water

The water used in the mix design was potable water from the water-supply network system; so, it was free from suspended solids and organic materials, which might have affected the properties of the fresh and hardened concrete.

**Table 5: Grading of coarse aggregate**

<table>
<thead>
<tr>
<th>Sieve size (mm)</th>
<th>Cumulative passing%</th>
<th>Limits of Iraqi specification No.45/1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>14</td>
<td>97.5</td>
<td>90-100</td>
</tr>
<tr>
<td>10</td>
<td>82.4</td>
<td>50-85</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0-10</td>
</tr>
<tr>
<td>2.36</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

**Table 6: Physical properties of coarse aggregate*  

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Test result</th>
<th>Limits of Iraqi specification No.45/1984&lt;sup&gt;(101)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
<td>2.65</td>
<td>/</td>
</tr>
<tr>
<td>Sulfate content</td>
<td>0.03%</td>
<td>≤ 0.1 %</td>
</tr>
<tr>
<td>Absorption</td>
<td>0.5%</td>
<td>/</td>
</tr>
<tr>
<td>Dry rodded density (kg/m³)</td>
<td>1635</td>
<td>/</td>
</tr>
</tbody>
</table>

*Physical tests were conducted by the constructional materials laboratory of University of Karbala

### 2.1.5 Super plasticizer

A chemical admixture based on modified poly carboxylic ether, which is known (Glenium 51) was used in producing HSC as a high range water reducing admixture (HRWRA). Glenium 51 is considered one of a new generation of copolymer-based super plasticizer that complies with ASTM C 494 type A and F. Typical properties of Glenium 51 are shown in Table 7.
Table 7: Typical properties of Glenium 51*

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form</td>
<td>Viscous liquid</td>
</tr>
<tr>
<td>Color</td>
<td>Light down</td>
</tr>
<tr>
<td>Relative density</td>
<td>1.1 at 20°C</td>
</tr>
<tr>
<td>pH</td>
<td>6.6</td>
</tr>
<tr>
<td>Viscosity</td>
<td>128 cps at 20°C</td>
</tr>
<tr>
<td>Chloride content</td>
<td>Free</td>
</tr>
</tbody>
</table>

*Given by manufacture.

2.2 Mix proportion

High strength concrete mix was designed to give a 28 days compressive strength 40 MPa. The design was made in accordance with American method for mix design method (ACI 211, 1998). The cement content was 550 kg/m³. After trial mixes, a mix proportion of (1:1.2:1.8) by weight was adopted through this work. Four mixes of HSC were investigated, direct casting HSC, HSC retempered by water after waiting 60 min. HSC retempered by G51 after waiting 60 min. and HSC retempering by remixing after waiting 60 min. Table (8) shows the details of these mixes.

Table 8: Details of mix proportion

<table>
<thead>
<tr>
<th>Concrete Mix</th>
<th>Cement (kg/m³)</th>
<th>Sand (kg/m³)</th>
<th>Gravel (kg/m³)</th>
<th>sp %</th>
<th>w/c</th>
<th>Retempering process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Cast</td>
<td>550</td>
<td>660</td>
<td>990</td>
<td>1.25</td>
<td>0.32</td>
<td>/</td>
</tr>
<tr>
<td>HSC retempered by water</td>
<td>550</td>
<td>660</td>
<td>990</td>
<td>1.25</td>
<td>0.32</td>
<td>Add 1.6% water</td>
</tr>
<tr>
<td>HSC retempered by G51</td>
<td>550</td>
<td>660</td>
<td>990</td>
<td>1.25</td>
<td>0.32</td>
<td>Add 0.05% G51</td>
</tr>
<tr>
<td>HSC retempering by remixing</td>
<td>550</td>
<td>660</td>
<td>990</td>
<td>1.25</td>
<td>0.32</td>
<td>/</td>
</tr>
</tbody>
</table>
2.3 Results and Discussions

2.3.1 Compressive Strength

The compressive strength is one of the most important properties of hardened concrete. To study the effect of retempering process on compressive strength of HSC, standard (10×10×10) cm cubes where used within this test. Table 9 and Fig.1 represent the results of compressive strength test at 7, 14, 28 and 56 days and it is shown that the compressive strength increases with age for all mixes. From the results of compressive strength shown in Fig.1, it can be seen that HSC-retempered by G51 exhibited a considerable increase in compressive strength at all ages relative to their reference HSC-direct cast. The percentage increase in compressive strength of HSC-retempered by G51 measured relative to HSC-direct cast were 55.4%, 23.5%, 27.6% and 28.3% at 7, 14, 28 and 56 days, respectively. This improvement could be attributed to inclusion the extra amount of HRWRA in HSC mix during retempering process which leads to a significant reduction in capillary porosity. As well as, SP surfactant prone to disagglomerate and disperse the cement grains in the mortar matrix, therefore, on continuing hydration there is a greater statistical chance of intermeshing of hydration products with fine and coarse aggregate to produce a system of higher internal integrity and hence higher strength (Mehta & Monteiro, 2006).

Also, the results indicate that the HSC-retempered by remixing yielded higher compressive strength when compared with HSC-direct cast, Fig.1. The compressive strength of HSC- retempered by remixing was 36.8, 44.7, 51.3 and 55MPa at 7, 14, 28 and 56 days, respectively. The improvement of compressive strength of HSC-retempered by remixing may be attributed to remixing process, which was agitated HRWRA in concrete mix, resulting in a uniform dispersion of cement grains in the concrete mixture. This leads to efficient hydration process and a higher early strength. Furthermore, the lower compressive strength for the four mixes was found for HSC-retempered by water. It showed a reduction in compressive strength values of 4.4%, 14.1%, 8.7% and 3.4% at 7, 14, 28 and 56 days, respectively compared with HSC-direct cast. This behavior is ascribed to the increase in the amount of water which was used to retempering the concrete mix. This tendency is in accord with what has been reported in (John & Ban, 2003) which indicates that retempering water is offsetting the effects of insufficient water batched initially or higher rates of evaporation and/or absorption than anticipated. However, once hydration has started a loss of strength can be expected.
Table 9: Results of compressive strength of HSC

<table>
<thead>
<tr>
<th>Age (days)</th>
<th>Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HSC Retempered by G51</td>
</tr>
<tr>
<td>7</td>
<td>59.7</td>
</tr>
<tr>
<td>14</td>
<td>63</td>
</tr>
<tr>
<td>28</td>
<td>71.2</td>
</tr>
<tr>
<td>56</td>
<td>73</td>
</tr>
</tbody>
</table>

Fig.1: Effect of retempering process on compressive strength of concrete

2.3.2 Splitting tensile strength

The results for splitting tensile strength have been presented in Table 10 and Fig. 2. The splitting tensile strength of the concrete increased continuously as the age of concrete progressed for all concrete mixes. The splitting tensile strength of HSC-retempered by G51 exhibited a noticeable strength decrease at 7 days of curing. The decreasing in splitting tensile strength at early ages for HSC-retempered by G51 is most properly attributed to high dosage of G51, which may lead to retardation of the cement hydration at early ages. Beyond this period, 7 day, a remarkable improve in splitting tensile strength was observed. After 56 days of curing, the percentage increases in tensile strength of HSC-retempered by G51 were 29.6%, 16.37% and 11.28% relative to HSC-retempered by water, HSC-direct cast and HSC-retempered by remixing, respectively. This improvement is imputed to the significant reduction in capillary porosity of the cement matrix as well as, a good dispersion of the cement grains throughout the mix (Neville, 1995).
Moreover, HSC-retempered by remixing showed a clear increase in splitting tensile strength compared to the direct cast concrete mixes. This increase is mainly associated with agitation process of HRWRA in concrete mix. Such process can contribute to the dispersion of cement agglomerates into primary particles to produce a more consistent system, thereby strengthening the transition zone and reducing the micro-cracking leading to a significant increase in tensile strength. The percentage increase in tensile strength of HSC-retempered by remixing relative to direct cast were 8.24%, 5.07%, 6.92% and 4.57% at 7, 14, 28 and 56 days, respectively.

On the other hand, the mixes that are retempered by water give lower splitting tensile strength than direct cast mixes. The percentage decrease in splitting tensile strength of HSC-retempered by water relative to HSC-direct cast were 12.36%, 20.07%, 7.69% and 10.21% at 7, 14, 28 and 56 days, respectively. These results are due to the water addition which increases the micro-cracking and weakening the transition zone between the cement paste and aggregate.

<table>
<thead>
<tr>
<th>Age(days)</th>
<th>HSC Retempered by G51</th>
<th>HSC Retempered by remixing</th>
<th>HSC Direct Cast</th>
<th>HSC Retempered by water</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4.47</td>
<td>4.99</td>
<td>4.61</td>
<td>4.04</td>
</tr>
<tr>
<td>14</td>
<td>5.43</td>
<td>5.39</td>
<td>5.13</td>
<td>4.1</td>
</tr>
<tr>
<td>28</td>
<td>5.83</td>
<td>5.56</td>
<td>5.2</td>
<td>4.8</td>
</tr>
<tr>
<td>56</td>
<td>6.61</td>
<td>5.94</td>
<td>5.68</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table 10: Results of splitting tensile strength of HSC
Fig. 2: Effect of retempering process on splitting tensile strength of concrete

2.3.3 Ultrasonic Pulse Velocity

The results of ultrasonic pulse velocity have been presented in Table 11 and Fig. 3. Test results show that the velocity of the ultrasonic waves for all specimens increases slightly with age increasing up to 56 days. This enhancement in strength is because of the progress of hydration which decreases the void space within the concrete mass. The increase in the gel/space ratio causes a rise in wave speed, since the velocity of ultrasonic through materials is larger than that if it transfers through space. Hence, the increase in the concrete mass within the same volume increases the ultrasonic pulse velocity.

HSC-retempered by G51 demonstrated a noticeable increase in pulse velocity at all ages compared with other mixes. For instance, the percentage increase in ultrasonic pulse velocity after 56 days of curing relative to HSC-retempered by remixing, HSC-direct cast and HSC-retempered by water were 1.64%, 2.76% and 3.14%, respectively. This behavior is attributed to the reduction in water due to evaporation and using G51 for retempering without adding any additional water. However, the HSC-retempered by water give lower pulse velocity than other mixes by about 3.05%, 1.45% and 0.36% for HSC-retempered by G51, HSC-retempered by remixing and HSC-direct cast, respectively.

Although, the pulse velocity is not related directly to compressive strength but it is agreed that as the concrete compressive strength increases, the pulse velocity increases. This increment is not linear and could be logarithmic relationship, as shown in Fig. 4.
Table 11: Results of UPV of HSC

<table>
<thead>
<tr>
<th>HSC Retempered by</th>
<th>HSC Retempered by remixing</th>
<th>HSC Direct Cast</th>
<th>HSC Retempered by water</th>
<th>Age(days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G51</td>
<td>5.46</td>
<td>5.28</td>
<td>5.24</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5.51</td>
<td>5.3</td>
<td>5.29</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>5.55</td>
<td>5.43</td>
<td>5.38</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>5.58</td>
<td>5.43</td>
<td>5.41</td>
<td>56</td>
</tr>
</tbody>
</table>

Fig. 3: Effect of retempering process on the ultrasonic pulse velocity of concrete
The surface hardness of the 10 cm HSC cubes is assessed by the, “Schmidt Rebound Hammer” test, according to the BS1881: Part 201: 1986. The results of the rebound number concrete specimens are shown in Table 12 and Fig.5.

Form the recoded results in Table 12, it can be seen that the rebound number for HSC specimens increased continuously, as the age of concrete progressed. It varied from 31.33 to 50.67 for various ages and different retempering methods. The increase in rebound hammer followed a trend similar to that of ultrasonic pulse velocity. However, the rate of increase in rebound hammer was lower than that observed in case of ultrasonic pulse velocity. It indicates that the Rebound Hammer is less sensitive to the micro-structural changes in concrete, as compared to ultrasonic pulse velocity.

HSC-retempered by G51 exhibited the highest level of rebound hammer. However, HSC-retempered by remixing provided slightly lower rebound hammer than HSC-retempered by G51, but it provided higher rebound hammer than reference concrete. On the contrary, HSC-retempered by water produced the lowest level of rebound hammer. The reasons are probably the same as discussed before. More precisely, the reduction in rebound hammer of HSC-retempered by water is related to the high water content of the mixture, resulting in microscopic pores that will reduce the final compressive strength of concrete. For instance, the percentage decrease in rebound number for HSC-retempered by water at 7, 14, 28 and 56 days were 18.41%, 7.56 %, 5.38% and 5.35 %, respectively measured relative to HSC-direct
cast. While, the percentage increase in rebound number for HSC-retempered by G51 at 7, 14, 28 and 56 days were 16.33%, 21.81%, 17.88% and 16.03, respectively measured relative to HSC- direct cast.

Fig. 6 shows the relationship between compressive strength and rebound number for HSC mixes. It appears from the figure that rebound number increases with increase in compressive strength of concrete mix.

Table 12: Results of Rebound hammer of HSC

<table>
<thead>
<tr>
<th>The Rebound Number</th>
<th>Age(days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC Retempered by G51</td>
<td>HSC Retempered by remixing</td>
</tr>
<tr>
<td>44.67</td>
<td>40.67</td>
</tr>
<tr>
<td>44.67</td>
<td>41.33</td>
</tr>
<tr>
<td>48.33</td>
<td>45.33</td>
</tr>
<tr>
<td>50.67</td>
<td>48</td>
</tr>
</tbody>
</table>

Fig. 5: Effect of retempering process on the Rebound Hammer of concrete
Fig. 6: Relationship between rebound hammer and compressive strength

2.4 Conclusions

The following conclusions can be drawn based on the experimental results and discussions of the study conducted are:

1. The most effective manner of retempering methods was retempering by G51. It is known to be an effective way to restore workability without adversely affecting other properties. As well as, it produced the highest mechanical properties, rebound numbers and pulse velocity.

2. HSC-retempered by G51 achieved a remarkable compressive strength compared to other mixes. The rate of increase in compressive strength at age 56 days were 32.72%, 28.29% and 4.73% relative to HSC-retempered by water, HSC-direct cast and HSC-retempered by remixing, respectively.

3. The results of splitting tensile strength of HSC-retempered by G51were 29.6%, 16.37% and 11.28% relative to HSC-retempered by water, HSC-direct cast and HSC-retempered by remixing, respectively.

4. The addition of water to HSC mix may result in a substantial reduction in strength. Therefore, it is strongly recommended that adding water to concrete mixes in order to compensate for loss of workability should be prohibited.

5. In compressive strength test, the maximum reduction in strength was assigned for the HSC- retempered by water. It exhibited a reduction in compressive strength values of 4.4%, 14.1%, 8.7% and 3.4% at 7, 14, 28 and 56 days, respectively compared with HSC-direct cast.
6. In splitting tensile strength, the maximum reduction in strength was assigned for the HSC- retempered by water. The percentage decrease in splitting tensile strength of HSC-retempered by water relative to HSC-direct cast were 12.36%, 20.07%, 7.69% and 10.21% at 7, 14, 28 and 56 days, respectively.

7. For HSC mixes, the compressive strength and splitting tensile strength are closely related to U.P.V with a high correlation coefficient, R2. This verifies the suitability of the proposed relationships for prediction of hardened HSC strengths from measured U.P.V values.

2.5 References


6. Iraqi Specification, No.45/1984, "Aggregate from Natural Sources for Concrete and Construction".


Analysis of Turbulent Free Convection in Enclosure with Conductive Partitions

By
Khudheyer S. Mushatat
College of Engineering
Thi-Qar University

Rafid M. Hannun
College of Engineering
Thi-Qar University

Qais A. Rishack
College of Engineering
Basra University

Mushtaq I. Hasan
College of Engineering
Thi-Qar University

Abstract: A numerical method is presented to investigate the turbulent free convection inside an enclosure with partitions. The conductive partitions were located at the bottom wall. The Navier-Stokes and energy equations besides to the kinetic and dissipation equations were discretized by using a finite volume method. The solution of these equations was made via constructing a Fortran 90 computer program. The turbulence in the flow was modeled by using a k-ε model. The obtained results show that the conductive partitions represented a key factor for enhancing the rate of heat transfer. Also the results showed that the partitions height has more effect on enhancing the rate of heat transfer compared with the partitions thickness. The height of partitions were ranged from 0.25≤ h ≤ 0.65 and the width from 0.07 ≤ w ≤ 0.13. The enclosure was partially heated and the range of the considered values of Rayleigh number was up to 10^{14}. The maximum rate of heat transfer is enhanced by 35% with increasing partitions height up to h= 0.65.

Keywords: turbulent natural convection, partitions, enclosure.

1. Introduction

Study the turbulent natural convection in enclosures or cavities represented a rich research material for many researchers during the recent years. The focus on this topic is increased due to increasing development at the experimental and numerical techniques such as cooling of electronic equipments, solar collectors and geothermal applications. Some studies were done on using partitions or baffles inside these enclosures to enhance the rate of heat transfer via enhancing the heat transfer coefficient. The location and geometry of these partitions were a crucial topic for the mentioned studies. However more comprehensive understanding of the turbulent flow and heat in the partitioned enclosure is needed.

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An experimental benchmark study on turbulent natural convection in an air-filled square cavity was made by Ampofo and Karayiannis [1]. The cavity was differentially heated and the study was performed for a Rayleigh number range up to of $1.58 \times 10^9$. The results were obtained by measuring the local velocities and temperature at multiple locations in the cavity. The local and average Nusselt numbers and the wall shear stress besides to the turbulent kinetic energy were presented. The laminar natural convection heat transfer inside a partially divided square box was investigated by Acharya and Jetli [2]. The results verified that the thermal stratification between the divider and cold wall indicated an important role. Moreover, it was found that the influence of the divider position on the overall heat transfer coefficient was small. Markatos and Pericleous [3] investigated the buoyancy-driven laminar and turbulent flow and heat transfer in a square cavity with differentially heated side walls. They used Donor-cell differencing scheme and grid refinements were used for the studied Rayleigh numbers. The obtained results were represented in tabular and graphical forms. Bilgen [4] investigated numerically the laminar and turbulent natural convection in a differentially heated enclosure. The tested Rayleigh numbers were ranged from $10^4$ to $10^{11}$. Different values of partitions location ratio, aspect ratio and height ratio were tested. The results were documented as isotherm contours and streamlines for different values of geometrical conditions. The two-dimensional numerical simulation of the two vertical plates with uniform heat generation was studied by Barozzi and Corticelli [5]. A rectangular heating block with constant wall temperature was placed in the center of the enclosure. The study was done for $Gr$ ranging from $4\times10^4$ to $10^8$. Khalifa and Sahib [6] studied numerically the natural convection in a rectangular enclosure fitted with adiabatic partitions. The enclosure was differentially heated. They used water as working fluid to get a Rayleigh number range of $10^{11}$ to $7 \times 10^{11}$. They obtained correlations for the studied configuration and the percentage reduction in heat transfer for each case was compared to that of a single room. Sey et al. [7] performed a numerical study on transient laminar mixed convection in a two-dimensional enclosure partitioned by a conducting baffle. The interaction between the external forced air stream and the buoyancy driven flow was exhibited by streamlines and isotherms. Kuyper et al. [8] investigated the laminar and turbulent natural convection in an inclined enclosure. The $k$-$\varepsilon$ model was used to model the turbulence. They verified that the angle of inclination showed a significant effect on the Nusselt number. The turbulent natural convection in a partitioned enclosure was studied by Said et al. [9]. The study presented Numerical solutions for the buoyancy driven flows in an inclined two-dimensional rectangular enclosure. One of the inclined walls was heated and the other was cold. The low Reynolds number $k$-$\varepsilon$ model was used to model the turbulence. The flow field and average Nusselt number was investigated for different angles of inclination and Rayleigh numbers. The Nusselt number was increased as Rayleigh number increases. The natural convection heat transfer in a partially divided enclosure was studied by Yucel and Ozdem [10]. In their results, they demonstrated that the average rate of heat transfer was decreased with decreasing number and height of partitions. Zekeriya and Ozen [11] performed a numerical study for laminar natural convection in tilted rectangular enclosures with a vertically situated hot plate. The plate was very thin and isothermal on both lateral ends and act as a heat source. Fu et al. [12] studied numerically the transient laminar natural convection in an enclosure partitioned by an adiabatic baffle. It was found that that stream function strength was strictly dependent on the position of the baffle and Rayleigh number. Khalifa and Abdulla [13] studied the turbulent natural convection in a partitioned rectangular enclosure. They performed their study for Rayleigh number range up to $1.5\times10^8$. The correlations and the effect of the enclosure inclination angle besides to the number of partitions on the flow and thermal fields were presented and discussed. Shi et al. [14] investigated numerically the laminar natural convection inside a differentially heated square cavity with a fin on the hot wall. It was found
that that the flow field is enhanced for high Rayleigh numbers regardless of baffle length and location.

So, this work aims to enhance the scientific research in this field. The natural convection is found in many engineering applications such as cooling of electronic devices, solar collectors and thermal insulation...etc. Many authors dealt with the turbulent natural convection inside enclosures.

In this work, a numerical study has been done to cover investigating the turbulent natural convection inside a partitioned square enclosure. Three conducting partitions are fixed on the bottom wall. The working fluid was air with Pr=1. As Fig. 1 shows, the vertical walls are fixed at different isothermal temperature while the top and bottom walls are insulated. The study covered a range of Rayleigh number as $10^8 \leq Ra \leq 10^{14}$, partitions height $0.25 \leq h \leq 0.65$ and partitions width $0.07 \leq w \leq 0.13$. The aim of the present study is to show how using conductive partitions can enhance the rate of heat transfer and change the flow behaviour.

The aim of the present study is to show how using conductive partitions can improve the flow and then enhance the rate of heat transfer.

![Fig. 1 the present domain of study](image)

**2. Mathematical and Numerical Analysis**

The governing partial differential equations of the turbulent flow and heat transfer for the working fluid (air). The properties of the working fluid are assumed to be constant except the density in the gravity force.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\rho u}{\partial x} + \frac{\rho v}{\partial y} = -\frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left(\mu_{\text{eff}} \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_{\text{eff}} \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial x}\left(\mu_{\text{eff}} \frac{\partial u}{\partial y}\right)
\]

\[
\frac{\rho u}{\partial x} + \frac{\rho v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu_{\text{eff}} \frac{\partial v}{\partial x}\right) + 2\frac{\partial}{\partial y}\left(\mu_{\text{eff}} \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu_{\text{eff}} \frac{\partial u}{\partial y}\right)
\]

\[
\frac{\rho u}{\partial x} + \frac{\rho v}{\partial y} = \frac{\partial}{\partial x}\left(\Gamma_{\text{eff}} \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma_{\text{eff}}^\prime \frac{\partial T}{\partial y}\right)
\]

\[
\mu_{\text{eff}} = \mu + \mu_i, T_0 = \left(T_c + T_h\right)/2 , \beta = 1/T_0
\]

\[
\Gamma_{\text{eff}, x} = \frac{\mu}{\text{Pr}} + \frac{\mu_i}{\text{Pr}}
\]
The turbulence was modeled through using the k-ε model [15].

\[
\rho u \frac{\partial k}{\partial x} + \rho v \frac{\partial k}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma_{\text{eff}, k} \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{\text{eff}, k} \frac{\partial k}{\partial y} \right) + G - \rho \varepsilon \tag{7}
\]

\[
\rho u \frac{\partial \varepsilon}{\partial x} + \rho v \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma_{\text{eff}, \varepsilon} \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{\text{eff}, \varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} G + C_{2\varepsilon} \frac{\varepsilon^2}{k} \tag{8}
\]

\[
G = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \tag{9}
\]

\[
\mu = \frac{\rho c_p k^2}{\varepsilon} \tag{10}
\]

The values of the above constants in the turbulence model are \((\sigma_k ; \sigma_\varepsilon ; C_{1\varepsilon} ; C_{2\varepsilon} ; C_\mu) = (1.0 , 1.3 , 1.44 , 1.92 , 0.09 )\) respectively.

The distribution of the stream function \((\psi)\) is obtained from the Poisson equation with the boundary condition \(\psi = 0\) at the solid walls.

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \tag{11}
\]

The distribution of temperature through the conducting solid baffles is gained by solving the steady state heat conduction equation using finite difference method.

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{12}
\]

2.1. Boundary Conditions

To obtain the target results according to the physical problem, the following boundary conditions are imposed

At the walls: \(u = v = 0 \). and wall functions laws were incorporated to deal with the near wall grid points [15].

For vertical walls: at \(x = 0, T = T_h\) at \(x = L, T = T_c\)

For the horizontal walls, \(\frac{\partial T}{\partial x} = 0 \) at \(0, y)H\).

The local dimensionless Nusselt number along the left vertical hot wall is obtained as follows:

\[
Nu = \frac{\partial \theta}{\partial X} = \frac{\partial T}{\partial X} \frac{H}{T_h - T_c} \tag{13}
\]

At the partitions boundaries:
\[
\frac{\partial \theta}{\partial n}\bigg|_f = \frac{k_f}{k_j} \frac{\partial \theta}{\partial n},
\]

where \( k_f \) and \( k_j \) are the thermal conductivities of the fluid and solid respectively, \( n \) is a unit normal vector.

The Nusselt number is a function of grid points and Rayleigh number. The number of grid points for \( 10^7 \leq Ra \leq 10^{10} \) is 41x41 and for \( 10^{10} \leq Ra \leq 10^{14} \) is 84x82. It is important to mention here that the increase in \( Ra \) requires more grid points and computational time to gain converged solutions. The high density of grid points for all the studied Rayleigh numbers was concentrated near the bounded walls.

The governing equations of continuity, momentum and energy for the working fluid (air) inside an inclined partitioned square enclosure were discretized to algebraic equations through using a finite volume method while that of steady state conduction equation is by a finite difference method. The gained algebraic discretized equations were solved by using a semi-implicit line by line Gause elimination scheme. Non-uniform grids in all directions were used and the densities of these grids were used near the bounding walls where steep gradients of the dependent variables are important value. The computational grids are staggered for vector variables and assigned in their original positions for the scalar variables. Because of the inherent coupling and non-linearity in the governing equations, underlaxation factors were used. The factors used for velocity components, energy and turbulence quantities are 0.5, 0.8, 0.7 respectively.

A computer program is developed to get the results using the pressure velocity coupling (SIMPLE algorithm)[16]. The residual sum for each of the variables is computed and stored at the end of each iteration. The criteria \( \max_{i,j,k} \left| \phi^i(i,j,k) - \phi^{i-1}(i,j,k) \right| \leq 10^{-5} \) was the convergence criteria required for all dependent variables. The accuracy of the present results were judged through comparison with published results shown in Fig (10).

3. Results and Discussion:

The obtained results of the present work have been reported as stream function distribution, temperature distribution and Nusselt number variation. The study covered a range of Rayleigh number up to \( 10^{14} \).

Fig. 2 shows the distribution of stream function for different values of dimensionless partitions height. It is evident that the partition height has a noticeable effect on distribution of stream lines and generated vortices. As \( h=0.25 \), three vortices besides to the secondary flow are formed behind the partitions while a large vorticity is found above the partitions. The stream lines are very closer to each other in the region above the portion especially on the first one and that lead increase the velocity. However this accelerating in the flow is decelerated after the first portion due strong recirculation behind this portion. The recirculation regions behind these partitions are expected to increase the heat losses and consequently enhance the rate of heat transfer. However the disadvantage of using these partitions is distorting the cone region. When the partition height increases to \( h=0.45 \), the site and location of the resulted vortices are became larger and the stream lines are shifted away from the insulated bottom wall. The vortices behind these partitions made the stagnation cone region to be smaller and consequently affecting the temperature distribution. This trend is enhanced as \( h=0.65 \) where the stagnation core region is disappeared and number of vortices is increased. The contours of dimensionless temperature distribution are presented in Fig. 3. It is clear that the heat is transferred from the hot wall to the cold wall and partitions through the working fluid (air) and conduction heat transfer occurs through the solid partitions. The rate of heat transfer is little at the third partition and in the region behind the third partition. This expected due to
weak recirculation behind this partition which made the working fluid to be trapped in this region. This trend is found in (b) and (c). It is expected that the rate of heat transfer is increased as the height of the partition is increased. The partition distorts the isotherm lines and the convection heat transfer from the hot fluid to the solid partition is decreased after the first partition.

The effect of Rayleigh number on stream function distribution is found in Fig. 4. It can be seen that the number of vortices and strength of these vortices are increased due to increase the Rayleigh number. The number and size of these vortices are changed as Rayleigh number increases. It can mention here that the cause behind increasing the strength and size of vortices is due to increase convection currents as a result to increase Rayleigh number.

The effect of Rayleigh number on dimensionless temperature distribution is found in Fig. 5. When Rayleigh number exceeds $10^8$, the slope of isotherm lines is increased and that leads to enhance the rate of heat transfer as shown in the next section. Also this parameter affected the heat transfer through the partitions as shown in (b) and (c). Increasing Rayleigh number leads to increase buoyancy forces and consequently acceleration of the fluid motion and enhancing the rate of heat transfer. However this trend is affected by the value of height of the partitions as is explained in the next section.

The effect of Rayleigh number on local rate of heat transfer for different values of partitions height is shown in Fig.6. It can be seen that the trend of local Nusselt number is noticeably changed as Rayleigh number increases for the considered values of partitions height. This behaviour trend is concentrated above the partitions for $0.5 \leq Y \leq 1$. It can be seen that the rate of heat transfer is significantly increased as Rayleigh number and partitions height increase. However when Ra$>10^{10}$, this trend is reflected for at $0 \leq y \leq 0.5$. As the figure shows the values of local Nusselt number on the left hot wall is decreased sharply for $y>0$ and this behaviour is reflected sharply as Rayleigh number increases especially at (c).

Fig.7 shows the effect of Rayleigh number on the local rate of heat transfer on the left hot wall for the considered values of partitions height. It can see that the local rate of heat transfer is increased significantly as Rayleigh number increases. The local rate of heat transfer is enhanced by nearly 15% when Ra$=10^{14}$ for $y<0.25$ and this rate is increased for $y>0.25$ and it reaches 45% at $0.75 \leq y \leq 1$. This large increase of local Nusselt number becomes very little as Ra$\geq10^{12}$. The trend of local Nusselt number variation is the same for all the partitions height.

The effect of partitions width on stream function and temperature distribution is described in Fig.8 and Fig.9 respectively. In Fig.8, it can be seen that the partitions width affected the size of the resulted vortices. However there is no significant change recorded in number of these vortices for $0.03 \leq w \leq 0.07$. This trend is applicable for the other considered values of partitions height. In Fig.9, the isotherm lines distribution shape demonstrates that the heat is transferred from the hot wall to the cold one and there is a heat transfer occurs by conduction and convection through the solid partition changes occurs. However there is no significant change occurs in the distribution of isotherm lines computed with that of partitions height.

The validation of the present code is examined with published studies [1] of this field and acceptable agreement has been obtained as shown in Fig. 10.

Fig. 10 shows the effect of partitions width on variation of local variation Nusselt number. It is shown that there is a slightly increase in Nusselt number with increasing partitions width for $Y>0.6$. It can be demonstrated here that the Nusselt number is increased up to W=0.1 for $Y<0.6$ after that it decreases. However it is noted that the effect of partitions width on variation of local Nusselt number is less than that of partition height.

4. Conclusions:

The following conclusions can be obtained from the present work:

1. The rate of heat transfer is enhanced as partitions height increases.
2. There is no significant change in heat transfer as the partitions thickness increases.
3. The partitions shifted the thermal boundary layer on the lower wall and this trend affected the variation of local Nusselt number.

4. The effect of partitions on enhancement of heat transfer is clearer at low values of Rayleigh number compared with those of high Rayleigh numbers. The rate of heat transfer is enhanced as Rayleigh number increases.

5. References


Nomenclature

\( \alpha \) \hspace{1cm} \text{thermal diffusivity, m}^2/\text{s} \\
G \hspace{1cm} \text{generation term by shear, kg/m.s}^3 \\
h \hspace{1cm} \text{relative height of the partion (L/H)} \\
H \hspace{1cm} \text{height of the enclosure, m} \\
k \hspace{1cm} \text{turbulent kinetic energy, m}^2/\text{s}^2 \\
L_1 \hspace{1cm} \text{length of the partion, m} \\
Nu \hspace{1cm} \text{local Nusselt number} \\
Nu_{av} \hspace{1cm} \text{average Nusselt number} \\
P \hspace{1cm} \text{pressure, N/m}^2 \\
Pr \hspace{1cm} \text{Prandtl number} \\
Ra \hspace{1cm} \text{Rayleigh number} \left( g\beta H^4 (T_h - T_c) \right) \\
T_C \hspace{1cm} \text{cold wall temperature, } ^\circ\text{C} \\
T_h \hspace{1cm} \text{hot wall temperature, } ^\circ\text{C} \\
x, y \hspace{1cm} \text{Cartesian coordinates, m} \\
X \hspace{1cm} \text{dimensionless Cartesian coordinate} \left( \frac{x}{H} \right) \\

Greek symbols

\( \epsilon \) \hspace{1cm} \text{turbulence dissipation rate, m}^2/\text{s}^3 \\
\mu \hspace{1cm} \text{dynamic viscosity, N.s/m}^2 \\
\mu_t \hspace{1cm} \text{turbulent viscosity, N.s/m}^2 \\
\theta \hspace{1cm} \text{dimensionless temperature} \left( \frac{T - T_c}{T_h - T_c} \right) 

Fig. 2. Effect of partitions height on stream function distribution for $Ra=10^8$ and $w=0.1$
Fig. 2. Effect of partitions height on dimensionless temperature distribution for Ra=10E8 and w=0.1
Fig. 4. Effect of Ra on stream function distribution for h=0.45 and w=0.1
Fig. 5. Effect of Rayleigh number on the dimensionless temperature distribution for $h=0.45$ and $w=0.1$.
Fig. 6. Variation of the local Nusselt number on vertical hot wall for different values of Rayleigh number and $w=0.1$
Fig. 7. Effect of Rayleigh number on the local Nusselt number on vertical hot wall for different values of partitions height and \( w=0.1 \)
Fig. 8 Effect of partitions width on stream function distribution for Ra=10E8
Fig. 9. Effect of partitions width on dimensionless temperature distribution for $Ra = 10^8$.
Fig. 10. Effect of partitions width on variation of local Nusselt number on hot wall for Ra=10E8.