Journal of Babylon University/Pure and Applied Sciences/ No.(3)/ Vol.(21): 2013

On $\delta\psi - p -$ Continuous Functions In Bitopological Spaces by

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Abstract

In this paper we introduce some new continuous functions in bitopological spaces by the notion of $\alpha\psi - p -$ open sets and $\delta\psi - p -$ open sets ,and studied the relationships of these sets from side and between these function from the other side ,many result are discussed in this paper.

Keywords
$\alpha\psi - p -$ open, $\delta\psi - p -$ open, $\delta\psi - p -$ continuous

ألخلاصه

من خلال البحث قمنا بدراسة بعض الدوال المستمرة الجديدة في الفضاءات الثنائية التو بولوجي والمعرفة بواسطة مجموعات $\alpha\psi - p -$ المفتوحة ومجموعات $\delta\psi - p -$ المفتوحة حيث قمنا بدراسة العلاقة بين تلك المجموعات من جهة والعلاقة بين الدوال ألمعرفه بواسطتها من جهة أخرى.

1.Introduction

[A.S. Mashhour, M.E. Abd El-monsef and S.N. El-deeb ,1982] introduced the notion of pre open sets and pre continuous functions in topological spaces ,[R. Devi , A. Selvakumar and M. Paramili,submitted] ,introduced the notion of $\alpha\psi$ - open sets which are weaker than open sets, [R.Devi and M. Parimala,2010],introduced the notion of $\alpha\psi - p -$ open sets and $\alpha\psi - p -$ continuity in topological spaces.

A sub set A of a topological space (X,τ) is called semi-open (resp. semi-closed) if $A \subseteq cl(\text{int}(A))$ (resp. $\text{int}(cl(A)) \subseteq A$) [N.Levine,1963], α -open(resp α -closed)if $A \subseteq \text{int}(cl(\text{int}(A)))$ (resp. $cl(\text{int}(cl(A))) \subseteq A$)[S.N.Maheshwari and R.Prasad,1977/78],pre open(resp .pre closed)if $A \subseteq \text{int}(cl(A))$ (resp. $cl(\text{int}(A)) \subseteq A$) [A.S. Mashhour, M.E. Abd El-monsef and S.N. El-deeb ,1982],regular-open ,if $A = \text{int}(cl(A))$ and it is called δ -open if for each $x \in X$,there exist a regular open set G such that $x \in G \subset A$ [N.V. Velicko,1968].

A sub set A of a topological space (X,τ) is called a semi-generalized-closed(briefly sg-closed)set [P. Bhattacharya and B.K .Lahiri,1987] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ).the complement of sg-closed is called sg-open ,and it is called ψ -closed set [R. Devi , A. Selvakumar and M. Paramili , submitted] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X,τ) [R. Devi , A. Selvakumar and M. aramili,submitted],the complement of ψ -closed set is called ψ -open ,and it is called $\alpha\psi -$ closed set [R. Devi and M. Parimala,2010],if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X,τ),the complement of $\alpha\psi -$ closed is called $\alpha\psi -$ open where $\psi cl(A)$ be the intersection of all ψ -closed sets containing A.

A sub set A of a topological space (X,τ) is called a $\alpha\psi - p$-open[R.Devi and M. Parimala,2010] if $A \subseteq \text{int}(cl_{\alpha\psi}(A))$,the complement of $\alpha\psi - p$-open is called $\alpha\psi - p$-closed ,where $(cl_{\alpha\psi}(A))$ be the intersection of all $\alpha\psi - p$-closed set containing A,in this paper we shall recall all these concepts in bitopological spaces and introduce the notion of $\delta\psi$-p-open sets and $\delta\psi$-p-continuous function in bitopological spaces.

2- Preliminaries

Definition(2.1)

For $i \neq j$ and $i, j \in \{1,2\}$, a sub set A of a bitopological space(X, τ_1, τ_2) is called:

i) ij - semi open set if $A \subseteq i - cl(j - \text{int}(A)$, and ij - semi closed set if $i - \text{int}(j - cl(A)) \subseteq A$.[S.N.Maheshwari and R.Prasad,1977/78]

ii) ij - α − open set if $A \subseteq i - \text{int}(j - cl(i - \text{int}(A)))$, and ij - α − closed set if $i - cl(j - \text{int}(i - \text{int}(A))) \subseteq A$.[M.Jelic,1990],[D.Andreijevic,1984]

iii) ij - pre open set if $A \subseteq i - \text{int}(j - cl(A))$, and ij - pre closed set if $i - cl(j - \text{int}(A) \subseteq A$.[M.Jelic,1990],[S.Sampath Kumar,1997]

iv) $ij - \delta -$ open set if , for all $x \in A$,there exist τ_j - regular open set G such that $x \in G \subset A, A \in \tau_i$,and it is complements called $ij - \delta -$ closed.[M.Jelic,1990]

v) ij -pre regular p -open set if $A = i - p\text{int}(j - pcl(A))$, and ij -pre regular p -closed set if $A = i - pcl(j - p\text{int}(A))$. [M.Jelic,1990]

vi) ij - semi generalized closed set(briefly ij - sg-closed) if $A \subseteq U \in ij -$ semi open , $\tau_j - scl(A) \subseteq U$, and it is complement is called ij - sg-open.[G. Thamizharasi and P. Thangavelu,2010]

vii) $ij - \psi -$ closed set if $A \subseteq U \in ij -$ sg-closed, $\tau_j - scl(A) \subseteq U$,it is complement is called $ij - \psi -$ open set.

viii) $ij - \alpha\psi -$ closed set if $\tau_j - \psi cl(A) \subseteq U$, $A \subseteq U \in ij - \alpha -$ open set, and it is complement is called $ij - \alpha\psi -$ open set.

ix) $ij - \delta -$ pre open set if $A \subseteq i - \text{int}(j - cl_\delta(A))$, and it is complement is called $ij - \delta -$ pre closed set[S. Raychaudhuri and M.N. Mukherijee,1993]

xii) $ij - \delta\psi -$ closed set if $\tau_j - \psi cl(A) \subseteq U$, $A \subseteq U \in ij - \delta$ -open set, and it is complement is called $ij - \delta\psi -$ open set

The intersection of all ij - semi closed (resp. ij - α − closed, ij - pre closed, $ij - \delta -$ closed, , ij -pre regular p -closed, ij - sg-closed, $ij - \psi -$ closed, $ij - \alpha\psi -$ closed, $ij - \delta -$ pre closed, , $ij - \delta\psi -$ closed) sets containing A is called ij - semi closure (resp. ij - $\alpha -$ closure, ij - pre closure, $ij - \delta -$ closure, , ij -pre regular p - closure, ij - sg- closure, $ij - \psi -$ closure, $ij - \alpha\psi -$ closure, $ij - \delta -$ pre closure, $ij - \delta\psi -$ closure) and it is denoted by ij - scl(A)(resp. ij - α cl(A), ij - pcl(A), $ij - \delta$ cl(A), , , ij - $cl_{sg}(A)$, $ij - \psi$ cl(A), $ij - cl_{\alpha\psi}(A)$, $ij - cl_{\delta p}(A)$).

The family of all ij - semi open (resp. ij - $\alpha -$ open, ij - pre open, $ij - \delta -$ open, , ij - pre regular p - open, ij - sg- open, $ij - \psi -$ open, $ij - \alpha\psi -$ open, $ij - \delta -$ pre open, $ij - \delta\psi -$ open) sets is denoted by ij -SO(X) (resp. ij - α O(X), ij -PO(X), ij - δ O(X), ij -PRO(X), ij -SGO(X), ij - ψ O(X), ij - $\alpha\psi$ O(X), ij - δ PO(X), ij - $\delta\psi$ O(X))).

3- Some types of $ij-\psi-$p- open sets

Definition(3.1)

A sub set A of a bitopological space (X,τ_1,τ_2) is called $ij-\alpha\psi-$p- closed set if $i-cl(j-\text{int}_{\alpha\psi}(A))\subseteq A$. The complement of $ij-\alpha\psi-$p- closed set is called $ij-\alpha\psi-$p- open set, the family of all $ij-\alpha\psi-$p- closed set (resp. $ij-\alpha\psi-$p- open set) is denoted by $ij-\alpha\psi PC(X)$ (resp. $ij-\alpha\psi PO(X)$) , For $i\neq j$ and $i,j\in\{1,2\}$

Definition(3.2)

A sub set A of a bitopological space (X,τ_1,τ_2) is called $ij-\delta\psi-$p- closed set if $i-cl(j-\text{int}_{\alpha\psi}(A))\subseteq A$. The complement of $ij-\delta\psi-$p- closed set is called $ij-\delta\psi-$p- open set, the family of all $ij-\delta\psi-$p- closed set (resp. $ij-\delta\psi-$p- open set) is denoted by $ij-\delta\psi PC(X)$ (resp. $ij-\delta\psi PO(X)$) , For $i\neq j$ and $i,j\in\{1,2\}$

The intersection of all $ij-\alpha\psi-$p- closed (resp. $ij-\delta\psi-$p- closed) sets containing A is called $ij-\alpha\psi-$p- closure (resp. $ij-\delta\psi-$p- closure) and it is denoted by $ij-pcl_{\alpha\psi}(A)$ (resp. $ij-pcl_{\delta\psi}(A)$).

Proposition(3.3)

Let A,B are two sub sets in bitopological space (X,τ_1,τ_2) , then the following properties hold:

i) A is $ij-\alpha\psi-$p- closed (resp. $ij-\delta\psi-$p- closed) if and only if $A=ij-pcl_{\alpha\psi}(A)$ (resp. $ij-pcl_{\delta\psi}(A)$).

ii) if $A\subset B$, then $ij-pcl_{\alpha\psi}(A)\subset ij-pcl_{\alpha\psi}(B)$ (resp. $ij-pcl_{\delta\psi}(A)\subset ij-pcl_{\delta\psi}(B)$)

iii) $ij-pcl_{\alpha\psi}(A)$ (resp. $ij-pcl_{\delta\psi}(A)$) is $ij-\alpha\psi-$p- closed (resp. $ij-\theta\psi-$p- closed , $ij-\delta\psi-$p- closed) and that is $ij-pcl_{\alpha\psi}(\,ij-pcl_{\alpha\psi}(A)\,)=ij-pcl_{\alpha\psi}(A)$ (resp., $ij-pcl_{\delta\psi}(\,ij-pcl_{\delta\psi}(A)\,)=ij-pcl_{\delta\psi}(A)$) .

Proposition(3.4)

Every ij -pre open is $ij-\alpha\psi-$p- open set.

Proof :

it follows from definition , notes that the converse of this proposition need not be true by the following example.

Example(3.5)

Let X={a,b,c} and $\tau_1=\tau_2=$ { X,Ø{a},{c},{a,c},{b,c} },here {b} and {a,b} are $12-\alpha\psi-$p- open set but not 12-pre open

Proposition(3.6)

i) Every $ij-\delta-$ open ij -pre open set .

ii) Every $ij-\delta-$ open is $ij-\alpha\psi-$p- open set.

Proof:

(ii) follows from (i) and proposition (3.4)above. And the converse of (i) and(ii) need not be true by the following example.

Example(3.7)

Let X={a,b,c} and $\tau_1 = \tau_2 =$ { X,Ø{a},{c},{a,c},{b,c} },here {a,c} is12-pre open but not12 $-\delta-$ open and {b},{a,b}and{a,c} are $12-\alpha\psi-$p- open set but not $12-\delta-$open.

Proposition(3.8)

i) Every $ij-\delta-$ open is $ij-\delta-$pre open set.

ii) Every $ij-\alpha\psi-$closed is $ij-\alpha\psi-$p- closed.

iii) Every $ij-\delta\psi-$closed is $ij-\delta\psi-$p- closed.

Proof:

Easy by definition of each one ,but the converse is not true for any one by the following example .

Example(3.9)

Let X={a,b,c} and $\tau_1 = \tau_2 =$ { X,Ø{a},{c},{a,c},{b,c} },here {a,c} is $12-\delta-$pre open but not12 $-\delta-$ open,{c} is $12-\alpha\psi-$p- closed but not12 $-\alpha\psi-$closed,{a,b} is $12-\delta\psi-$p- closed but not12 $-\delta\psi-$closed set.

Proposition(3.10)

Every $ij-\delta\psi-$p- open set is $ij-\alpha\psi-$p- open set.

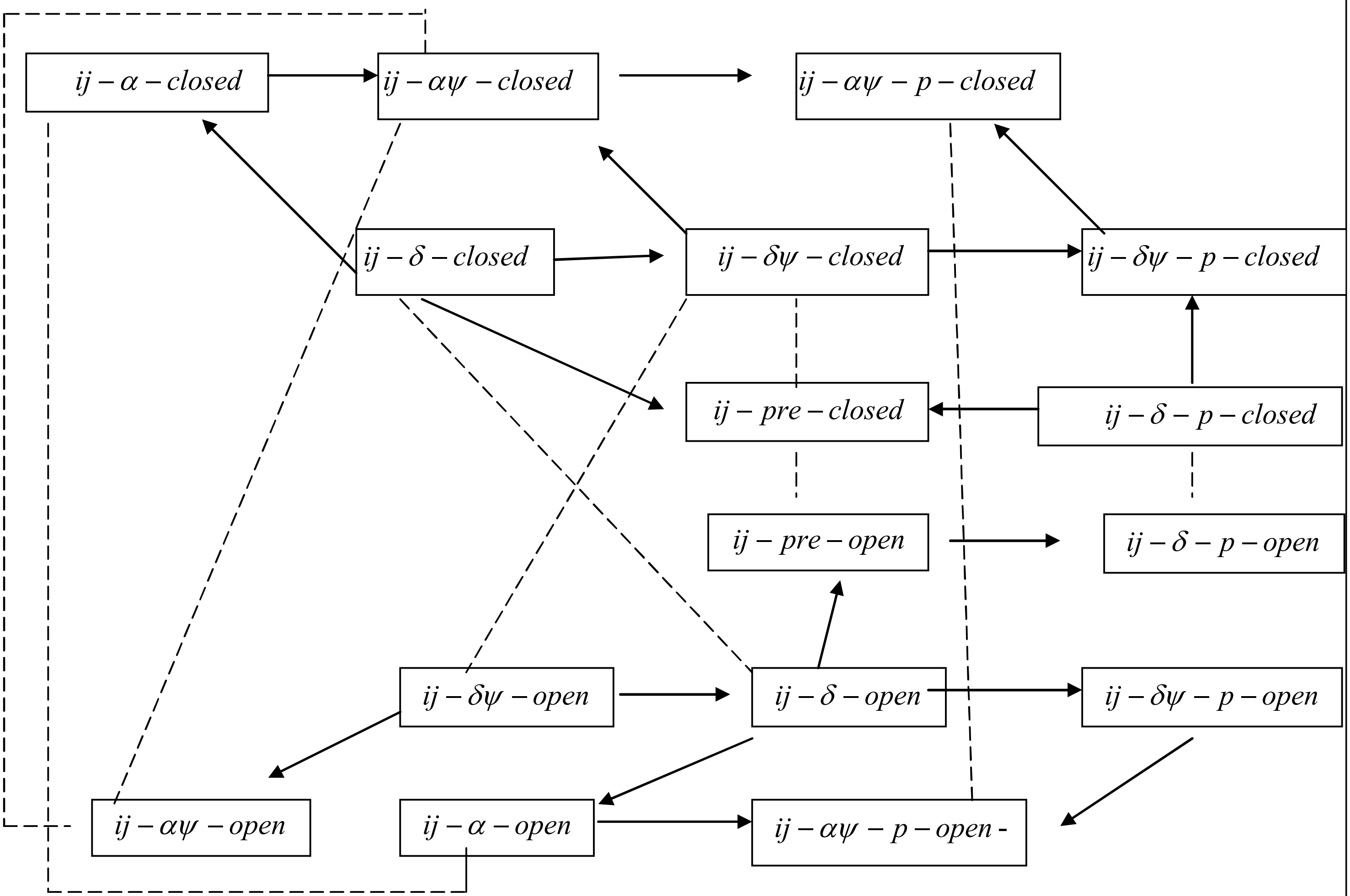
Proof :

by definition and the fact that every $ij-\delta-$open set is $ij-\alpha-$open set , the converse of this proposition need not true by the following example.

Example(3.11)

Let X={a,b,c} and $\tau_1 = \tau_2 =$ { X,Ø{a},{c},{a,c},{b,c} },here {b} is $12-\alpha\psi-$p- open but not $ij-\delta\psi-$p- open set

The following diagram shows the relationships between these concepts.
set but it is not $12-\delta\psi-$p- open



Where none of these implication is reversible and(-------) means independent.

4- Some types of functions defending by $ij-\alpha\psi$ and $ij-\delta\psi$ sets

Definition(4.1)
For $i \neq j$ and $i,j \in \{1,2\}$ a function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\alpha C(Y)$ [13].
ii) $ij-\delta-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\delta C(Y)$.
iii) $ij-\alpha\psi-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\alpha\psi C(Y)$.
iv) $ij-\delta\psi-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\delta\psi C(Y)$.
v) $ij-\alpha\psi-p-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\alpha\psi pC(Y)$.
vi) $ij-\delta\psi-p-closed$ if ,for all $F \in i-C(X), f(F) \in ij-\delta\psi pC(Y)$.
Where $i-C(X)$ be the set of all closed sets in $\tau_i, i=1$ or 2

Proposition(4.2)
i) Every $ij-\alpha-closed$ set is $ij-\alpha\psi-closed$ and the last is $ij-\alpha\psi-p-closed$.
ii) Every $ij-\delta-closed$ set is $ij-\delta\psi-closed$ and the last is $ij-\delta\psi-p-closed$.
Proof:
Easy by definition of each one ,but the converse is not true by the following example.

Example(4.3)
Let X={a,b,c} and τ_1= { X,Ø{a}}, τ_2 ={ X,Ø{b,c}},here {a}is $ij-\alpha\psi-closed$ and $ij-\alpha\psi-p-closed$ but it is not $ij-\alpha-closed$ also {b,c} is $ij-\delta\psi-closed$ and $ij-\delta\psi-p-closed$ but it is not $ij-\delta-closed$.

Proposition(4.5)
i) Every $ij-\alpha-closed$ function $ij-\alpha\psi-closed$ function and any $ij-\alpha\psi-closed$ function is $ij-\alpha\psi-p-closed$ function.
ii) Every $ij-\delta-closed$ function is $ij-\delta\psi-closed$ function and any $ij-\delta\psi-closed$ function is $ij-\delta\psi-p-closed$ function
proof:
(i) let $f:(X,\tau_1,\tau_2) \rightarrow (Y,\rho_1,\rho_2)$ be $ij-\alpha-closed$ function, then ,for all $F \in i-C(X), f(F) \in ij-\alpha C(Y)$,by (i) of proposition(4.2) above, $f(F)$ is $ij-\alpha\psi-closed$ and it is $ij-\alpha\psi-p-closed$,hence the result,
(ii) same of (i) and using (ii) of proposition(4.2) above.

Proposition(4.6)

Every $ij-\delta\psi-closed$ is $ij-\alpha\psi-closed$ set.

Proposition(4.7)
i)Any $ij-\delta-closed$ function is $ij-\alpha-closed$ function
ii)Any $ij-\delta\psi-closed$ function is $ij-\alpha\psi-closed$ function.

Proof:
ii)It follows by definition and proposition (4.6) above.

5- Some types of continuous functions defending by $ij-\alpha\psi$, and $ij-\delta\psi$ sets

Definition (5.1)

For $i \neq j$ and $i, j \in \{1,2\}$, a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha-continuous$ if ,for each $x \in X$ and each $V \in i-O(Y)$, there exists $U \in ij-\alpha O(X)$ such that $f(U) \subset V$.

ii) $ij-\delta-continuos$ if ,for each $x \in X$ and each $V \in i-O(Y)$, there exists $U \in ij-\delta O(X)$ such that $f(U) \subset V$.

Where $i-O(Y)$, be the set of all open sets in ρ_i, $i=1$ or 2.

Proposition(5.2)

For $i \neq j$ and $i, j \in \{1,2\}$, a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha-continuous$ if ,for all $F \in i-O(Y), f^{-1}(F) \in ij-\alpha O(X)$.

ii) $ij-\delta-continuos$ if ,for all $F \in i-O(Y), f^{-1}(F) \in ij-\delta O(X)$.

Where $i-O(Y)$, be the set of all open sets in ρ_i, $i=1$ or 2.

Proposition(5.3)

For $i \neq j$ and $i, j \in \{1,2\}$, a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha-continuous$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\alpha C(X)$. Where $C(Y)$ be the set of all closed sets in Y. [O.A. EL-Tantawy and H.M. Abu-Donia,2004]

ii) $ij-\delta-continuos$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\delta C(X)$.

Where $i-C(Y)$, be the set of all closed sets in ρ_i, $i=1$ or 2

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Proposition(5.4)

Any $ij-\delta-continuos\ \ function$ is $ij-\alpha-continuous\ \ function$.

Proof:

It follows by definition and the fact that any $ij-\delta-$closed set is $ij-\alpha-$closed set ,and the converse is not true by the following example.

Example(5.5

Let X={a,b,c} and τ_1= { X,Ø{a}}, τ_2={ X,Ø{b,c}},and Y={1,2,3} and ρ_1={Y, Ø ,{1} }, ρ_2={ Y, Ø ,{2,3} }and $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is defined by f (a)=1, f (b)=2 and f (c)=3.

Definition (5.6)

For $i \neq j$ and $i, j \in \{1,2\}$, a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha\psi-continuos$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\alpha\psi C(X)$.

ii) $ij-\delta\psi-continuos$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\delta\psi C(X)$.

Proposition(5.7)

Any $ij-\delta\psi-continuos\ \ function$ is $ij-\alpha\psi-continuous\ \ function$

Proof:

It follows by definition and the fact that any $ij-\delta\psi-$closed set is $ij-\alpha\psi-$closed set.

Definition (5.8)

For $i \neq j$ and $i, j \in \{1,2\}$ a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called:

i) $ij-\alpha\psi-p-continuos$ if ,for each $x \in X$ and each all such $V \in i-O(Y), there exists U \in ij-\alpha\psi pO(X)$ such that $f(U) \subset V$.
ii) $ij-\delta\psi-p-continuos$ if ,for all $V \in i-O(Y), there exists U \in ij-\delta\psi pO(X)$ such that $f(U) \subset V$.

Proposition(5.9)

For $i \neq j$ and $i, j \in \{1,2\}$ a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called be:
i) $ij-\alpha\psi-p-continuos$ if ,for all $F \in i-O(Y), f^{-1}(F) \in ij-\alpha\psi pO(X)$.
ii) $ij-\delta\psi-p-continuos$ if ,for all $F \in i-O(Y), f^{-1}(F) \in ij-\delta\psi pO(X)$.

Proposition(5.10)

For $i \neq j$ and $i, j \in \{1,2\}$ a function $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ is called be:
i) $ij-\alpha\psi-p-continuos$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\alpha\psi pC(X)$.
ii) $ij-\delta\psi-p-continuos$ if ,for all $F \in i-C(Y), f^{-1}(F) \in ij-\delta\psi pC(X)$.

Proposition(5.11)

Any $ij-\delta\psi-p-continuos\ \ function$ is $ij-\alpha\psi-p-continuous\ \ function$.

Proof:

It follows by definition and the fact that any $ij-\delta\psi-$p-closed set is $ij-\alpha\psi-$p-closed set ,and the converse is not true by the following example.

Proposition(5.12)

let $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$,then the following are equivalent.
i) $ij-\alpha\psi-p-continuos$.
ii) $B \in i-C(Y), f^{-1}(B) \in ij-\alpha\psi pC(X)$
iii) $f(pcl_{\alpha\psi}(A)) \subset pcl_{\alpha\psi}(f(A))$,for each sub set A of X.
iv) $pcl_{\alpha\psi}(f^{-1}(B)) \subset f^{-1}(pcl_{\alpha\psi}((B))$,for each sub set B of Y.

Proof: (i) $\Leftrightarrow$ (ii): obvious
(iii) $\Leftrightarrow$ (iv) , let B be any sub set of Y ,then by (iii) , we have
$f(pcl_{\alpha\psi}(f^{-1}(B)) \subset\ (pcl_{\alpha\psi}(f(f^{-1}((B)))$,this implies that
$f^{-1}(f(pcl_{\alpha\psi}(f^{-1}(B))) \subset\ f^{-1}((pcl_{\alpha\psi}(f(f^{-1}((B))))$. And hence (iv)
Conversely ,let $B = f(A)$ where A is a sub set of X , then by (iv),we have
$pcl_{\alpha\psi}(A) \subset pcl_{\alpha\psi}(f^{-1}(f(A)))$ thus $f(pcl_{\alpha\psi}(A)) \subset pcl_{\alpha\psi}(f(A))$.
(ii) $\Leftrightarrow$ (iv) let $B \in i-C(Y) then f^{-1}(B) \in ij-\theta\psi pC(X)$ and
$f^{-1}(B) \subset f^{-1}(pcl_{\alpha\psi}(B)$ then $pcl_{\alpha\psi}(f^{-1}(B)) \subset f^{-1}(pcl_{\alpha\psi}((B))$.
Conversely ,let $K \in i-C(Y)$,by (iv)
$pcl_{\alpha\psi}(f^{-1}(K)) \subset f^{-1}(pcl_{\alpha\psi}((K)) = f^{-1}(K)$ thus, $f^{-1}(K) \in ij-\theta\psi pC(X)$

Proposition(5.13)

let $f:(X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$,then the following are equivalent.
i) $ij-\delta\psi-p-continuos$.
ii) $B \in i-C(Y), f^{-1}(B) \in ij-\delta\psi pC(X)$
iii) $f(pcl_{\delta\psi}(A)) \subset pcl_{\delta\psi}(f(A))$,for each sub set A of X.
iv) $pcl_{\delta\psi}(f^{-1}(B)) \subset f^{-1}(pcl_{\delta\psi}((B))$,for each sub set B of Y.

Proof:

same of proposition(5.12).

Proposition(5.14)

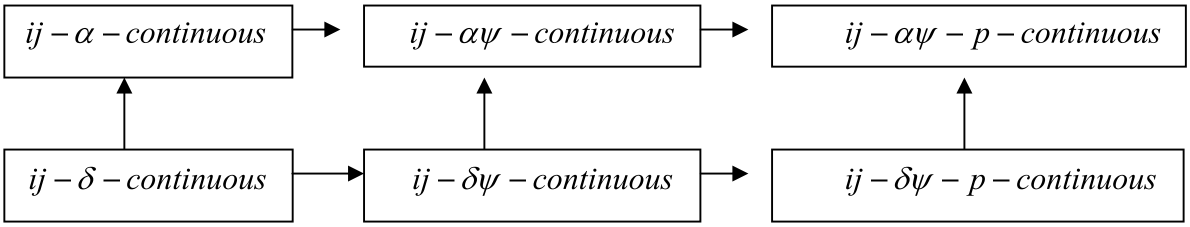
i) Every $ij-\alpha-continuous$ function is $ij-\alpha\psi-continuous$ and the last is $ij-\alpha\psi-p-continuous$.

ii) Every $ij-\alpha-continuous$ function is $ij-\delta\psi-continuous$ and the last is $ij-\delta\psi-p-continuous$..

Proof:

it follows from definition and proposition(4.2)

The following diagram shows the relationships between these functions where none of these implication is reversible



6. $ij-\alpha(ij-\delta, ij-\alpha\psi, ij-\delta\psi)$ **Connectivity**

Definition(6.1)

A bitopological space (X,τ_1,τ_2) is said to be $ij-\alpha$ (resp $ij-\delta, ij\alpha\psi, ij\delta\psi$) connected if it is not union of two disjoint $ij-\alpha$ (resp $ij-\delta, ij\alpha\psi, ij\delta\psi$) open non-empty subsets A and B of X such that X=A$\cup$B, $i \neq j$ and $i,j \in \{1,2\}$.

Proposition(6.2)

let $f:(X,\tau_1,\tau_2)\to(Y,\rho_1,\rho_2)$ be an $ij-\alpha-continuous$ function and (X,τ_1,τ_2) is $ij-\alpha-connected$ then $f(X)$ is $ij-\alpha-connected$.

Proof:

Assume that $f(X)$ is not connected then there exists G,H both $ij-\alpha-$open in Y ,such that $G\cap f(X)\neq\phi$,$H\cap f(X)\neq\phi$ and $(G\cap f(X))\cap(H\cap f(X))=\phi$ and $(G\cap f(X))\cup(H\cap f(X))=f(X)$

Now, $\phi=f^{-1}(\phi)=f^{-1}((G\cap f(X))\cap(H\cap f(X)))=f^{-1}($
$(G\cap H)\cap f(X))$=$f^{-1}(f(X))=f^{-1}(G\cap f(X))\cup(H\cap f(X))$=
$f^{-1}((G\cup H)\cap f(X))=(f^{-1}(G)\cup f^{-1}(H))\cap f^{-1}(f(X)))=$
$f^{-1}(G)\cup f^{-1}(H)\cap X=f^{-1}(G)\cup f^{-1}(H)$, since f is $ij-\alpha-continuous$ then $f^{-1}(G), f^{-1}(H)$ are $ij-\alpha-$open in X which is mean that X is not $ij-\alpha-connected$,which is not contradiction.

Proposition(6.3)

let $f:(X,\tau_1,\tau_2)\to(Y,\rho_1,\rho_2)$ be an $ij-\delta-continuous$ function and (X,τ_1,τ_2) is $ij-\delta-connected$ then $f(X)$ is $ij-\delta-connected$.

Proposition(6.4

let $f:(X,\tau_1,\tau_2)\to(Y,\rho_1,\rho_2)$ be an $ij-\alpha\psi-continuous$ function and (X,τ_1,τ_2) is $ij-\alpha\psi-connected$ then $f(X)$ is $ij-\alpha\psi-connected$.

Proposition(6.5)

let $f:(X,\tau_1,\tau_2)\to(Y,\rho_1,\rho_2)$ be an $ij-\delta\psi-continuous$ function and (X,τ_1,τ_2) is $ij-\delta\psi-connected$ then $f(X)$ is $ij-\delta\psi-connected$.

Remark(6.6)

The proof of propositions(6.3),(6.4) and (6.5) is the same manner of proposition(6.2) and the definition of each one.

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