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Study Even-Even ^{188,190}W Isotopes In The Framework Of The Interacting Boson Model(IBM-)

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Abstract

In this study, we determined the most appropriate Hamiltonian that is needed for the present calculations of energy levels, quadrupole moment and *B(E2)* values of ^{188,190}W nuclei which have mass number (*A*=188,190) using the interacting boson model (IBM-1) . Using the best fitted values of parameters of the Hamiltonian of the IBM-1, We have calculated energy levels, quadrupole moment and *B(E2)* values of a number of transitions in ^{188,190}W. The results were compared with the recent available experimental data and it was observed that they are in good agreement. The results show the Hamiltonian parameter (P.P) used in the present work for the IBM-code decreases as the mass number increases ,while the (J.J) and (Q.Q) parameters increase slightly as the mass number increase .Also the ^{188,190}W isotopes have more O(6) properties . The energy levels calculated in this work at β band (k=2) are not calculated experimentally .

الخلاصة

في هذه الدراسة، تم تحديد المؤثر الهاملتوني الاكثر تناسبا لمتطلبات الحسابات الحالية لكلاً من مستويات الطاقة، وعزم رباعي القطب، وقيم احتمالية الانتقال الكهربائي لنـوى ^{188,190}W ذات العدد الكتلي(*A*=188,190) باستعمال انموذج البوزونات المتفاعلة الاول (IBM-1)، باستعمال القيم الافضل ملائمة لبارامترات المؤثر الهاملتوني لـ IBM-1، تم حساب كلاً من مستويات الطاقة وعزم رباعي القطب واحتمالية الانتقال الكهربائي لعدد من الانتقالات ^{188,190}W. النتائج قورنت مع البيانات العملية الحديثة المتوفرة ولوحظ انها في توافق جيد مع نتائج البحث. أظهرت النتائج بأن المؤثر الهاملتوني (P.P) المستخدم في هذا العمل لـ IBM-1 يقل بزيادة العدد الكتلي ، بينما المؤثرات (J.J) و(Q.Q) ازدادت قليلا بزيادة العدد الكتلي . كذلك فأن نـوى ^{188,190}W تمتلك خواص تحديد كاما الغير مستقرة . مستويات الطاقة التي تم حسابها في الحزمة بيتا (k=2) في هذا العمل لم تحسب عملياً بعد .

1-Introduction

In 1975, the interacting boson model (IBM-1) that was used in this present work, was proposed by F. Iachello and A. Arima where interacting bosons are used to describe collective excitations in nuclei. From the symmetry properties of the model's boson operators, three types of idealized nuclei were found whose properties can be calculated analytically. These three limits of nuclei can be used as benchmarks with which to classify different nuclei. It was found that different regions of the nuclear chart exhibit properties that are similar to one of these idealized limits[Arima and Iachello, 1975] The interacting boson model is a very effective phenomenological model for describing low-lying collective properties of nuclei across an entire major shell. The building blocks are the *s*- and *d*-boson, which are interpreted as the approximation to the correlated fermion pairs with $L^{\pi}= 0^{+}, 2^{+}$, respectively [Arima and Iachello, 1987]. In the original model (IBM-1), only one kind of *s* and *d* bosons were considered [Casten and Warner,1988].The structure of the ^{200}Hg, $^{182-186}$W and $^{172-180}$Hf isotopes, were examined in the context of the interacting boson approximation (IBA) and the dynamic deformation model (DDM). The characteristics of the 2j level in ^{200}Hg were found to be shared with nearby 2+ levels [Zając and Szpikowski ,1986] The first predicted excitation spectra and B(E2) ratios of exotic Os and W isotopes with (N = 114 – 120) have been carried out in terms of the IBM Hamiltonian constructed by the constrained Hartree-Fock-Bogoliubov HFB calculations with Gogny-D1S Energy Density Functional (EDF). The prolate-to-oblate shape/phase transition had been examined as functions of neutron number N in the considered isotopic chains [Nomura *et al.*, 2011].

2. (IBM-1) model

2.1. Levels Energies

The IBM-1 model describes the low-lying energy state of the even –even tungsten nuclei as a system of interacting s-bosons, and d-bosons. The π and υ bosons are treated as one boson. Introducing creation $(s^{\dagger} d^{\dagger})$ and annihilation $(s\tilde{d})$ operators for s and d bosons, the most general Hamiltonian [Arima and Iachello, 1987] which includes one-boson term in boson-boson interaction has been used in calculating the levels energy is:

$$H = \varepsilon'_d \hat{n}_d + \frac{1}{2}\sum_J C_J (d^{\dagger}d^{\dagger})^{(J)}(\tilde{d}\tilde{d})^{(J)} + \frac{v_2}{\sqrt{10}}\left[(d^{\dagger}d^{\dagger})^{(2)}(\tilde{d}s) + H.c.\right] + \frac{v_0}{2\sqrt{5}}\left[(d^{\dagger}d^{\dagger})^{(0)}(ss)^{(0)} + H.c.\right] \quad (1)$$

Here two *d* bosons couple to produce angular momentum, *J*, equal to 0, 2, and 4, giving the six parameters: boson energy (ε'_d), coupling coefficients of the *d* bosons ($C_{0,2,4}$), and coefficients that describe the interaction between the *s* and *d* bosons (v_0 and v_2). The second group of terms describes the interaction between *d* boson pairs and does not alter the relative number of *s* and *d* bosons. The third and fourth groups of terms change the number of d bosons by ±1 or ±2, and therefore mix basis states. The $\tilde{d}$ operator is defined as $\tilde{d}_m = (-1)^m d_{-m}$ and introduces a phase factor. The term, *H.c.*, represents the Hermitian conjugates of the corresponding operators. However, it is more common to write the Hamiltonian of the IBM-1 as a multipole expansion, grouped into different boson-boson interactions [Casten, 1990].

$$H = \varepsilon'' \hat{n}_d + a_o \hat{P}^{\dagger}\cdot\hat{P} + a_1 \hat{J}\cdot\hat{J} + a_2 \hat{Q}\cdot\hat{Q} + a_3 \hat{T}_3\cdot\hat{T}_3 + a_4 \hat{T}_4\cdot\hat{T}_4 \quad (2)$$

The operators are defined as [Casten,1990];

$$\begin{aligned} \hat{n}_d &= (d^{\dagger}\cdot\tilde{d}), \\ \hat{P}^{\dagger} &= \tfrac{1}{2}(d^{\dagger}\cdot d^{\dagger} - s^{\dagger}\cdot s^{\dagger}) \\ \hat{P} &= (\hat{P}^{\dagger})^{\dagger} = \tfrac{1}{2}(d\cdot d - s\cdot s) \\ \hat{J} &= \sqrt{10}[d^{\dagger}\times\tilde{d}]^{(1)} \\ \hat{Q} &= [d^{\dagger}\times s + s^{\dagger}\times\tilde{d}]^{(2)} + \chi[d^{\dagger}\times\tilde{d}]^{(2)}, \\ \hat{T}_3 &= [d^{\dagger}\times\tilde{d}]^{(3)}, \\ \hat{T}_4 &= [d^{\dagger}\times\tilde{d}]^{(4)} \end{aligned} \quad (3)$$

The $\hat{n}_d$ operator gives the number of *d* bosons, $\hat{P}$ is the pairing operator for the *s* and *d* bosons, $\hat{J}$ is the angular momentum operator, $\hat{Q}$ is the quadrupole operator, $\hat{T}_3$ and $\hat{T}_4$ are the octupole and hexadecapole operators, respectively. The $\hat{n}_d$, $\hat{J}$, $\hat{T}_3$, and $\hat{T}_4$ operators have $\Delta n_d = 0$, while $\hat{P}^{\dagger}\cdot\hat{P}$ has $\Delta n_d = 0, \pm 2$ and $\hat{Q}\cdot\hat{Q}$ has $\Delta n_d = 0, \pm 1, \pm 2$.

Table (1) Shows the Parameters used in IBM-1 Hamiltonian (all in MeV)

nucleus	N	EPS	$\hat{P}.\hat{P}$	$\hat{J}.\hat{J}$	$\hat{Q}.\hat{Q}$	$\hat{T}_3.\hat{T}_3$	$\hat{T}_4.\hat{T}_4$	CHI	SO6	E2DD (e b)	E2SD (e b)
^{188}W	10	0.0000	0.03417	0.02062	-0.00854	0.0000	0.0000	-1.3228	1.0000	-0.33783	0.114208
^{190}W	9	0.0000	0.02055	0.03257	-0.00513	0.0000	0.0000	-1.3228	1.0000	-0.30956	0.104653

2.2. Transition Rates and Quadruple Moment

Another important property that can be deduced and calculated using the IBM-1 called the *reduced transition probability* B(E2).

The general linear E2 operator of the IBM-1 is the *J*=2 tensor operator given by [Casten and Warner, 1988]:

$$\hat{T}_{\mu}^{(E2)} = \alpha_2 \left[d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d} \right]_{\mu}^{(2)} + \beta_2 \left[d^{\dagger} \times \tilde{d} \right]_{\mu}^{(2)} \quad (4)$$

where α_2 and β_2 are free parameters.

The important physical quantities calculated with the E2 operator are the reduced transition probability $B(E2; J_i \rightarrow J_f)$, which is defined by the expression [Arima and Iachello, 1987]:

$$B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \left\langle J_f \left\| \hat{T}_{\mu}^{(E2)} \right\| J_i \right\rangle \right|^2 \quad (5)$$

where $\left| \left\langle J_f \left\| \hat{T}_{\mu}^{(E2)} \right\| J_i \right\rangle \right|$ is the matrix element of E2 transition .

$$B(E2; J+2 \rightarrow J) = \alpha_2^2 \frac{3}{4} \frac{(J+2)(J+1)}{(2J+3)(2J+5)} (2N-J)(2N+J+3) \quad (6)$$

The electric quadrupole moment of states in the ground state band is [Arima and Iachello, 1987]:

$$Q(J) = -\alpha_2 \left(\frac{16\pi}{40} \right)^{1/2} \frac{J}{2J+3} (4N+3) \quad (7)$$

While the results obtained by the geometric model [Bohr and Mottelson, 1975]are;

$$B(E2; J+2 \rightarrow J) = \frac{5}{16\pi} e^2 Q_{\circ}^2 \frac{3}{2} \frac{(J+2)(J+1)}{(2J+3)(2J+5)} \quad (8)$$

$$Q(J) = -eQ_{\circ} J/(2J+3) \quad (9)$$

3. Results and Discussion

3.1.Energy Levels

IBM-1 model has been used in calculating the energy of the positive low -lying levels of tungsten isotopes. A comparison between the experimental data [Singh, 2002, Singh, 2003] and our calculations, using the values of the model parameters given in table 1 for the ground bands, is illustrated in Fig. 1. and Fig. 2 . The agreement between the theoretical and their corresponding experimental values for all the nuclei are slightly higher but reasonable.

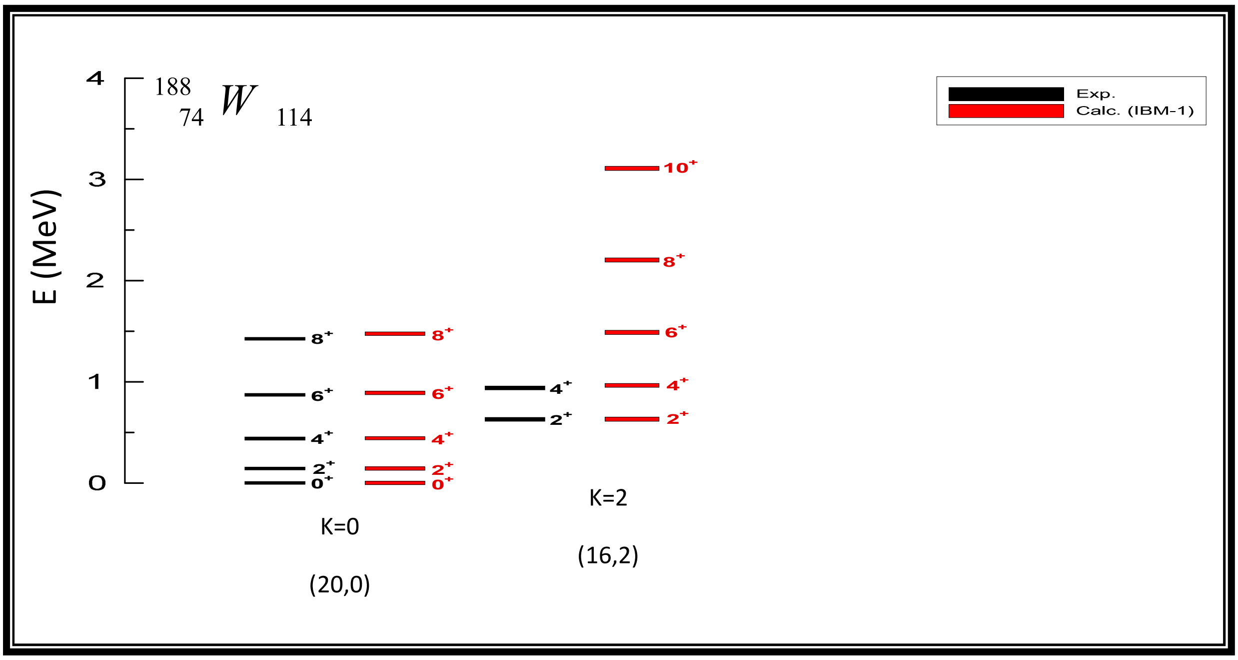

Figure 1: Comparison between Experiment [Singh, 2002,] and Calculated (IBM-1)

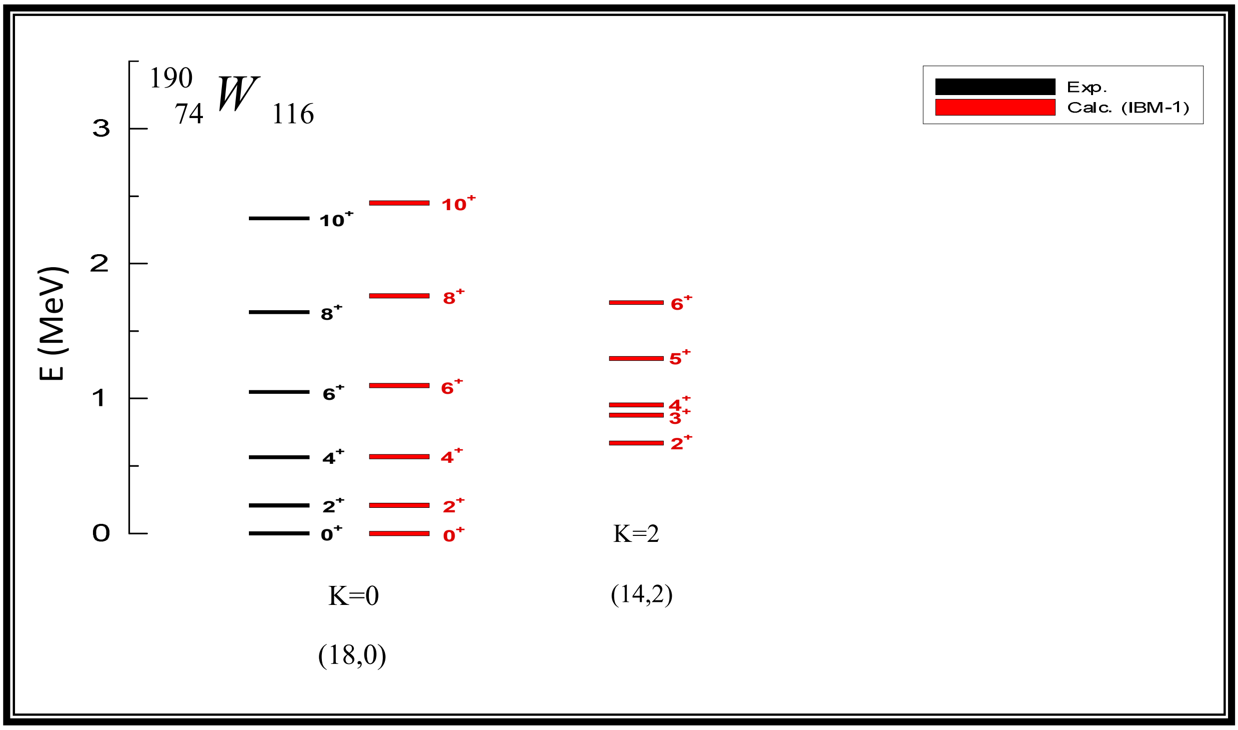

Figure 2 : Comparison between Experiment [Singh,2003] and Calculated (IBM-1)

Figures (1) and (2) show excellent agreement for the g-band (low) and in reasonable agreement with β-band (middle) .This agreement is due to that the IBM-1 succeed in lowing levels . Some of the energy levels calculated in this work at β band (k=2) are not calculated experimentally .

3.2. Electromagnetic Transition Rates B(E2) and Electric Quadrupole Moment $Q_{2_1^+}$

There is no enough measurements of B(E2) rates for these ^{188,190}W nuclei. The only measured $B(E2;\ 0_1^+ \rightarrow 2_1^+)$'s are presented, in table 2, for comparison with the calculated values. The parameters E2SD and E2DD used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in table 1.

Table 2 : The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment $Q_{2_1^+}$ for ^{188,190}W Isotopes

$J_i - J_f$	B(E2)↓ e^2b^2 ^{188}W		B(E2)↓ e^2b^2 ^{190}W	
	Exp. [Tuli,2011]	Calc.	Exp. [Tuli,2011]	Calc.
$2_1^+ \rightarrow 0_1^+$	0.6	0.586	0.414 (5)	0.410
$2_1^+ \rightarrow 0_2^+$	------	0.002	------	0.000
$2_2^+ \rightarrow 0_1^+$	------	0.000	------	0.000
$2_2^+ \rightarrow 0_2^+$	------	0.006	------	0.007
$2_2^+ \rightarrow 2_1^+$	------	0.000	------	0.000
$2_3^+ \rightarrow 0_1^+$	------	0.001	------	0.000
$2_3^+ \rightarrow 0_2^+$	------	0.355	------	0.295
$2_3^+ \rightarrow 2_1^+$	------	0.000	------	0.000
$2_4^+ \rightarrow 0_3^+$	------	0.006	------	0.030
$2_1^+ \rightarrow 2_2^+$	------	0.000	------	0.000
$4_1^+ \rightarrow 2_1^+$	------	0.819	------	0.575
$4_1^+ \rightarrow 2_3^+$	------	0.000	------	0.000
$4_2^+ \rightarrow 2_1^+$	------	0.000	------	0.000
$4_2^+ \rightarrow 2_2^+$	------	0.249	------	0.170
$4_1^+ \rightarrow 2_3^+$	------	0.000	------	0.000
$6_1^+ \rightarrow 4_1^+$	------	0.866	------	0.603
$Q(2_1^+)$	------	-1.57	------	-1.306

Conclusions

The IBA-1 model has been applied successfully to $^{188-190}$W isotopes and we have got:

1. The ground state bands are successfully reproduced and compared with recent available experimental data.
2. Electromagnetic transition rates B(E2) are calculated .

3. Quadrupole $Q(2_1^+)$ moment also is calculated and compared with the corresponding experimental data.
4. The results show the Hamiltonian parameter (P.P) used in the present work for the IBM-code decreases as the mass number increases ,while the (J.J) and (Q.Q) parameters increase slightly as the mass number increase .
5. The ^{188,190}W isotopes have more O(6) properties .
6. The energy levels calculated in this work at β band (k=2) are not calculated experimentally .

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