The Artin's Exponent of Special Linear Group SL(4,p), p = 5, 7

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Abstract: *The main purpose of this work is to find Artin's exponent of finite special linear group from any arbitrary characters of cyclic subgroups of these special linear groups and denoted by a(SL(4,p)) where p is any prime number such that p = 5, 7 and we found that a(SL(4,p)) is equal to 2.*

Key Words: Special linear group, Artin's exponent, conjugacy class, cyclic group.

1. Introduction

In 1968, Lam.T., [1] proved a sharp form of Artin's theorem, he determined that least positive integer A(G) such that A(G)χ is an integral linear combination of the induced principle characters of cyclic subgroups for any rational valued character χ of G, A(G) is called the Artin exponent of G.

In 1978, David Gluix, [2] considered integral Linear combinations of any arbitrary characters induced from cyclic subgroups of G, he determined a(G)χ is an integral linear combination of characters induced from cyclic subgroups, for all χ of G.

Recentely, Mohammed Serdar and Simaa Hassan introduced and discussion new concept of Artin's exponent for any arbitrary character of finite linear group in 2008, [3], and Mohammed Serdar and Lemia Abd Alameer are find Artin's exponent of special linear group SL(2,2k), k is an natural, k > 1 in 2009, [4].

This paper concentrates on the constructing of the character table of the irreducible rational representation and Artin's character induced from all cyclic subgroups of SL(4,p) where p is prime number, p = 5, 7.

2. Some Basic Concepts of SL(4,p)

We give some basic concept of SL(3,p), p is prime number, p ≥ 3 with some properties of these set and some theorems.

<u>Definition 2.1</u> : [5]

The general linear group of degree n is the set of $n\times n$ invertible (non-singular) matrices, together with the operation of ordinary matrix multiplication.

<u>Definition 2.2</u> : [2]

The general linear group over the field F is the group of invertible $n\times n$ matrices denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F*. Thus

SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one and it is called special linear group.

Theorem 2.3 : [5]

The order of SL(2,p), where p is prime number, p ≥ 3 is $p(p^2-1)$ denoted by $|SL(2,p)| = p(p^2-1)$.

Lemma 2.4 :

$|SL(4,p)| = p(p^2-1)$, p is prime number, p ≥ 3.

Proof :

$$SL(4,p)=\left\{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&x_1&x_2\\0&0&x_3&x_4\end{pmatrix}, x_1,x_2,x_3,x_4\in F \ \& \ x_1,x_4-x_2x_3=1\right\}, p=5,7$$

Where $SL(4,p)$ generated by $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&x_1&x_2\\0&0&x_3&x_4\end{pmatrix}$

One can now count the element of SL(4,p)
Thus two cases: the sum of which will five the required result

Case 1:
$x_3 = 0$
Then $x_1x_4-x_2x_3=1$
$x_1x_4=1$
If $x_1 \neq 0$ is fixed

and x_4 is determined is the multiplicative inverse of x_1. Then there are p-1 choices for x_1 & non for x_4 on the order hand x_2 can be chosen arbitrary,
i.e. p choices for x_2 in total we have counted p(p-1) element for case 1 .

Case 2 :
$x_3 \neq 0$
from $x_1x_4-x_2x_3=1$
hence $x_2=(x_1x_4-1)/x_3$.

Now we have p-1 choices for x_3 . One may choose x_1 & x_4 arbitrary and x_2 can then be determined hence we have p choices for x_1, p choices for x_4 & non for x_2 then case 2 covers $\varphi^2(p-1)$ element of SL(4,p) , p = 5 ,7 .
Sum (1) and (2)

$$p(p-1) + p^2(p-1) = p^2 - p + p^3 - p^2$$
$$= p^3 - p$$
$$= p(p^2 - 1)$$

<u>Examples 2.5 :</u>

The order of SL(4,5) = $5(5^2 - 1) = 5(24) = 120$.

The order of SL(4,7) = $7(7^2 - 1) = 7(48) = 336$.

<u>Theorem 2.6 : [4]</u>

Let G=SL(2,p) has exactly p + 4 conjugacy classes namely 1, *z*, *c*, *d*, *zc*, *zd*, *a*, a^2, …, $a^{\frac{p-3}{2}}$, *b*, b^2, …, $b^{\frac{p-1}{2}}$

- *v* be the generator of the cyclic multiplicative group F^*
- $1 \leq \ell \leq (p-3)/2$
- $1 \leq m \leq (p-1)/2$.

Thus this conjugacy classes which we denoted by C_g.

So table (2.1) represented these conjugacy classes.

Table (1). conjugacy classes of $SL(2,p^k)$, k > 1

$g \in G$	Notation	C_g	$\mid C_g \mid$	$\mid C_G(g) \mid$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p(p^2-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	Z	C_z	1	$p(p^2-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	C	C_c	$(p^2-1)/2$	2p
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	D	C_d	$(p^2-1)/2$	2p
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	Zc	C_{zc}	$(p^2-1)/2$	2p
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	Zd	C_{zd}	$(p^2-1)/2$	2p
$\begin{pmatrix} \nu^{\ell} & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^{ℓ}	Ca^{ℓ}	$p(p+1)$	$p-1$
Element of order $(p^k+1)m$	b^m	Cb^m	$p(p-1)$	$p+1$

<u>Example 2.7 :</u>

To compute the conjugacy classes of the group G = SL(4,5)

$|SL(4,5)| = 5(5^2-1) = 5(24) = 120.$

This group has exactly 5 + 4 = 9 conjugacy classes, $v = 2$,

$1 \le \ell \le (5-3)/2 \Rightarrow 1 \le \ell \le 1$, $1 \le m \le (5-1)/2 \Rightarrow 1 \le m \le 2.$

So these conjugacy classes are: 1, *z*, *c*, *d*, *zc*, *zd*, *a*, *b*, b^2.
These conjugacy classes are given in table (2).

Table (2). conjugacy classes of SL(3,5)

$g \in G$	Notation	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	1	C_1	1	120
$\begin{pmatrix}1&0&0\\0&4&0\\0&0&4\end{pmatrix}$	z	C_z	1	120
$\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}$	c	C_c	12	10
$\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}$	d	C_d	12	10
$\begin{pmatrix}1&0&0\\0&4&0\\0&4&4\end{pmatrix}$	zc	C_{zc}	12	10
$\begin{pmatrix}1&0&0\\0&4&0\\0&3&4\end{pmatrix}$	zd	C_{zd}	12	10
$\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}$	a	C_a	30	4
$\begin{pmatrix}1&0&0\\0&3&2\\0&4&3\end{pmatrix}$	b	C_b	20	6
$\begin{pmatrix}1&0&0\\0&2&2\\0&4&2\end{pmatrix}$	b^2	$C_b{}^2$	20	6

Example 2.8 :

To compute the conjugacy classes of the group G = SL(4,7)

This group has exactly 7 + 4 = 11 conjugacy classes, $v = 3$,

$1 \le \ell \le (7-3)/2 \Rightarrow 1 \le \ell \le 2$, $1 \le m \le (7-1)/2 \Rightarrow 1 \le m \le 3$.

So these conjugacy classes are: 1, z, c, d, zc, zd, a, a^2, b, b^2, b^3.
These conjugacy classes are given in table (3).

Table (3). conjugacy classes of SL(4,7)

$g \in G$	Notation	C_g	$\mid C_g \mid$	$\mid C_G(g) \mid$
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix}$	1	C_1	1	336
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&6&0\\0&0&0&6 \end{pmatrix}$	Z	C_z	1	336
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1 \end{pmatrix}$	c	C_c	24	14
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1 \end{pmatrix}$	d	C_d	24	14
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&6&0\\0&0&6&6 \end{pmatrix}$	zc	C_{zc}	24	14
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&6&0\\0&0&4&6 \end{pmatrix}$	zd	C_{zd}	24	14
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&3&0\\0&0&0&5 \end{pmatrix}$	a	C_a	56	6
$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&2&0\\0&0&0&4 \end{pmatrix}$	A^2	$C_a{}^2$	56	6

$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&6\\0&0&0&3\end{pmatrix}$	b	C_b	42	8
$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&6&4\\0&0&3&1\end{pmatrix}$	B^2	${C_b}^2$	42	8
$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&6\\0&0&1&0\end{pmatrix}$	B^3	${C_b}^3$	42	8

3. Artin Exponent of SL(4,p), p =5,7

In this part we study the method to find Artin exponent of induced any arbitrary characters from cyclic subgroup of the finite special linear group and denoted by A(G) the least integer.

Definition 3.1 : [6]

Let $H \leq G$ and χ a character of G, well-known that $\chi\downarrow H$ is a character of H by restriction we consider now the process, where charcters of G are induced from characters of H.

Definition 3.2 : [2]

The character induced from the trivial character of a subgroups of G is called Artin character.

Definition 3.3 : [6]

Let G be a finite group and let χ be any rational valued character on G. The smallest positive integer number n is such that:

$$n\chi = \sum_{c} A_c \phi_c ,$$

where $ac \in Z$ and ϕc is Artin's character, is called the Artin exponent of G and denoted by A(G).

Theorem 3.4 : [1]

Let G be a finite group of order pq, where p and q are primes (not necessarily distinct). Then:

$$A(G)=\begin{cases}1 & \text{if } G \text{ is cyclic}\\ \min(p,q) & \text{if } G \text{ is not cyclic}\end{cases}$$

<u>Example 3.5</u> :

For the finite special linear group SL(4,5):

$\omega^5 = 1 \Rightarrow \omega^5 - 1 = 0$ i.e. $\omega = e^{\frac{2\pi i}{5}}$

$\omega^4 + \omega^3 + \omega^2 + \omega = -1.$

In addition, this group has exactly 5 + 4 = 9 conjugacy classes and $v = 2$, $1 \le \ell \le 1$, $1 \le m \le 2$.

Therefore these conjugacy classes are: 1, z, c, d, zc, zd, a, b, b^2.

Now:

(1) $I = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \Rightarrow |I| = 1$

Order of I: o(I) = 1

(2)

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$\therefore$ o(z) = 2

I = 1, z = 1

(3) $c = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$\therefore \quad o(c) = 5$

1	χ	χ^2	χ^3	χ^4
1	1	1	1	1
1	ω	ω^2	ω^3	ω^4
1	ω^2	ω^4	ω	ω^3
1	ω^3	ω	ω^4	ω^2
1	ω^4	ω^3	ω^2	ω

$I = 1$

$c = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

(4) $$d = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&v&1\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$\therefore$ o(d) = 5

1	**χ**	**χ^2**	**χ^3**	**χ^4**
1	1	1	1	1
1	ω	ω^2	ω^3	ω^4
1	ω^2	ω^4	ω	ω^3
1	ω^3	ω	ω^4	ω^2
1	ω^4	ω^3	ω^2	ω

I = 1, $d = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

(5) $zc=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&-1&-1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&2&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&2&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&1&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&1&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$\therefore\ o(zc) = 10$

1	χ	χ^2	χ^3	χ^4	χ^5	χ^6	χ^7	χ^8	χ^9
1	1	1	1	1	1	1	1	1	1
1	ω	ω^2	ω^3	ω^4	-1	ω	ω^2	ω^3	ω^4
1	ω^2	ω^4	ω	ω^3	1	ω^2	ω^4	ω	ω^3
1	ω^3	ω	ω^4	ω^2	-1	ω^3	ω	ω^4	ω^2
1	ω^4	ω^3	ω^2	ω	1	ω^4	ω^3	ω^2	ω

$I = 1,\ z = 1$

$c = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

$zc = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

(6)

$$zd=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&-v&-1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&-2&-1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&4&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&4&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&3&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&1&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&1&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&2&4\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&3&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&2&4\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$\therefore\ o(zd) = 10$

1	χ	χ^2	χ^3	χ^4	χ^5	χ^6	χ^7	χ^8	χ^9
1	1	1	1	1	1	1	1	1	1
1	ω	ω^2	ω^3	ω^4	-1	ω	ω^2	ω^3	ω^4
1	ω^2	ω^4	ω	ω^3	1	ω^2	ω^4	ω	ω^3
1	ω^3	ω	ω^4	ω^2	-1	ω^3	ω	ω^4	ω^2
1	ω^4	ω^3	ω^2	ω	1	ω^4	ω^3	ω^2	ω

I = 1, $z = 1$

$d = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

$zd = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

(7) $$a^{\ell} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & vl & 0 \\ 0 & 0 & 0 & v-l \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore\ o(a^{\ell}) = 4$

1	χ	χ^2	χ^3
1	1	1	1
1	ω	ω^2	ω^3
1	ω^2	1	ω^2
1	ω^3	ω	ω

I = 1, $z = 1$

$a^{\ell} = 2 \Rightarrow a = 2(-1) = -2.$

(8) $b^m = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 4 & 3 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore \ o(b^m) = 6$

1	χ	χ^2	χ^3	χ^4	χ^5
1	1	1	1	1	1
1	ω	ω^2	ω^3	ω^4	-1
1	ω^2	ω^4	ω	ω^3	1
1	ω^3	ω	ω^4	ω^2	-1
1	ω^4	ω^3	ω^2	ω	1

I = 1, $z = 1$

$b^m = \omega + \omega^2 + \omega^3 + \omega^4 = -1$

$\Rightarrow b = 2(-1) = -2.,\qquad b^2 = 2(-1) = -2.$

Then the Artin character table for SL(4,5) is:

Table (4). Artin Character of SL(4,5)

C_g	1	*z*	*c*	*d*	*zc*	*zd*	*a*	*b*	b^2
$\mid C_g \mid$	1	1	12	12	12	12	30	20	20
$\mid C_G(g) \mid$	120	120	10	10	10	10	4	6	6
θ_1	120	0	0	0	0	0	0	0	0
θ_2	60	60	60	0	0	0	0	0	0
θ_3	24	0	– 2	0	0	0	0	0	0
θ_4	24	0	0	– 2	0	0	0	0	0
θ_5	12	12	– 1	0	– 1	0	0	0	0
θ_6	12	12	0	– 1	0	– 1	0	0	0
θ_7	30	30	0	0	0	0	– 2	0	0
θ_8	40	40	0	0	0	0	0	– 2	– 2

Now, from the Artin character:

$$-\frac{1}{2}\theta_8 - \frac{1}{2}\theta_7 - \theta_6 - \theta_5 + \theta_2 = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$\begin{array}{lcccccccccc}
-\frac{1}{2}\theta_8 & : & -20 & -20 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
-\frac{1}{2}\theta_7 & : & -15 & -15 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-\theta_6 & : & -12 & -12 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
-\theta_5 & : & -12 & -12 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
+\theta_2 & : & 60 & 60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
 & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}$$

Now, from the above result we get, the Artin exponent of SL(4,5) is equal to 2

A(SL(4,5)) = 2.

<u>Example 3.6</u> :

On the same way to compute the Artin's exponent of SL(4,7), see table (5)

Table (5). Artin Character of SL(4,7)

C_g	1	z	c	D	zc	zd	a	a^2	b	b^2	b^3
$\vert C_g \vert$	1	1	24	24	24	24	56	56	42	42	42
$\vert C_G(g) \vert$	336	336	14	14	14	14	6	6	8	8	8
Θ_1	336	0	0	0	0	0	0	0	0	0	0
Θ_2	168	168	0	0	0	0	0	0	0	0	0
Θ_3	48	0	– 2	0	0	0	0	0	0	0	0
Θ_4	48	0	0	– 2	0	0	0	0	0	0	0
Θ_5	24	24	– 1	0	– 1	0	0	0	0	0	0
Θ_6	24	24	0	– 1	0	– 1	0	0	0	0	0
Θ_7	112	112	0	0	0	0	– 2	– 2	0	0	0
Θ_8	126	126	0	0	0	0	0	0	– 2	– 2	– 2

Now, from the Artin character:

$$-\frac{1}{2}\theta_8 \quad : \quad -63 \quad -63 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1$$

$$-\frac{1}{2}\theta_7 \quad : \quad -56 \quad -56 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0$$

$$-\theta_6 \quad : \quad -24 \quad -24 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$-\theta_5 \quad : \quad -24 \quad -24 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$+\theta_2 \quad : \quad 168 \quad 168 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

Now, from the above result we get, the Artin exponent of SL(4,7) is equal to 2

A(SL(4,7)) = 2.

4. Result:

The Artin's exponent of the special linear group SL(4,p), $p \geq 3$ is equal to 2.

5. Open problems:

We suggest some open problems

1- We can found the Artin's exponent of the special linear group $SL(3,3^k), k \geq 1$.

2- We can found the Artin's exponent of the special linear group $SL(5,p), p \geq 1$.

3- We can found the Artin's exponent of the special linear group $SL(n,p^k), k \geq 1, n \geq 6$.

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أس آرتن للزمرة الخطية الخاصة
SL (4,p) عندما P= 5, 7

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المستخلص:

الهدف الرئيسي من العمل هو إيجاد أس آرتن للزمرة الخطية الخاصة المنتهية الناتج من الزمر الجزئية الدائرية لأي شواخص أختيارية من هذه الزمر الخطية الذي يرمز له A(SL(4,p)) حيث p أي عدد أولي بحيث أن p = 5, 7 وقد وجدنا أن A(SL(4,p)) مساويا الى 2.