

## Optimization of Microchannel Geometries to Enhance Convection Cooling of Parallel Plate Duct Flow

Faris A. Badawy

Institute of Technology, Foundation of Technical Education /Baghdad

[Faris.badawy@yahoo.com](mailto:Faris.badawy@yahoo.com)

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### ABSTRACT

Forced convection heat transfer from arrays of microchannels was considered for fixed volume and fixed pressure drop constraints. Order of magnitude relationships were developed for the optimal dimension of a number of fundamental duct shapes including the parallel plate channel with rectangular, square, rhombus, isosceles right triangle, circular duct and elliptic duct. Using the exact analysis method to produce an approximate expression for the optimal duct shape was developed for all ducts considered. Where approximate analytical results show that the optimal shape is the parallel plate channel duct array but with addition a triangular or rectangular duct to the parallel plate, where the rectangular model give a percent of (52%) increasing in the heat dissipation higher than the circular at an aspect ratio about ( $b/a=0.4$ ), while give a percent of increase about (87%) in the same aspect ratio compared with rhombus duct model in heat dissipation due to their ability to provide the most efficient arrays in a fixed volume. Comparison of the approximate results with exact results from the literature show excellent agreement for the optimal duct dimensions.

**Keywords:** Micro channel, Forced Convection, Optimal Geometry.

### تحسين انتقال الحرارة بالحمل القسري لتبريد القنوات الصغيرة باستخدام اشكال جديدة

#### الخلاصة

تم في هذا البحث اعتبار دراسة انتقال الحرارة بالحمل القسري من خلال صفوف للقنوات الميكروية وباستخدام نظام الحجم المحددة لانخفاض الضغط الثابت على طول القناة , وبترتيب اهمية العلاقات المطورة للحصول على البعد الامثل لعدد من الاشكال الاصلية للمجرى وتتضمن صفيحة مستوية اضيف اليها اما مجرى مستطيل , مجرى مربع , مجرى معيني , مجرى مثلث , مجرى دائري او مجرى بيضوي. وباستخدام طريقة التحليل الدقيق للحصول على تعابير تقريبية لشكل المجرى الامثل, واوضحت نتائج التحليل التقريبي ان الشكل المتمثل بالصفيحة المستوية هو الافضل من بين جميع اشكال القنوات الاخرى , ولكن عند اضافة الشكل المستطيل الى الصفيحة المستوية او الشكل المثلث بحيث يعطي الشكل المستطيل نسبة زيادة في الحرارة المشتتة تصل الى (52%) عند نسبة شكل 0.4 مقارنة مع الشكل الدائري بينما يعطي نسبة زيادة تقدر ب (87%) لنفس نسبة الشكل مقارنة مع الشكل المعيني لتشتيت الحرارة وذلك لقدرته على توفير صفوف فعالة في الحجم الثابتة , وقد قورنت النتائج التقريبية مع نتائج دراسات سابقة واعطت تقارب جيد للبعد الامثل للمجرى.

### GREEK SYMBOLS

$\alpha$	thermal diffusivity, $m^2/s$
$\varepsilon$	aspect ratio, $b/a$
$\mu$	dynamic viscosity, $Ns/m^2$
$\nu$	kinematic viscosity, $m^2/s$
$\rho$	fluid density, $kg/m^3$
$\tau$	wall shear stress, $N/m^2$
$\Delta P$	Pressure drop across microchannel, Pa.
$\mu$	Absolute viscosity of fluid, $kg/m \cdot s$ .
$\nu$	Kinematic viscosity of fluid, $m^2/s$ .
$\rho$	Fluid density, $kg/m^3$ .

### ABBREVIATED SYMBOLS

PRM	Parallel plate with rectangular duct
PSM	Parallel plate with square duct
PPM	Parallel plate with rhombus duct
PTM	Parallel plate with triangular duct
PCM	Parallel plate with circular duct
PEM	Parallel plate with ellipse duct

### NOMENCLATURE

A	Total heating surface area, $mm^2$ .
a,b	major and minor axes, m
Be	Bejan number, $\Delta P L^2 / \alpha \mu$
Cp	specific heat, $J/kgK$
d	diameter of circular duct, m
$D_h$	hydraulic diameter, $4A/P$
f	friction factor $=\tau/(1/2\rho U^2)$
G	Volume flow rate, $m^3/s$ .
k	Thermal conductivity of solid, $W/m \cdot K$ .
$k_f$	Thermal conductivity of fluid, $W/m \cdot K$ .
$k_{ce}$	Sum of entrance and exit losses.
H	height, m reference length scale, m
L	duct length, m
m	Fin parameter, $m^{-1}$ .
n	number of sides of polygon
N	number of channels or ducts
P	perimeter, m
p	pressure, $N/m^2$
Po	Poiseuille number
Pr	Prandtl number.
Q	heat transfer rate, W
$Q_w$	dimensionless Q.
r	radius, m
$Re_L$	Reynolds number.
Ts	wall or surface temperature, K

T <sub>i</sub>	fluid inlet temperature, K
U	average velocity, m/s
W	width, m
w <sub>w</sub>	Half of the fin thickness ,mm.

#### SUFFIFFIBFFI SCRIPTS

P	based upon square root of flow area
c	circumscribed
D <sub>h</sub>	based upon the hydraulic diameter
f	fluid
i	inscribed
L	based upon the arbitrary length L
s	small

#### INTRODUCTION

Microelectromechanical systems (MEMS) based devices find their applications in a wide variety of emerging technologies, ranging from the microactuators, microsensors, microreactors to the microchannel heat sinks and the thermo-mechanical data storage systems.. Microfluidic systems typically have characteristic lengths of the order of 1–100  $\mu\text{m}$ . **Kumar, (2009)** [1].

The predictions of this theory agree fairly well with known experimental data related to heat transfer in the conventional size straight channels **Kays, Crawford, (1993)** [2-3]. The development of micro-mechanics, stimulated during the last decades, a great interest to study flow and heat transfer in micro-channels. A number of theoretical and experimental investigations studied the effect of the geometry on the performance of the microchannel ducts devoted to this problem were performed during **Tso and Mahulikar (1995–2005)** [4–8]. The data on heat transfer in laminar and turbulent flows in microchannels with different geometry were obtained. Several special problems related to heat transfer in micro-channels were discussed: effect of axial conduction in the wall and viscous dissipation effect **Tso and Mahulikar, (1998)** [9–11].

There are a lot of experimental study The microchannel heat sink under the Micro-Electro-Mechanical Systems (MEMS) has first been illustrated in **Tuckerman and Pease (1981)** [12], their theoretical analyses and experimental tests were conducted to investigate the characteristics of heat transfer in the microchannel heat sink. Up to 790 W/cm<sup>2</sup> of heat flux was implemented to Very Large Scale Integration (VLSI) with high power density.

**Yu-Tang Chen et. al. (2004)** [13] presented a experimental investigation of fluid flow and heat transfer in microchannels Methanol was used as the working fluid and flowed through microchannels with different hydraulic diameters ranging from 57–267 $\mu\text{m}$  in the experiments. The phenomenon shows that the surface roughness, viscosity, and channel geometry have great effects on flow characteristics in microchannels.

And the numerical and analytical studies including **Khan et. al. (2007)** [14] used entropy generation minimization method is applied as a unique measure to study the thermodynamic losses caused by heat transfer and pressure drop for a fluid in cross flow with tube banks. And also **Khan 2009**[15] presented an entropy generation minimization (EGM) procedure is employed to optimize the overall

performance of microchannel heat sinks. This allows the combined effects of thermal resistance and pressure drop. They developed new general expressions for the entropy generation rate by considering an appropriate control volume and applying mass, energy, and entropy balances.

**Jung Yim Min et al. (2004) [16]** studies the effect of tip clearance on the cooling performance of the microchannel heat sink under the fixed pumping power condition. From the numerical results, they have shown that the thermal resistance of the flow is decreased. As a result the overall thermal resistance, attains a minimum value when  $H_c=w_c$  is about 0.6. Another numerical study presented by **Muzychka (2005) [17]** considered a heat transfer from arrays of microchannels for fixed volume and fixed pressure drop constraints. Order of magnitude relationships were developed for the optimal reference dimension of a number of fundamental duct shapes including the rectangle, ellipse, and regular polygons. **Buyukalaca, et al. [18]** applied an exact method of analysis to obtain results for a single duct for the equilateral triangle, square, circular, and parallel plate geometries. The method of analysis is quite involved due to the use of complex generalized empirical correlations developed by one of the authors. **Bejan and Sciubba (1992) [19]** first considered this problem for an array of parallel plates with application to the cooling of electronic systems. Using the intersection of asymptotes method, obtained expressions for the optimal plate spacing to channel length ratio,  $b_{opt}/L$ , and the maximum heat transfer per unit volume in terms of a dimensionless parameter which is now referred to as the **Bejan number S. Petrescu (1994) [20]**,

In the present work, the approximate analysis method of **Bejan and Sciubba (1992) [19]** is applied to several other channel shapes to determine the optimal passage size to length ratio in terms of the Bejan number. It will be shown that these optimal dimensions are independent of the array configuration and thus represent a basic constructional unit for built up systems. Depending upon the passage shape, several potential packing arrangements can be chosen, each with its own characteristic performance. Results for individual shapes are compared with those reported in **Muzychka (2005), [17]**.

## MODEL DEVELOPMENT

The geometry of a finite volume microchannel heat sink also contains an array of a non-circular of flat plate with square passages is shown in Figure (1). The length of the heat sink is (L), the width is (W) and the height is (H). A coolant passes through a number of microchannels along the z-axis and takes heat away from the heat dissipating electronic component attached below. There are channels and each channel has a height and width. The thickness of each fin is ( $2w_w$ ) whereas the thickness of the base is ( $t_b$ ). The temperature of the channel walls is assumed constant. At the channel wall, the slip flow velocity and temperature jump boundary conditions were applied to calculate friction and heat transfer coefficients. The system under consideration consists of a fixed volume to be cooled by means of laminar forced convection. These passages may be arranged in such a manner that their number is maximized. By using a several construction of flat plate particular passages with rectangular duct, square duct, rhombus duct, isosceles right triangle duct, circular duct, elliptic duct. The following assumptions are made throughout the analysis: the duct walls are isothermal (negligible conduction resistance in the array), uniform flow distribution (equal flow in all

ducts or channels), laminar flow, constant cross-sectional duct area, no inlet or exit plenum losses, Prandtl number range  $Pr > 0.1$ , and a finite control volume ( $V = HWL$ ).

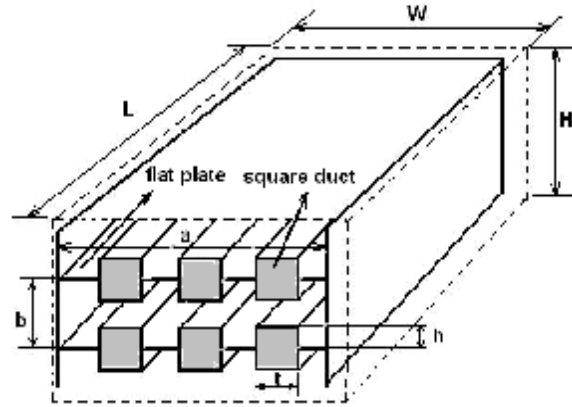


Figure (1) Convectively cooled finite volume for the flat plate with square duct geometry microchannels.

**HYDRODYNAMIC ANALYSES**

The pressure drop associated with flow across the channel is given by **Khan, et.al (2009), [15]**.

$$\Delta p = \frac{\rho U_{\infty}}{2} \left[ k_{ce} + \left( f \frac{L}{D_h} \right) \right] \quad \dots (1)$$

Where the friction factor depends on the Reynolds number, channel aspect ratio, and slip velocity coefficient **Khan, et.al (2010), [21]**.

$$f = \frac{24}{Re_{D_h}} \left[ \left( \frac{1}{1+\alpha} \right) + \left( \frac{1}{1+6} \right) \right] \quad \dots(2)$$

From data of **Kays and London (1964) [22]** and derived the following empirical correlation for the entrance and exit losses ( $k_{ce}$ ) in terms of channel width and fin thickness.

$$k_{ce} = 1.79 - 2.32 \left( \frac{w_c}{w_c + w_w} \right) + 0.53 \left( \frac{w_c}{w_c + w_w} \right)^2 \quad \dots (3)$$

If the (G) is the volume flow rate, then the total mass flow rate can be written as **Khan, et.al (2009), [15]**

$$\dot{m} = \rho G$$

Average velocity in the channel is given by **Kays and London (1964) [22]**.

$$U_{\infty} = \frac{\dot{m}}{Nr(2w_c)H_c} \quad \dots (4)$$

The value of N for a given array must be determined for the cross-section, HW, in terms of a characteristic dimension of the duct or channel in the array. Where N is the number of channels given by **Khan, et.al (2009), [15]**.

$$N = \frac{W - w_w}{w_c - w_w} \quad \dots (5)$$

**THERMAL ANALYSIS**

An array of ducts or channels with small cross-sectional characteristic reference length scale, the heat transfer rate for fully developed flow gives.

$$Q_s = \rho U_{\infty} N A C_p (T_s - T_f) \quad \dots (6)$$

The mean velocity,  $U_{\infty}$  in any one duct or channel assuming uniform flow distribution, may be determined from the fully developed flow Poiseuille number defined.

$$U_{\infty} = \frac{A \Delta p L_c}{\mu P L P o_{Lc}}$$

And can write the heat transfer rate in terms of the fundamental flow quantities

$$Q_s = \frac{\rho C_p A^2 \Delta p L_c (T_s - T_f)}{\mu P L P o_{Lc}} \quad \dots (7)$$

Where A is the cross-sectional area of an elemental duct or channel, N is the total number of ducts or channels,  $T_s$  is the mean wall temperature, and  $T_f$  is the fluid inlet temperature. The mean velocity,  $U_{\infty}$ , in any one duct or channel assuming uniform flow distribution, may be determined from the fully developed flow Poiseuille number defined as **Liu and Garimella, (2005) [22]**.

$$P o_{Lc} = \frac{(A/P)(\Delta p/L)L_c}{m \bar{U}} \quad \dots (8)$$

Equation. (7) may be written in alternate form **Liu and Garimella, (2005) [22]**.

$$Q_{System} = \frac{\rho C_p (T_s - T_f) \Delta p L_c}{\mu} \quad \dots (9)$$

$$Q_{Geometry} = \frac{N A^2 L_c}{P L P o_{Lc}} \quad \dots (10)$$

The heat transfer rate is determined from

$$Q_1 = 0.7611 \frac{k_f \text{Pr}^{1/3} \Delta p^{1/3} (T_s - T_f)}{\rho^{1/3} \nu^{2/3}} N(A^2 PL)^{1/3} \quad \dots (11)$$

The optimal duct or channel size may be found by means of the method of intersecting asymptotes [23]. The exact shape of the heat transfer rate curve may be found using more exact methods such as expressions found in **Shah and London [22]** for individual geometries an approximate value for the reference duct dimension may be found. Intersecting Eqs. (7) and (11) gives after simplifying and collecting the system and geometry terms, the above equation may be written in the following form:

$$\text{Be}^{1/4} = 0.9027 \frac{L}{\left(\frac{Lc}{\text{Po}_{Lc}}\right)^{3/8} \left(\frac{A}{P}\right)^{5/8}} \quad \dots (12)$$

Where ( $\text{Be} = \text{Dp}L^2/\alpha\mu$ ) is the Bejan number as defined in [20]. The right hand side is only a function of the duct shape and aspect ratio, while the left hand side is a system parameter which is constant and independent of duct shape or aspect ratio once a cooling volume,  $V = \text{HWL}$ , is specified. When the hydraulic diameter is chosen,  $Lc = 4A/P$ , the optimal solution is determined by solving:

$$\text{Be}^{1/4} = 0.5365 \frac{PL}{A} \text{Po}_{\text{Dh}}^{3/8} \quad \dots (13)$$

Substituting Equation. (13) into Equation. (12) gives the following results for the optimal plate spacing:

$$\frac{b_{\text{opt}}}{L} = 2.726 \text{Be}^{-1/4} \quad \dots (14)$$

The maximum heat transfer rate for a fixed volume can be obtained from Eq. (7) using the optimal result determined by Eq. (13). The number of ducts or channels  $N$ , which appears in the final result, may then be cast in terms of the cooling volume cross-section  $\text{HW}$ . In this way, the maximum heat transfer per unit volume may be determined. Subsequent results may then be presented in terms of the following dimensionless heat transfer per unit volume:

$$Q_{\text{max}} = \frac{\zeta L^2}{k(T_s - T_f)} \quad \dots (15)$$

Where  $z = \frac{Q}{\text{HWL}}$  is the heat transfer per unit volume, and can be consider the flat plate microchannel as the rectangular fin, to compute the efficiency of the heat dissipation.

$$h_{fin} = \frac{\tanh(mH)}{mH} \quad \dots (16)$$

Where

$$m = \sqrt{\frac{2h_{av}}{kw_w}}$$

**ELEMENTAL GEOMETRIES**

Consider several common geometries which are convenient in electronics cooling and compact heat exchanger design. These include arrays of parallel plates, circular tubes, rectangular channels, elliptic ducts, polygonal ducts, and triangular ducts, as shown Table (1) and Figure (2) method of intersecting asymptotes.

**Table(1) Elemental geometries model.**

Models Under study	surface area, mm.	perimeter, m	hydraulic diameter, 4A/P
Parallel plates with rectangular shape	$A = ab - (n_r ht)$	$P = 2[(a - n_r t) + n_r (t + h)]$	$D_h = \frac{2(ab - n_r h)}{(a + n_r h)}$
Parallel plates with square shape	$A = ab - n_s h^2$	$P = 2a - n_s h + 4h$	$D_h = \frac{4(ab - n_s h^2)}{2a - n_s h + 4h}$
Parallel plates with rhombus shape	$A = ab - n_p h^2$	$P = 2a - 2n_s h \cos 45 + 4n_s h$	$D_h = \frac{ab - n_p h^2}{2a - 2n_s h \cos 45 + 4n_s h}$
Parallel Plate with triangular shape	$A = ab - \frac{1}{2} n_t h^2 \sin 60$	$P = 2a - n_t h + 3n_t h$	$D_h = \frac{2ab - n_t h^2 \sin 60}{a - n_t h}$
Parallel plates with circular shape	$A = ab - \frac{\pi d^2 n_c}{4}$	$P = 2a - n_c d + \pi n_c d$	$D_h = \frac{4ab - \pi n_e ht}{2a - n_e t + 2n_e tE(e)}$

**RESULT AND DISCUSSION**

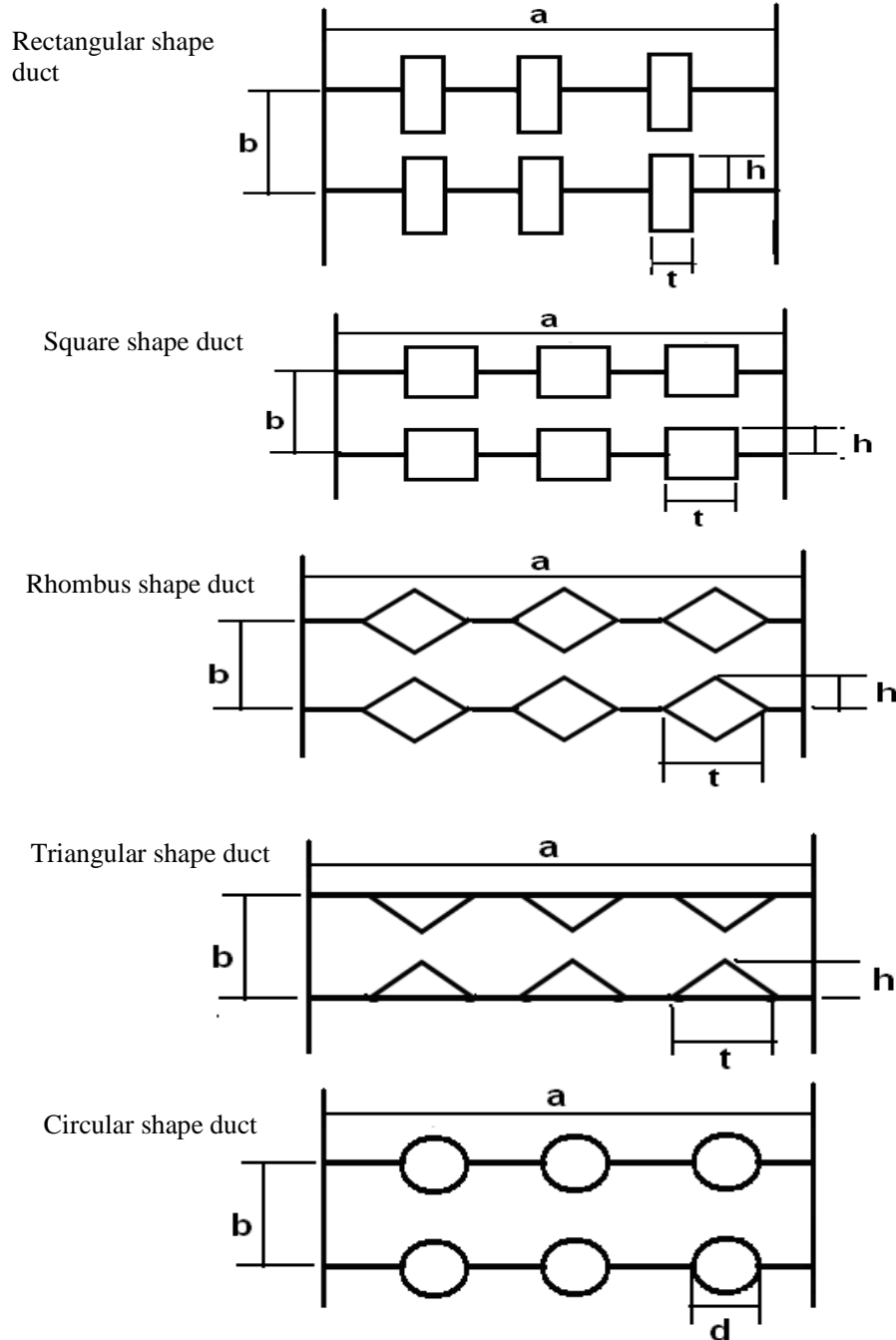
The approximate solutions for the optimal duct geometry have been obtained using the simple approach of **Bejan and Sciubba (1992)** [19]. By using simple (MATLAP-7) program to solve above equation to study the effects of the (Reynolds numbers, aspect ratios, Poiseuille numbers, Prandtl number geometry and average velocity) on the parameter of (maximum heat transfer, heat dissipation, optimal plate spacing ( $b_{obt} / Lc$ ), optimal heat transfer and plate spacing ( $Q_{max} / b_{obt}$ ). For the most common and useful shapes examined, we see that arrays of Parallel plate with rectangular and Parallel plate with triangular duct appear to yield maximum heat transfer per unit volume. Intuition would tend to agree with these results, as Parallel plate with circular duct (PCM)

Parallel plate with ellipse duct (PEM) have the lowest Poiseuille numbers and hence lower flow resistance, while at the same time, these duct shapes can also



provide for a large surface area per unit volume. see that from Figure (3) effect of channel aspect ratio on the Poiseuille numbers at superior performance where  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models. The arrays of parallel plate with rectangular channels and equilateral or isosceles right triangles appear to yield maximum Poiseuille numbers and the lowest one is the parallel plate with circular channels. And Figure (4) shows the effect of Reynolds number ( $Re$ ) on the efficient of heat dissipation at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models. The arrays of parallel plate with rectangular and right triangles channels give the highest heat dissipation compared with the another models and the ellipse give low value of the heat dissipation at the high Reynolds number and show that with increasing the value of  $Re$ , the heat dissipation well increasing for all models. But from Figure (5) effect of channel aspect ratio on the optimal plate spacing ( $b_{opt} / Lc$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models show that the models of parallel plate with circular and ellipse will increasing the value of channels ( $b_{opt} / Lc$ ) with increasing of channel aspect ratio while the models of parallel plate with rectangular will decrease with increasing of aspect ratio but another model will seem constant in this range. And from the Figure (6) the average velocity increase with increasing of the optimal heat transfer plate spacing ( $Q_{max} / b_{opt}$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models. Figure (7) illustrate the effect of channel aspect ratio on the heat transfer rate ( $QS$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models, show the decreasing of heat transfer rate with increase the aspect ratio and the heat transfer rate ( $Q_{max}$ ) increasing with channel aspect ratio at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models with maximum value of heat transfer rate at the model of rectangular channel. Show in Figures (8, 9) illustrates the effect of channel aspect ratio on the Poiseuille numbers at  $Pr=10$  and the number of passages equal to  $n=8$  for all models show the decreasing the Poiseuille numbers with increasing the channel aspect ratio. And present study present that the heat dissipation of the ellipse model increasing the Reynolds number  $Re$  at  $Pr=10$  and the number of passages equal to  $n=8$  but another model seem constant in Figure (10). The optimal heat transfer plate spacing ( $Q_{max} / b_{opt}$ ) increase with average velocity on the at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models in Figure (11). To study the effect of channel aspect ratio on the heat transfer rate ( $QS$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models using Figure (12) show that the rectangular model higher than the circular about 52% at  $\epsilon$  aspect ratio,  $b/a=0.4$  and by 87%compard with rhombus duct model. Figure (13) shows the effect of Reynolds number on the maximum heat transfer rate ( $Q_{max}$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models, the model of rectangular appear increasing about 20% compared rhombus duct model but by 14% with circular model at  $Re=3125$  the relation between channel aspect ratio and the efficient of heat dissipation by the effect of ( $Pr$ ) and the number of passages ( $n$ ) for the models of rectangular and square passages shapes. Use Figure (16) to study the relation between Reynolds number  $Re$  and the heat transfer rate ( $QS$ ) by the effect of ( $Pr$ ) and the number of passages ( $n$ ) for the models of rectangular and square passages shapes and to compare present work with anther studies using Figure (17) comparison the numerical results given by the present study flat plate with ellipse duct with the results of Y.S. Muzychka [17] for ellipse duct, the effect of channel aspect ratio on the maximum heat transfer rate ( $Q_{max}$ ) at

$pr=0.1$  for all model of ellipse the results show that the arrays of parallel plate with ellipse channels of the present show increasing about 57% at the same ( $\epsilon$  aspect ratio  $b/a=0.5$ ).



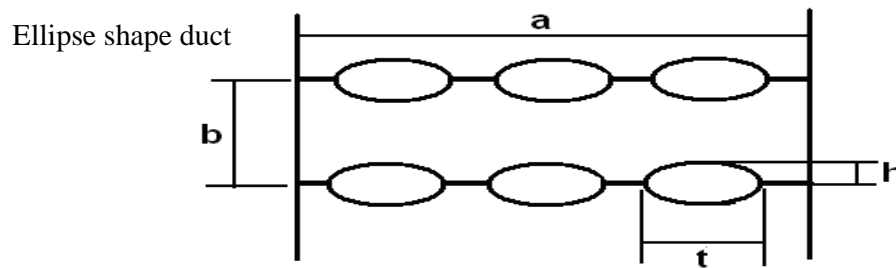


Figure (2) Elemental geometries being considered.

### CONCLUSIONS

The numerical result show that the heat transfer rate ( $Q_{max}$ ) at  $pr=0.1$  increasing with increasing renoldes number and the Poiseuille numbers decreasing with thechannel aspect ratio at  $pr=0.1$  and the number of passages equal to  $n=2$  for all models. The optimal heat transfer plate spacing ( $Q_{max} / b_{opt}$ ) increasing with average velocity show that the rectangular model higher than the circular about 52% at  $\epsilon$  aspect ratio,  $b/a=0.4$  and by 87% compard with rhombus duct model.

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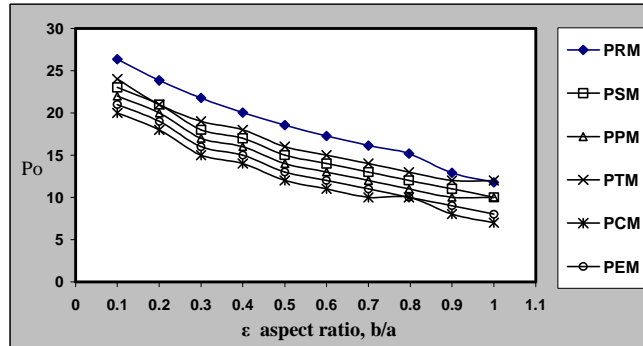


Figure (3) Effect of channel aspect ratio on the Poiseuille numbers at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

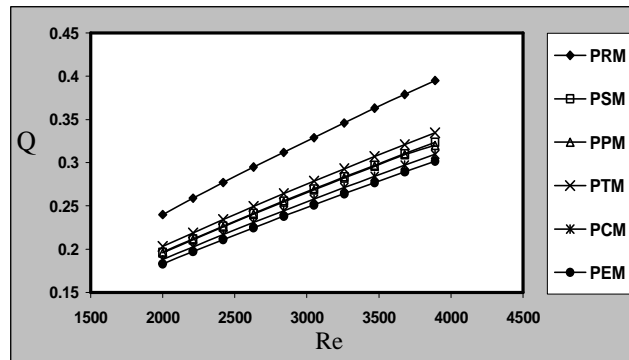


Figure (4) Effect of Reynolds number  $Re$  on the efficient of heat dissipation at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

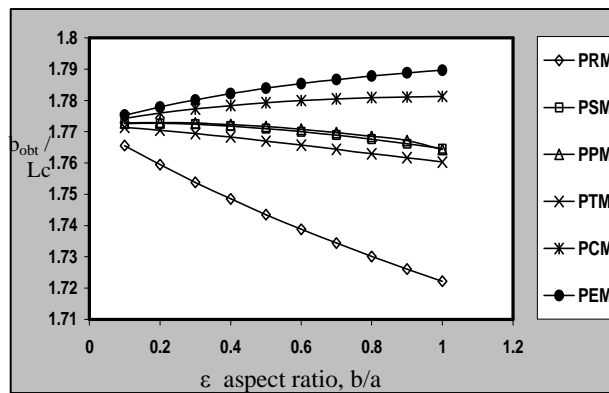


Figure (5) Effect of channel aspect ratio on the optimal plate spacing ( $b_{obt} / L_c$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

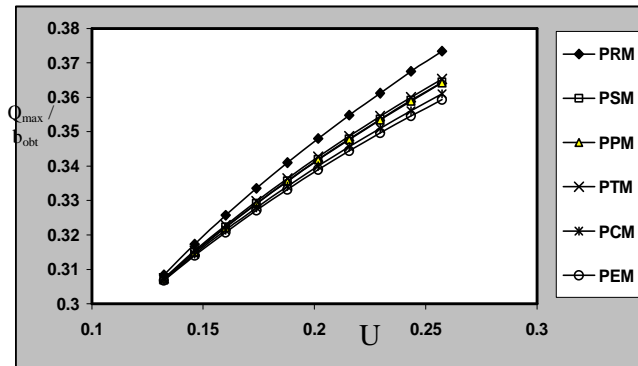


Figure (6) Effect of average velocity on the optimal heat transfer plate spacing ( $Q_{max} / b_{opt}$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

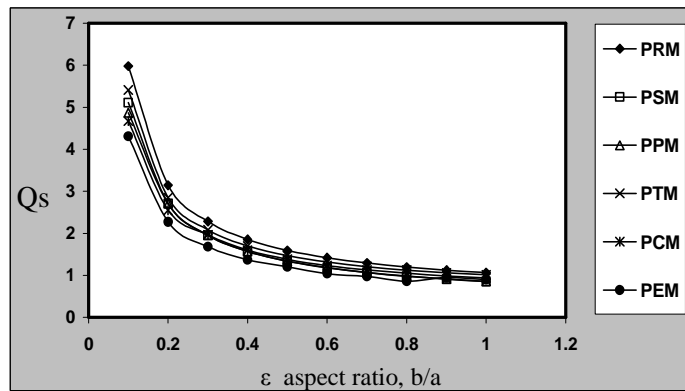


Figure (7) Effect of channel aspect ratio on the heat transfer rate ( $Q_s$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

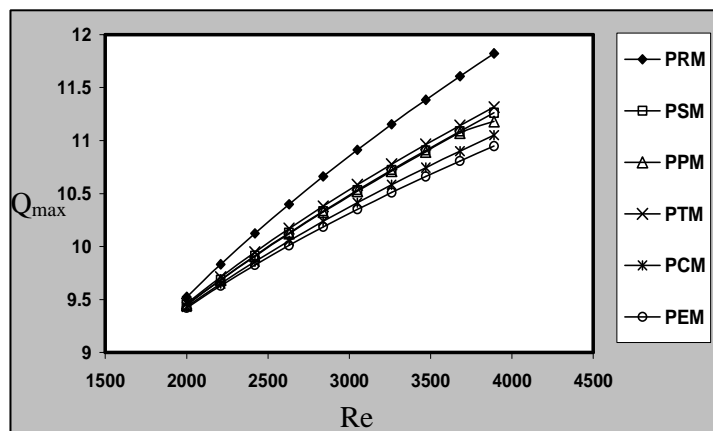


Figure (8) Effect of channel aspect ratio on the heat transfer rate ( $Q_{max}$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

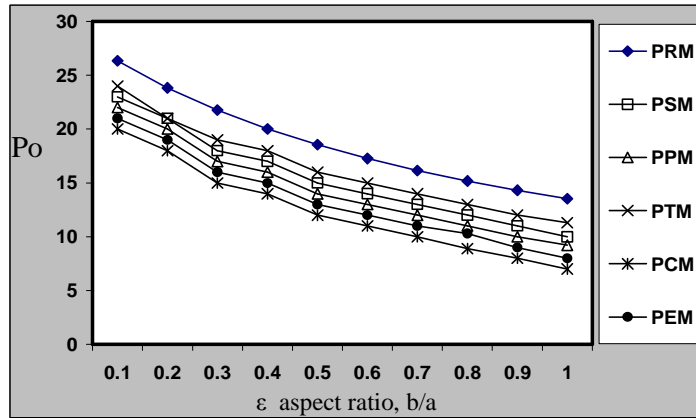


Figure (9) Effect of channel aspect ratio on the Poiseuille numbers at Pr=10 and the number of passages equal to n=8 for all models.

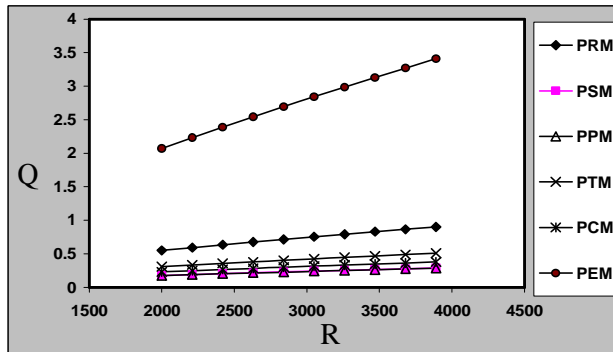


Figure (10) Effect of Reynolds number Re on the efficient of heat dissipation at Pr=10 and the number of passages equal to n=8 for all models.

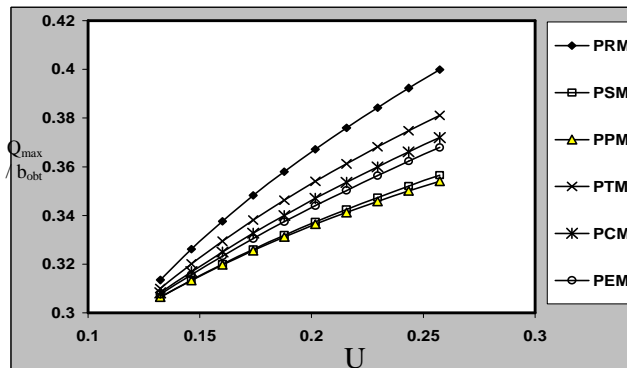


Figure (11) Effect of average velocity on the optimal heat transfer plate spacing ( $Q_{max} / b_{obt}$ ) at Pr=0.1 and the number of passages equal to n=2 for all models.

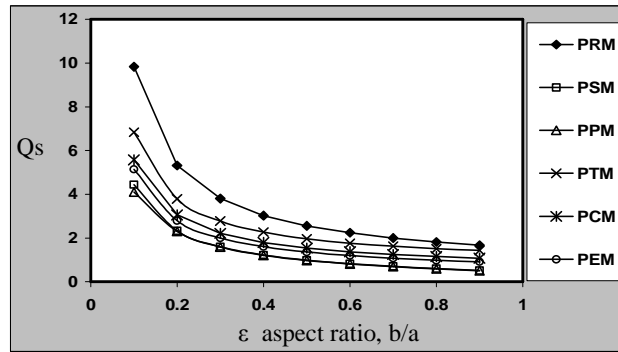


Figure (12) Effect of channel aspect ratio on the heat transfer rate ( $Q_s$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

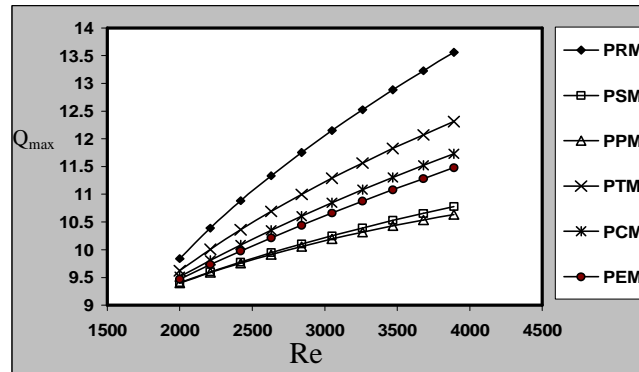


Figure (13) Effect of channel aspect ratio on the maximum heat transfer rate ( $Q_{max}$ ) at  $Pr=0.1$  and the number of passages equal to  $n=2$  for all models.

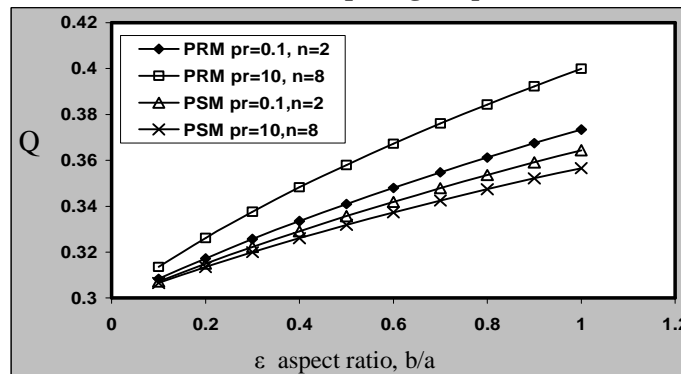


Figure (14) the relation between channel aspect ratio and the efficient of heat dissipation for the models of rectangular and square passages shapes.



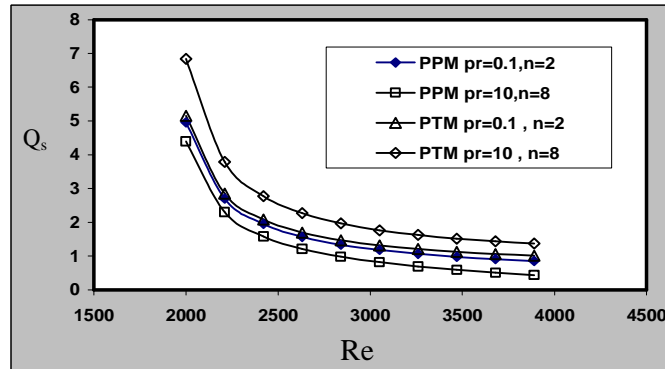


Figure (15) the relation between Reynolds number  $Re$  and the heat transfer rate ( $Q_s$ ) for the models of rectangular and square passages shapes.

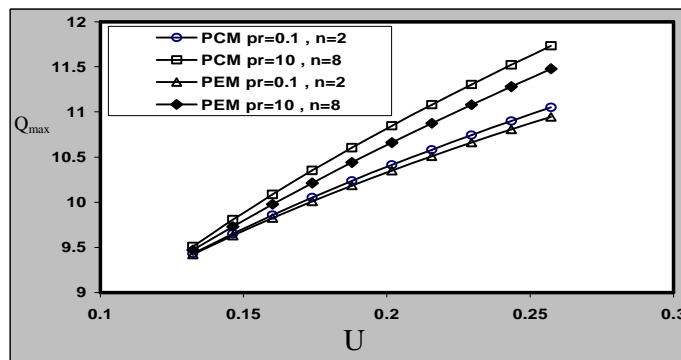


Figure (16) the relation between maximum heat transfer rate ( $Q_{max}$ ) and the average velocity  $U$  for the models of rectangular and square passages shapes.

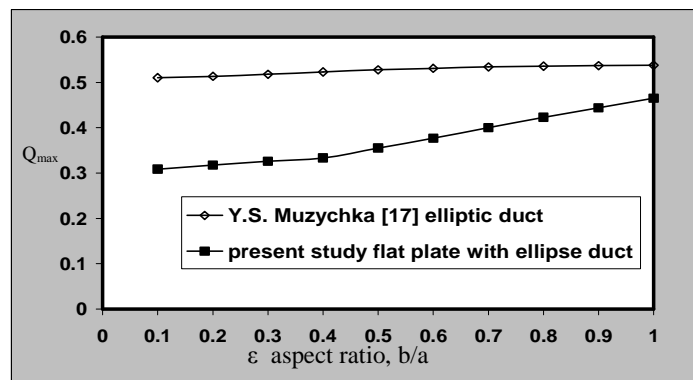


Figure (17) comparison present study flat plate with ellipse duct with the results of Muzychka (2005)[17].