Certain Types of Separation Axioms in Intuitionistic Bitopological Spaces

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The aim of this paper is to present and study some new separation axioms using the b-open sets in intuitionistic bitopological spaces.

INTRODUCTION

The concept of intuitionistic fuzzy set has been introduced by Atanassov in 1984. Later in 1996 Coker introduced the concept of intuitionistic set and intuitionistic topology as special case of intuitionistic topological spaces. In 1963 J.C.Kelly introduced the concept of bitopological space on his work "Bitopological spaces" in 1996, Andrijevic [2] introduced b-open sets in topological spaces. In 1963 Levine [1], introduced semi-open sets in topological spaces, and these semi-open sets were used to define three new separation axioms called semi-T₀, semi-T₁ and semi-T₂ by Maheswari and Prasad in 1975. The purpose of this paper is to introduce some new separation properties by using the (1,2)b-open sets,(1,2)semi-open sets and (1,2)pre-open sets in intuitionistic bitopological spaces and to investigate the relationships between them.

Preliminaries-2:

Definition 2-1:[6] A subset A of a space X is said to be:
1- Semi-open if \( A \subseteq \text{cl}(\text{int}(A)) \).
2- Pre-open if \( A \subseteq \text{int}(\text{cl}(A)) \).
3- b-open if \( A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \).

Definition 2-2: [3] Let X be a non-empty set, an intuitionistic set (IS`s for short) A is an object having the form A = \( < x, A_1, A_2 > \) where \( A_1 \) and \( A_2 \) are disjoint subsets of X, the set \( A_1 \) is called the set of members of A while \( A_2 \) is called the set of nonmembers of A.

Definition 2-3: [3] Let X be a non-empty set, and let A, B be two IS`s having the form A= \( < x, A_1, A_2 > \), B=\( < x, B_1, B_2 > \) and let \( \{ A_i : i \in I \} \) be an arbitrary family of IS`s in X, where \( A_i = \{ x, A_i^{(1)}, A_i^{(2)} \} \) then:
1- \( \widetilde{X} = < x, X, \phi > \) and \( \widetilde{\phi} = < x, \phi, X > \).
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2. \( A \subseteq B \iff A_i \subseteq B_i \) and \( B_2 \subseteq A_2 \).

3. \( A = B \iff A \subseteq B \) and \( B \subseteq A \).

4. The complement of \( A \) is denoted by \( \overline{A} \) and defined by \( \overline{A} = \{ x, A_2, A_i \} \).

5. \( \bigcup_{i \in I} A_i = \left\{ x, \bigcup_{i \in I} A_{i}^{(1)}, \bigcap_{i \in I} A_{i}^{(2)} \right\} \) and
   \( \bigcap_{i \in I} A_i = \left\{ x, \bigcap_{i \in I} A_{i}^{(1)}, \bigcup_{i \in I} A_{i}^{(2)} \right\} \)

**Remark 2-4:** [4] Every crisp set \( A \) on a non-empty set \( X \) is obviously an IS having the form \( \langle x, A, A' \rangle \).

**Definition 2-5:** [3] An intuitionistic topology (IT, for short) on a non-empty set \( X \), is a family \( T \) of Is in \( X \) containing \( \emptyset \) and \( X \) and closed under arbitrary union and finitely intersection. In this case the pair \( (X,T) \) is called intuitionistic topological space (ITS, for short). Any IS in \( T \) is known as an intuitionistic open set (IOS, for short) in \( X \). The complement of IOS is called intuitionistic closed set (ICS, for short).

**Definition 2-6:** [3] Let \( (X,T) \) be an ITS and let \( A = \{ x, A_1, A_2 \} \) be an IS in \( X \), then the interior and closure of \( A \) are denoted by \( \text{int} (A) \) and \( \text{cl} (A) \), respectively, and defined by:

\[
\text{int}(A) = \bigcup \left\{ G_i : G_i \in T \text{ and } G_i \subseteq A \right\}
\]

\[
\text{cl}(A) = \bigcap \left\{ F_i : F_i \in T \text{ and } A \subseteq F_i \right\}
\]

So \( \text{int}(A) \) is the largest IOS contained in \( A \), and \( \text{cl}(A) \) is the smallest ICS containing \( A \).

**Intuitionistic bitopological spaces -3**

In [5] further definitions appear. In his section we introduce a new concept concerning intuitionistic bitopological spaces. First we recall the following definitions.

**Definition 3.1:** Let \( (X,T_1,T_2) \) be an intuitionistic bitopological space. A subset \( A \) of \( X \) is intuitionistic \( T_1T_2 \)-open set if \( A \in T_1 \bigcup T_2 \) and intuitionistic \( T_1T_2 \)-closed set if its complement in \( X \) is intuitionistic \( T_1T_2 \)-open set. The intuitionistic \( T_1T_2 \)-closure of \( A \) is denoted by \( T_1T_2 - \text{cl} (A) \) and

\[
T_1T_2 - \text{cl}(A) = \bigcap \left\{ F : F \text{ is intuitionistic } T_1T_2 \text{-closed set in } X \text{ and } A \subseteq F \right\}
\]

**Definition 3.2:** An intuitionistic topological space \( (X,T) \) is said to be \( R_0 \) if for each \( G \in T, x \in G \) implies \( T - \text{cl} (\{x\}) \subseteq G \) and it is \( R_1 \) if for each pair of points \( x, y \in X \), such that \( T - \text{cl}(\{x\}) \neq T - \text{cl}(\{y\}) \) there are disjoint intuitionistic open sets \( U \) and \( V \), such that \( x \in U, y \in V \).
**Definition 3.3:** An intuitionistic bitopological space $X$ is pair wise $R_0$ if for each $G \in T_i, x \in G$ implies $T_j - cl(\{x\}) \subset G$ for $i, j = 1, 2, i \neq j$ and pair wise $R_i$ if for $x, y \in X$, such that $T_j - cl(\{x\}) \neq T_i - cl(\{y\})$ there are $T_i$ intuitionistic open set $U$ and $T_j$ intuitionistic open set $V$, such that $x \in U, y \in V$ and $U \cap V = \emptyset$ for $i, j = 1, 2, i \neq j$.

**Definition 3.4:** An intuitionistic bitopological space $X$ is pairwise- $R_i$ if for each $x, y \in X$, such that $x \not\in T_i - cl(\{y\})$, there are $T_i$ intuitionistic open set $U$ and $T_j$ intuitionistic open set $V$, such that $x \in U, y \in V$ and $U \cap V = \emptyset, i, j = 1, 2, i \neq j$.

**Definition 3.5:** An intuitionistic subset $A$ of $X$ is called

(i) a $(1,2)$ intuitionistic b-open set $[(1,2) IBOS$, for short] if $A \subset T_1 - int (T_1 T_2 - cl(A)) \bigcup T_1 T_2 - cl(T_1 - int(A))$.

(ii) a $(1,2)$ intuitionistic semi-open set $[(1,2) ISOS$ for short] if $A \subset T_1 T_2 - cl(T_1 - int(A))$.

(iii) a $(1,2)$ intuitionistic pre-open set $[(1,2) IPO$ for short] if $A \subset T_1 - int(T_1 T_2 - cl(A))$.

The family of all $(1,2)$ IBO(resp. $(1,2)$ ISO, $(1,2)$ IPO) sets of $X$ is denoted by $(1,2)$ IBO$(X)$ (resp. $(1,2)$ ISO$(X)$, $(1,2)$ IPO$(X)$).

The complement of a $(1,2)$ IBO (resp. $(1,2)$ ISO, $(1,2)$ IPO) set in $X$ is called $(1,2)$ IBC (resp. $(1,2)$ ISC, $(1,2)$ IPC) sets in $X$.

$(1,2)$ IBcl$(A)$ (resp. $(1,2)$ IScl$(A)$, $(1,2)$ Ipcl$(A)$) denotes $(1,2)$ b-closure (resp. $(1,2)$ semi-closure, $(1,2)$ pre-closure) of $A$ in $X$.

**Proposition 3.6:** Let $X$ be an intuitionistic bitopological space. Then $(1,2)$ ISO$(X) \bigcup (1,2)$ IPO$(X) \subset (1,2)$ IBO$(X)$.

Proof: The proof is obvious.

But the converse may be not true in general, for example.

**Example 3.7:** Let $X = \{a, b, c\}$,

$$T_1 = \left\{ T \in \mathcal{P}(X), \langle x, \{a\}, \{b, c\}\rangle, \langle x, \{b\}, \{c\}\rangle, \langle x, \{a, b\}, \{c\}\rangle, \langle x, \phi, \{b, c\}\rangle \right\}$$

$$T_2 = \left\{ T \in \mathcal{P}(X), \langle x, \{a\}, \{c\}\rangle \right\}$$

then:

$(1,2)$ ISO$(X) = \left\{ T_1, \langle x, \{a\}, \{b, c\}\rangle, \langle x, \{a\}, \phi\rangle, \langle x, \{b\}, \{c\}\rangle, \langle x, \{b\}, \phi\rangle, \langle x, \{a, b\}, \phi\rangle, \langle x, \{a\}, \{b\}\rangle, \langle x, \phi, \{b, c\}\rangle, \langle x, \phi, \{b\}\rangle, \langle x, \phi, \{c\}\rangle, \langle x, \phi, \{b, c\}\rangle \right\}$

$(1,2)$ IPO$(X) = \left\{ T_1, \langle x, \{a\}, \{b, c\}\rangle, \langle x, \{a\}, \phi\rangle, \langle x, \{b\}, \{c\}\rangle, \langle x, \{a, b\}, \phi\rangle, \langle x, \{a\}, \{b\}\rangle, \langle x, \phi, \{b, c\}\rangle, \langle x, \phi, \{b\}\rangle, \langle x, \phi, \{c\}\rangle, \langle x, \phi, \{b, c\}\rangle \right\}$.
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(1.2)\(IBO(X) = \langle \tilde{X}, \tilde{\phi}, \langle x, \{a\}, \{b, c\} \rangle, \langle x, \{a\}, \{b\} \rangle, \langle x, \{a\}, \{c\} \rangle, \langle x, \{b\}, \{c\} \rangle \rangle \)
for all \( x \in X \)

Proposition 3.8: An intuitionistic subset \( A \) of intuitionistic bitopological space \( X \)

then \( (1.2)IBO(X) = (1.2)ISO(X) \cup (1.2)IPO(X) \)

Proof:

let \( S \in IBO(X) \). If \( T_1 \cap \{ T_1 \cap \text{cl}(A) \} \subseteq T_1 \cap \text{cl}(T_1 \cap \text{int}(A)) \) or \( T_1 \cap \text{cl}(T_1 \cap \text{int}(A)) \subseteq T_1 \cap \text{cl}(T_1 \cap \text{int}(A)) \)

then \( S \in ISO(X) \) 

and so \( S \in ISO(X) \) If \( T_1 \cap \{ T_1 \cap \text{cl}(A) \} \subseteq T_1 \cap \text{cl}(T_1 \cap \text{int}(A)) \),

then \( S \subseteq T_1 \cap \text{cl}(T_1 \cap \text{int}(A)) \) and so \( S \in IPO(X) \).

Intuitionistic ultra \( b-T_i(i = 0, 1) \) space. -4

Definition 4-1: An intuitionistic bitopological space \( X \) is said to be intuitionistic ultra \( b-T_0 \) space (resp. Intuitionistic ultra semi-\( T_0 \) space and Intuitionistic ultra pre-\( T_0 \) space) if for \( x, y \in X, x \neq y \) there exists \( U \in (1.2)IBO(X) \) (resp \( (1.2)ISO(X) \) and \( (1.2)IPO(X) \)) such that \( x \in U, y \notin U \) or \( x \notin U, y \in U \).

Example 4-2: let \( X = \{a, b\} \)

\[ T_i = \{ \tilde{X}, \tilde{\phi}, \langle x, \{a\}, \{b\} \rangle, \langle x, \{a\}, \{c\} \rangle \} \]

\[ (1.2)ISO(X) = \{ \tilde{X}, \tilde{\phi}, \langle x, \{a\}, \{b\} \rangle, \langle x, \{b\}, \{c\} \rangle, \langle x, \{c\}, \{b\} \rangle \} \]

\[ (1.2)IPO(X) = \{ \tilde{X}, \tilde{\phi}, \langle x, \{a\}, \{b\} \rangle, \langle x, \{b\}, \{c\} \rangle, \langle x, \{c\}, \{b\} \rangle \} \]

It is clear that \( X \) is Intuitionistic ultra \( b-T_0 \) space, Intuitionistic ultra semi-\( T_0 \) space, but not Intuitionistic ultra pre-\( T_0 \) space.

Definition 4-3: An intuitionistic bitopological space \( X \) is said to be intuitionistic ultra \( b-T_i \) space (resp. Intuitionistic ultra semi-\( T_i \) space, Intuitionistic ultra pre-\( T_i \) space) if for \( x, y \in X, x \neq y \) there exists \( U, V \in (1.2)IBO(X) \) resp. \( (1.2)ISO(X), (1.2)IPO(X) \) s.t \( x \in U, y \notin U, x \notin V \).

Example 4-4: let \( X = \{a, b\} \)

\[ T_i = \{ \tilde{X}, \tilde{\phi}, \langle x, \{b\}, \{a\} \rangle, \langle x, \{a\}, \{b\} \rangle \} \]

\[ (1.2)ISO(X) = \{ \tilde{X}, \tilde{\phi}, \langle x, \{b\}, \{a\} \rangle, \langle x, \{b\}, \{c\} \rangle \} \]

\[ (1.2)IPO(X) = \{ \tilde{X}, \tilde{\phi}, \langle x, \{a\}, \{b\} \rangle, \langle x, \{b\}, \{a\} \rangle \} \]
(1,2)\(IBO(X) = \{\bar{x}, \tilde{\phi}, \langle x, \{a\}, \{x, b\}, \phi \rangle, \langle x, \{b\}, \phi \rangle, \langle x, \phi, \phi \rangle\}\)

Therefore \(X\) is Intuitionistic ultra \(b-T_1\) space, Intuitionistic ultra pre-\(T_0\) space but not Intuitionistic ultra semi \(T_1\) space.

**Example 4-5:** see example 4-2, \(X\) is Intuitionistic ultra semi-\(T_1\) space.

**Remarks 4-6:**

1- Every Intuitionistic ultra \(b-T_0\) space is Intuitionistic ultra \(b-T_0\) space.

2- Every Intuitionistic ultra semi- \(T_1\) space is Intuitionistic ultra semi- \(T_0\) space.

3- Every Intuitionistic pre- \(T_0\) space is Intuitionistic ultra pre-\(T_0\) space.

But the converse of remark 4-6 is not true,

**Example 4-7:** see example 4-4 is Intuitionistic ultra semi- \(T_0\) but not is Intuitionistic ultra semi- \(T_1\) and example 4-2 to explain \(X\) is Intuitionistic ultra \(pre-T_0\) but not is Intuitionistic ultra \(pre-T_1\).

**Definition 4-8:** A Intuitionistic bitopological space \(X\) is said to be Intuitionistic ultra \(b-T_2\) space (resp. Intuitionistic ultra semi- \(T_2\) space, Intuitionistic ultra pre- \(T_2\) space) if for \(x, y \in X, x \neq y\) there exists \(U, V \in (1,2)IBO(X) (resp (1,2)ISO(X), (1,2)IPO(X))\).

Such that \(x \in U, y \in V, U \cap V = \emptyset\).

**Remark 4-9:** Every Intuitionistic ultra \(b-T_2\) space (resp. Intuitionistic ultra semi- \(T_2\) space and intuitionistic ultra pre-\(T_2\) space) is Intuitionistic ultra \(b-T_1\) space (resp.Intuitionistic ultra semi -\(T_1\) space, Intuitionistic ultra pre-\(T_1\) space).

**Proposition 4-10:**

1- Every Intuitionistic ultra semi- \(T_1\) space is Intuitionistic ultra \(b-T_1\) space.

2- Every Intuitionistic ultra pre- \(T_1\) space is Intuitionistic ultra \(b-T_1\) space, where \(i = 0, 1, 2\).

Proof: since Every \((1,2)\) Intuitionistic semi- open set is \((1,2)\) Intuitionistic \(b\)– open set and every \((1,2)\) Intuitionistic pre- open set is \((1,2)\) Intuitionistic \(b\)– open set.

The converse of proposition 4-10 is not true, see example 4-4, \(X\) is Intuitionistic ultra \(b-T_i\) space but is not Intuitionistic ultra semi - \(T_i\) space, where \(i = 0, 1\).

And example 4-2, \(X\) is Intuitionistic ultra \(b-T_i\) space, but not Intuitionistic ultra pre- \(T_i\) space, where \(i=0, 1\).
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Theorem 4-11: An intuitionistic bitopological space X is Intuitionistic ultra \( b-T_i \) space if and only if \( \langle x, \{a\}, \{a^c\} \rangle \) is (1.2) intuitionistic \( b- \) closed in X, for every \( a \in X \).

Proof: if \( \langle x, \{a\}, \{a^c\} \rangle \) is (1,2)intuitionist \( b- \)closed in X, for \( a,b \in X \) with \( \{a\}/\{b\} \), \( \{a\}/\{b\} \) are (1,2)Intuitionistic \( b- \)open sets such that \( b \in \{a\}/\{a^c\} \) and \( a \in \{b\}/\{b^c\} \). therefore, X is Intuitionistic ultra \( b-T_i \) space.

Conversely, if X is Intuitionistic ultra \( b-T_i \) space and \( b \in X \setminus \{a\} \) such that \( \{a\}/\{b\} \), \( \{a\}/\{b\} \), \( \{a\}/\{b\} \), \( \{a\}/\{b\} \) then \( a \neq b \) therefore, there exist (1,2) Intuitionistic b-open sets \( U_a, V_b \) in X such that \( a \in U_a, b \notin U_a, \) and \( b \in V_b, a \notin V_b \).

Let \( G \) be union of all such \( V_b \) then \( G \) is (1,2) Intuitionistic b-open set in X and \( G \subset X \setminus \{a\} \subset X \), therefore, \( X \setminus \{a\} \) is (1,2) Intuitionistic b-open set in X.

Intuitionistic ultra \( b-R \) and Intuitionistic ultra semi - \( R_i \) space, \( i = 0.1-5 \):

Definition 5-1: An Intuitionistic bitopological space X is said to be.

1- Intuitionistic ultra \( b-R_0 \) (resp. Intuitionistic ultra semi-\( R_0 \)) if \( (1,2)IBcl(\{x\}) \subset U [resp. (1,2)IScl(\{x\}) \subset U] \)

whenever, \( x \in U \in (1,2)IBO(X) [resp. x \in U \in (1,2)ISO(X)] \)

2- Intuitionistic ultra \( b-R_i \) (resp .Intuitionistic ultra semi-\( R_i \)) if for \( x, y \in X \) such that \( x \notin (1,2)IBcl(\{y\}) [resp. (1,2)IScl(\{y\})] \) there exist disjoint \( (1,2)IB-[resp.(1,2)IS-open] \) \( U, V \) in X such that \( x \in U \) and \( y \in V \)

3- Intuitionistic weakly ultra \( b-R_0 \) (resp Intuitionistic weakly ultra semi-\( R_0 \))

if \( \bigcap_{x \in X}(1,2)IBcl(\{x\}) = \phi \) [resp. \( \bigcap_{x \in X}(1,2)IScl(\{x\}) = \phi \)]

Example 5-2: : Let X= \( \{a, b, c\} \)

\( T_1 = \{X, \phi, \langle x, \{a\}, \{b, c\} \rangle, \langle x, \{a, b\}, \{c\} \rangle\} \)

\( T_2 = \{X, \phi, \langle x, \{b, c\}, \{a\} \rangle\} \), then

\( (1,2)IBO(X) = \{X, \phi, \langle x, \{a\}, \{b, c\} \rangle, \langle x, \{a\}, \{b\} \rangle, \langle x, \{a\}, \{c\} \rangle, \langle x, \{a, b\}, \{c\} \rangle, \langle x, \{b\}, \{c\} \rangle, \langle x, \{a, c\}, \{b\} \rangle, \langle x, \{b, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a, c\} \rangle, \langle x, \{b\}, \phi \rangle, \langle x, \{c\}, \phi \rangle, \langle x, \{b, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a, c\} \rangle, \langle x, \{b\}, \{a\} \rangle, \langle x, \{b\}, \{a, c\} \rangle, \langle x, \{c\}, \{a\} \rangle, \langle x, \{c\}, \{a, b\} \rangle \}

\( (1,2)IBcl(\{a\}) = \{X, \phi, X, \phi, \langle x, \{a\}, \{b, c\} \rangle, \langle x, \{a\}, \{b\} \rangle, \langle x, \{a\}, \{c\} \rangle, \langle x, \{a, b\}, \{c\} \rangle, \langle x, \{b\}, \{c\} \rangle, \langle x, \{a, c\}, \{b\} \rangle, \langle x, \{a, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a, c\} \rangle \}

\( (1,2)IBcl(\{b\}) = \{X, \phi, X, \phi, \langle x, \{a\}, \{b, c\} \rangle, \langle x, \{b\}, \{a\} \rangle, \langle x, \{b\}, \{a, c\} \rangle, \langle x, \{a\}, \{b\} \rangle, \langle x, \{a\}, \{c\} \rangle, \langle x, \{a, b\}, \{c\} \rangle, \langle x, \{a, c\}, \{b\} \rangle, \langle x, \{a, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a\} \rangle, \langle x, \{b, c\}, \{a, c\} \rangle \}

\( \bigcap_{x \in X}(1,2)IBcl(\{x\}) = \{X, \phi, X, \phi\}, therefore \)
X is Intuitionistic weakly ultra $b - R_0$, but X is not Intuitionistic ultra $b - R_0$ since
\[(1.2)IBcl\{a\} = \tilde{X} \subseteq \langle x,\{a\},\{b,c\}\rangle \in (1.2)IBO(X) \text{ for } a \in \{a\} = \langle x,\{a\},\{b,c\}\}.
\]

**Proposition 5-3:**
1. Every Intuitionistic ultra semi - $R_0$ is Intuitionistic ultra $b - R_0$.
2. Every Intuitionistic ultra semi - $R_i$ is Intuitionistic ultra semi- $R_o$.

**Proof:** clear

**Proposition 5-4:** If X is Intuitionistic ultra $b - R_i$ then it is Intuitionistic ultra $b - R_0$.

**Proof:** let X is Intuitionistic ultra $b - R_i, U \in (1.2)IBO(X)$ and $x \in U$ for each $y \in X \setminus U$ therefore, there exist (1,2) IB-open sets $U_x, V_y$ in X such that $x \in (1.2)IBcl\{y\}, x \in U_x, y \in V_y, U_x \cap V_y = \emptyset$, let $A = \cup\{V_y : y \in X \setminus U\}$. Then $X \setminus U \subseteq A$ and $x \notin A$ which is a(1,2)b-open set so that
\[(1.2)IBcl\\{x\}\subset X \setminus A \subset U.
\]
Therefore, X is intuitionistic ultra $b - R_0$.

**Theorem 5-5:** An Intuitionistic bitopological space X is intuitionistic ultra $b - T_2$ if and only if it is intuitionistic ultra $b - T_i$ and intuitionistic ultra $b - R_i$.

**Proof:**
If X is intuitionistic ultra $b - T_2$ then it is intuitionistic ultra $b - T_i$ (by Remark 4.9). We prove X is intuitionistic ultra $b - R_i$.
If $x, y \in X$ such that $x \notin (1.2)IBcl\{y\}$ then $x \neq y$. Therefore, there exist disjoint (1.2)IB-open sets $U,V$ in X such that $x \in U \cap V$. Therefore, X is intuitionistic ultra $b - R_i$.

Conversely; if X is intuitionistic ultra $b - T_i$ and intuitionistic ultra $b - R_i$ and $x, y \in X$ such that $x \notin (1.2)IBcl\{y\}$ there exist disjoint (1,2)IB open sets $U,V$ in X such that $x \in U$ and $y \in V$. Since X is intuitionistic ultra $b - T_i$, (1,2)IBcl\{y\} = \{y\}$ (by theorem 4.11) Thus for $x \neq y$ and $V \in (1,2)IBO(X)$ such that $x \in U$ and $y \in V$, $U \cap V = \emptyset$.

Therefore, X is intuitionistic ultra $b - T_2$.

**Corollary 5-6:** An intuitionistic bitopological space X is intuitionistic ultra semi $T_2$ if and only if it is intuitionistic ultra semi $T_i$ and intuitionistic ultra semi- $R_i$.

**Proposition 5-7:** Every intuitionistic weakly ultra semi $R_0$ or intuitionistic weakly ultra pre- $R_0$ is intuitionistic weakly ultra $b - R_0$.

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Proof: if \( X \) is intuitionistic weakly ultra semi - \( R_0 \) or intuitionistic weakly ultra pre - \( R_0 \), \( \bigcap (1,2)\text{IScl} (\{x\}) = \emptyset \) or \( \bigcap (1,2)\text{IPcl} (\{x\}) = \emptyset \) therefore \( \bigcap (1,2)\text{IBcl} (\{x\}) = \emptyset \).

We can explain the relation among the above concepts by following diagram over,

1- intuitionistic ultra \( b-T_2 \)  
\[ \text{\rightarrow} \]  
intuitionistic ultra \( b-T_1 \)

\[ \downarrow \quad \uparrow \]

3- intuitionistic ultra \( b-T_0 \)

4- intuitionistic ultra \( b-R_1 \)
\[ \downarrow \]

5- intuitionistic ultra \( \text{semi}-T_1 \)

6- intuitionistic ultra \( b-R_0 \)
\[ \downarrow \]

7- intuitionistic ultra \( \text{semi}-R_1 \)

8- intuitionistic ultra \( \text{semi}-T_0 \)
\[ \downarrow \]

9- intuitionistic ultra \( \text{semi}-R_0 \)

Where \( A \rightarrow B \) (resp. \( A \rightarrow B \)) represents that A implies B (resp. A does not imply B).

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