A Remark on Generalized Semi- Preregular Closed Sets

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الخلاصة

إن الهدف من هذا البحث هو برهنة أن كل مجموعة من فضاء تبولوجي $(X, \tau)$ تكوَّن مغلقة من النمط $gspr$. (تكون مغلقة من النمط $gspr$)

ABSTRACT

The aim of this note is to show that every subset of a given topological space is a generalized semi- preregular closed set.

1. INTRODUCTION.

Navatagi [3] introduced and studied the class of $gspr$- closed sets. The notions of $gspr$- continuity and $gspr$- irresoluteness and $spr$- $T_{1/2}$ are introduced. The purpose of our note is to show that every subset of any topological space is $gspr$- closed set and therefore every function $f : (X, \tau) \to (Y, \sigma)$ is $gspr$- continuous and $gspr$- irresolute. We have felt the need to point out explicitly this observation since recently several papers have investigated concepts depending on $gspr$- closed sets which do not have any nontrivial meaning, we will point out that most results of [3] and [4] are either trivial or false.

Let A be a subset of a topological $(X, \tau)$. If $A \subseteq cl(int(cl(A)))$, then A is called semi- preopen[2] ($\beta$ - open[1]) set. The semi- preclosure of a set A, denoted by $spcl(A)$, is the intersection of all semi- preclosed supersets of A. Since the union of semi- preopen sets is also semi- preopen, the semi-preclosure of every set is in fact a semi- preclosed set. The notion of generalized semi-preregular closed set (briefly $gspr$-closed) is introduced in [3], thus a set A is called a $gspr$- closed if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in X. A function $f : (X, \tau) \to (Y, \sigma)$ is called $gspr$- continuous (resp. $gspr$- irresolute) [3] if the preimage of every closed (resp. $gspr$- closed) subset of Y is $gspr$- closed in X.

2. Every set is $gspr$- closed.

The following Lemmas will be useful in the sequel.

Lemma 2.1.

For any subset A of X, the following result hold.

(1) $spcl(A)= A \cup Int(cl(int(A)))$[1].

(2) $A \subseteq B \Rightarrow spcl(A) \subseteq spcl(B)$.

The proof of the following result is easy.

Lemma 2.2.

For every subset A of X, $int(cl(A)) = int(cl(int(cl(A))))$.

We have following result

Proposition 2.3.
If $A$ is regular open subset of $X$, then $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(\text{cl}(A))$.

**Proof.**

Since $A$ is regular open, so $A = \text{int}(\text{cl}(A))$. Hence $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(\text{cl}(\text{int}(A)))$. Therefore $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(\text{cl}(A))$, by Lemma 2.2.

**Theorem 2.4.**

If $A$ is a regular open set, then $\text{spcl}(A) = A$ and hence $A$ is semi-preclosed.

**Proof.**

Let $A$ be a regular open subset of $X$. Then by Lemma 2.1, $\text{spcl}(A) = A \cup \text{int}(\text{cl}(\text{int}(A)))$. Since $A$ is regular open, so $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(\text{cl}(A))$. Hence $\text{spcl}(A) = A \cup \text{int}(\text{cl}(A)) = A \cup A = A$. Thus $A$ is semi-preclosed.

**Corollary 2.5.**

(1) Every subset $A$ of a space $(X, \tau)$ is gspr-closed.

(2) Every function $f : (X, \tau) \rightarrow (Y, \sigma)$ is gspr-continuous. Further the definition of gspr-irresolute is trivial.

**Proof.**

(1) Let $A$ be a subset of $X$ and $U$ be regular open subset of $X$ such that $A \subseteq U$. Then $\text{spcl}(A) \subseteq \text{spcl}(U)$. By Theorem 2.4, $\text{spcl}(U) = U$. Hence $\text{spcl}(A) \subseteq U$. Therefore $A$ is gspr-closed set.

(2) Follows from (1).

**Remark 2.5.**

(1) Corollary 2.5 makes most of the results in [3] and [4] trivial.

(2) Remark 5.14 and 5.15 from [3] are wrong, since by corollary 2.4, the intersection and the union of gspr-closed sets are gspr-closed.

(3) Definition 5.22 in [3] becomes: A space $X$ is semipre-regular $T_{\frac{1}{2}}$ space if every subset is semipre-closed.

(4) Theorem 6.10 of [3] is trivial.

(5) Theorem 6.13, parts (d) and (e) [3] are trivial.

(6) Remark 2.4 and Example 2.8 from [4] are trivial.

**REFERENCES**


