The present study focused mainly on the analysis of stiffened and unstiffened composite laminated plates subjected to buckling load. Analytical, numerical and experimental analysis for different cases has been considered. The experimental investigation is to manufacture the laminates and to find mechanical properties of glass-polyester such as longitudinal, transverse young modulus, shear modulus. The compressive test was carried to find the critical buckling load of plate. The design parameters of the laminates such as aspect ratio, thickness ratio, boundary conditions and number of stiffeners were investigated using high order shear deformation theory (HOST) and Finite element coded by ANSYS. The main conclusion was the buckling load could increase and decrease depending on the boundary conditions, thickness ratio, and, the aspect ratio and number of stiffeners of the plate.

**Keywords:** composite laminated plate, glass-polyester, bucking load, high order shear deformation, stiffened and unstiffened plate.

**ABSTRACT**

Buckling Analysis of Stiffened and Unstiffened Laminated Composite Plates

Dr. Adnan Naji Jameel
Professor
University of Baghdad
College of Engineering
Mechanical Engineering Dep.
adnanaji2004@yahoo.com

Dr. louay Sabah yousuf
Assistant Professor
University of Baghdad
College of Engineering
Mechanical Engineering Dep.
louaysabah79@yahoo.com

Eng.Ahmed Mahdi Salih
Researcher
University of Baghdad
College of Engineering
Mechanical Engineering Dep.
flightstar8888@yahoo.com

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**Keywords:** composite laminated plate, glass-polyester, bucking load, high order shear deformation, stiffened and unstiffened plate.
1. INTRODUCTION

1.1 General

During the last decades, needs for composite materials which contain two or more types of materials mixed together homogenously have appeared.

Composite materials have many advantages such as high strength with low weight compared with traditional engineering materials; furthermore, their properties can be controlled during mixing of their components to meet the suitable design requirements. When a flat plate is subjected to low in-plane compressive loads, it remains flat and is in equilibrium condition. As the magnitude of the in-plane load increases, however, the equilibrium configuration of the plate is eventually changed to a non-flat configuration and the plate becomes unstable and begins to deflect to the middle portion. , Reddy J.N. 2004.

Many researches had studied buckling analysis of stiffened and unstiffened plate. Guedes Soares C. and Gordo J. M.1997, performed of three methods to design stiffened panels under predominantly in plane uniaxial compressive loading is compared by referring to numerical and experimental results. Satish Kumar Y.V., Madhujit Mukhopadhyay 1999, presented the basic plate element is a combination of Allman's plane stress triangular element and a discrete Kirchhoff-Mindlin plate bending element. The element includes transverse shear effects. The model accommodates any number of arbitrarily oriented stiffeners within the plate element and eliminates constraints on the mesh division of the plate. Rikards R. etal 2001, concentrated on the development of triangular finite element for buckling and vibration analysis of laminated composite stiffened shells. For the laminated shell, an equivalent layer shell theory is employed. The first-order shear deformation theory including extension of the normal line is used. In order to take into account a non-homogeneous distribution of the transverse shear stresses a correction of transverse shear stiffness is employed. Results of vibration and buckling analysis of stiffened plates and shells are discussed. Samuel Kidane2002, investigated buckling loads of stiffened composite cylinders under uniaxial loading condition by using analytical and experimental approaches. The stiffness contributions of the stiffeners are computed by analyzing the moment and force effect of the stiffener on a unit cell. Then the equivalent stiffness of the stiffener/shell panel is computed by superimposing the stiffness contribution of the stiffeners and the shell. Once the equivalent stiffness parameters are determined for the whole panel, the buckling load is calculated using the energy method. Aseel Jasim Mohammed Al-Hassani 2005, investigated the vibration characteristic of composite plates with and without holes manufactures of fiber glass+ polyester in the presence of steel layers reinforcement, stiffeners subjected to various load conditions and constraints (simply supported and clamped).

An experimental programme was also conducted in order to obtain the material properties of the used plates and study the plate behavior under static load and measuring the strain and deflection from which the structures are obtained and compared with the numerical results. Akl W. etal 2008, selected the orientation angles of stiffeners arranged in the form of isogrid configuration over a flat plate to optimize the static and dynamic characteristics of these plates/stiffeners assemblies. The static characteristics are optimized by maximizing the critical buckling loads of the isogrid plate, while the dynamic characteristics are optimized by maximizing multiple natural frequencies of the stiffened plate.
Tran Ich Thinh and Tran Huu Quoc 2010, studied free vibration and bending failure of laminated stiffened glass fiber/polyester composite plates with laminated open section (rectangular or T-shaped) and closed section (hat shaped) of stiffeners by finite element method and experiment. The results calculated by computational model for above plates under different boundary conditions are in good agreement with experiments. The failure problems of these stiffened glass fiber/polyester composite plates are also investigated.

Hasanain Ibrahim Nsaif 2011, investigated Buckling analysis of composite laminates for critical thermal and mechanical loads. The analytical investigation involved certain mathematical preliminaries, a study of equations of orthotropic elasticity for classical laminated plate theory (CLPT), higher order shear deformation plate theory (HSDT), and numerical analysis (Finite element method).

The present work is focused on how to derive the analytical solution of critical buckling load for stiffened and unstiffened composite laminated plates by high order shear deformation theory by applied different type of boundary condition on the symmetric cross-ply composite laminated plates using Levy solution. Mechanical properties, buckling load for composite plate made from (glass-polyester) with fiber volume fraction (0.3) are determined experimentally. Also Finite element coded by ANSYS12.1 used to find critical buckling load of stiffened and unstiffened composite laminate plate.

2. ANALYTICAL SOLUTION (HIGHER ORDER SHEAR DEFORMATION PLATE THEORY)

2.1. Unstiffened Laminated Plate

2.1.1 Displacement

High order shear deformation plate theory (HSDT) based on assuming the straight line perpendicular to the mid surface before deformation become curve line after deformation, Reddy, J.N. 2004.

\[ u(x,y,z,t) = u_o (x,y,t) + z \phi_x (x,y,t) - \frac{4}{3h^2} z^3 (\phi_x + \frac{\partial w_o}{\partial x}) \] (1. a)

\[ v(x,y,z,t) = v_o (x,y,t) + z \phi_y (x,y,t) - \frac{4}{3h^2} z^3 (\phi_y + \frac{\partial w_o}{\partial y}) \] (1. b)

\[ w(x,y,z,t) = w_o (x,y,t) \] (1. c)

2.1.2 Stress and strain

The total strains can be written as follows

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{xx}^{(0)} \\
\varepsilon_{yy}^{(0)} \\
\gamma_{xy}^{(0)}
\end{bmatrix} + z\begin{bmatrix}
\varepsilon_{xx}^{(1)} \\
\varepsilon_{yy}^{(1)} \\
\gamma_{xy}^{(1)}
\end{bmatrix} + z^2\begin{bmatrix}
\varepsilon_{xx}^{(3)} \\
\varepsilon_{yy}^{(3)} \\
\gamma_{xy}^{(3)}
\end{bmatrix}
\] (2. a)

\[
\begin{bmatrix}
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
= \begin{bmatrix}
\gamma_{yz}^{(0)} \\
\gamma_{xz}^{(0)}
\end{bmatrix} + z\begin{bmatrix}
\gamma_{yz}^{(1)} \\
\gamma_{xz}^{(1)}
\end{bmatrix} + z^2\begin{bmatrix}
\gamma_{yz}^{(2)} \\
\gamma_{xz}^{(2)}
\end{bmatrix}
\] (2. b)
The transformed stress-strain relations of an orthotropic lamina in a plane state of stress are; for $Q_{ij}$ see \text{Reddy J.N. 2004}.

\[
\begin{pmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{pmatrix}_k =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix}_k
\begin{pmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{pmatrix}
\tag{3.a}
\]

\[
\begin{pmatrix}
\sigma_{yz} \\
\sigma_{xz}
\end{pmatrix}_k =
\begin{bmatrix}
\bar{Q}_{44} & \bar{Q}_{45} \\
\bar{Q}_{45} & \bar{Q}_{55}
\end{bmatrix}_k
\begin{pmatrix}
\gamma_{yz} \\
\gamma_{xz}
\end{pmatrix}
\tag{3.b}
\]

The result stress $N_{xx}$, $N_{yy}$ and $N_{xy}$, $M_{xx}$,$M_{yy}$ and $M_{xy}$, shear force $Q_{xz}$, $Q_{yz}$, $R_{xz}$, $R_{yz}$, and the higher order stress resultants $P_{xx}$, $P_{yy}$ and $P_{xy}$ acting on a laminate are obtained by integration of the stress in each layer or lamina through the laminate thickness. Knowing the stress in terms of the displacement, the stress resultants $N_{xx}$, $N_{yy}$, $N_{xy}$, $M_{xx}$, $M_{yy}$, $M_{xy}$, $Q_{xz}$, $Q_{yz}$, $P_{xx}$, $P_{yy}$, $P_{xy}$, $R_{xx}$ and $R_{yy}$ can be obtained.

The inplane force resultants are defined as

\[
\begin{pmatrix}
N_{xx} \\
N_{yy} \\
N_{xy}
\end{pmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{pmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{pmatrix}_k \, dz \tag{4.a}
\]

Where $\sigma_x$, $\sigma_y$ and $\sigma_{xy}$ are normal and shear stress.

\[
\begin{pmatrix}
N_{xx} \\
N_{yy} \\
N_{xy}
\end{pmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{pmatrix}
+ \begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^1 \\
\varepsilon_{yy}^1 \\
\gamma_{xy}^1
\end{pmatrix}
- \begin{bmatrix}
E_{11} & E_{12} & E_{16} \\
E_{12} & E_{22} & E_{26} \\
E_{16} & E_{26} & E_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^{(3)} \\
\varepsilon_{yy}^{(3)} \\
\gamma_{xy}^{(3)}
\end{pmatrix}
\tag{4.b}
\]

\[
\begin{pmatrix}
M_{xx} \\
M_{yy} \\
M_{xy}
\end{pmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{pmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{pmatrix}_k \, z \, dz \tag{5.a}
\]

\[
\begin{pmatrix}
M_{xx} \\
M_{yy} \\
M_{xy}
\end{pmatrix} =
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^0 \\
\varepsilon_{yy}^0 \\
\gamma_{xy}^0
\end{pmatrix}
+ \begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
D_{12} & D_{22} & D_{26} \\
D_{16} & D_{26} & D_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^1 \\
\varepsilon_{yy}^1 \\
\gamma_{xy}^1
\end{pmatrix}
- \begin{bmatrix}
F_{11} & F_{12} & F_{16} \\
F_{12} & F_{22} & F_{26} \\
F_{16} & F_{26} & F_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^{(3)} \\
\varepsilon_{yy}^{(3)} \\
\gamma_{xy}^{(3)}
\end{pmatrix}
\tag{5.b}
\]

\[
\begin{pmatrix}
P_{xx} \\
P_{yy} \\
P_{xy}
\end{pmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{pmatrix}_k \, z^2 \, dz \tag{6.a}
\]

\[
\begin{pmatrix}
P_{xx} \\
P_{yy} \\
P_{xy}
\end{pmatrix} =
\begin{bmatrix}
E_{11} & E_{12} & E_{16} \\
E_{12} & E_{22} & E_{26} \\
E_{16} & E_{26} & E_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^0 \\
\varepsilon_{yy}^0 \\
\gamma_{xy}^0
\end{pmatrix}
+ \begin{bmatrix}
F_{11} & F_{12} & F_{16} \\
F_{12} & F_{22} & F_{26} \\
F_{16} & F_{26} & F_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^1 \\
\varepsilon_{yy}^1 \\
\gamma_{xy}^1
\end{pmatrix}
- \begin{bmatrix}
H_{11} & H_{12} & H_{16} \\
H_{12} & H_{22} & H_{26} \\
H_{16} & H_{26} & H_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{xx}^{(3)} \\
\varepsilon_{yy}^{(3)} \\
\gamma_{xy}^{(3)}
\end{pmatrix}
\tag{6.b}
\]
\[
\begin{align*}
\{Q_{yz}\} &= \int_{-h/2}^{h/2} \{\sigma_{yz}\} \, dz \\
\{Q_{xz}\} &= \left[\begin{array}{cc}
A_{44} & A_{45} \\
A_{45} & A_{55}
\end{array}\right] \left[\begin{array}{c}
(0) \\
(0)
\end{array}\right] - c_2 \left[\begin{array}{cc}
D_{44} & D_{45} \\
D_{45} & D_{55}
\end{array}\right] \left[\begin{array}{c}
(2) \\
(2)
\end{array}\right] \\
\{R_{yz}\} &= \int_{-h/2}^{h/2} \{\sigma_{yz}\} \, z^2 \\
\{R_{xz}\} &= \left[\begin{array}{cc}
D_{44} & D_{45} \\
D_{45} & D_{55}
\end{array}\right] \left[\begin{array}{c}
(0) \\
(0)
\end{array}\right] - c_2 \left[\begin{array}{cc}
F_{44} & F_{45} \\
F_{45} & F_{55}
\end{array}\right] \left[\begin{array}{c}
(2) \\
(2)
\end{array}\right]
\end{align*}
\]

Where

\[A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} (1, z, z^2, z^3, z^4, z^5) \, dz\] (9)

2.1.3 Equation of motion

The Euler-Lagrange equations are obtained by setting the coefficient of \(\delta u_0, \delta v_0, \delta w_0, \delta \phi_x, \delta \phi_y\) to zero separately

\[\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \] (10.a)

\[\frac{\partial N_{yx}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \] (10.b)

\[\frac{\partial Q_x}{\partial x} - c_2 \frac{\partial R_x}{\partial y} + \frac{\partial Q_y}{\partial y} - c_2 \frac{\partial R_y}{\partial y} + c_4 \left( \frac{\partial^2 P_{xx}}{\partial x^2} + 2 \frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_{yy}}{\partial y^2} \right) + \bar{N}_{xx} \frac{\partial^2 w}{\partial x^2} + \bar{N}_{yy} \frac{\partial^2 w}{\partial y^2} + 2 \bar{N}_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0 \] (10.c)

\[\frac{\partial M_{xx}}{\partial x} - c_1 \frac{\partial P_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - c_1 \frac{\partial P_{xy}}{\partial y} - Q_x + c_2 R_x = 0 \] (10.d)

\[\frac{\partial M_{yx}}{\partial x} - c_1 \frac{\partial P_{yx}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - c_1 \frac{\partial P_{yy}}{\partial y} - Q_y + c_2 R_y = 0 \] (10.e)

These equations of motion (10a-e) can be expressed in terms of displacements (\(\delta u_0, \delta v_0, \delta w_0, \delta \phi_x, \delta \phi_y\)) by substituting the forces results from eqs. (4,5,6,7,8) into eq. (10.a) to (10.e) and get partial differential equations, then the analytical solution done by levy method as derived in[9].

2.2 Stiffened Laminated Plate

2.2.1 Stress and strain

The displacements field of laminated plate are as given in Eqs (1(a-c)). The distribution of the normal strains over the depth of the stiffened plate is shown in Fig. (1), [10]

Then, the stress strain relations for laminated plate are as in Eqs. (3,a-b) and the stress strain relations for stiffeners can be defined as:

\(\sigma_{xx}) = E_{xx}^{st} \cdot \varepsilon_{xx} \), \(\sigma_{yy}) = E_{yy}^{st} \cdot \varepsilon_{yy} \)

\(\tau_{xy}) = \tau_{yx}) = 0 \), \(\tau_{yz}) = G_{yz}^{st} \cdot \gamma_{yz} \)

\(\tau_{xz}) = G_{xz}^{st} \cdot \gamma_{xz} \) (11)
Where, $G_{st}^x$ and $G_{st}^y$ are shear modules of elasticity of stiffeners in $x$ and $y$ directions, respectively.

The force and moment relation for stiffened plate are:

$$
\begin{bmatrix}
N \\
M \\
P
\end{bmatrix} = \begin{bmatrix}
N \\
M \\
P
\end{bmatrix}_{unst} + \begin{bmatrix}
N \\
M \\
P
\end{bmatrix}_{st}
$$

And,

$$
\begin{bmatrix}
Q \\
R
\end{bmatrix} = \begin{bmatrix}
Q \\
R
\end{bmatrix}_{unst} + \begin{bmatrix}
Q \\
R
\end{bmatrix}_{st}
$$

(12)

Where, $\begin{bmatrix}
N \\
M
\end{bmatrix}_{unst}$ and $\begin{bmatrix}
Q \\
R
\end{bmatrix}_{unst}$ are force and moment relations for laminated plate as in Eqs. (4, 5, 6, 7, 8(a-b)).

And $\begin{bmatrix}
N \\
M
\end{bmatrix}_{st}$ and $\begin{bmatrix}
Q \\
R
\end{bmatrix}_{st}$ are forces and moment relations for stiffeners can be defined as:

$$
N_{xx}(st) = \int_{Z_i(x^{st})}^{Z_{i-1}(x^{st})} \sigma_{x}(st) \frac{tx}{b_x} . dZ
$$

$$
N_{yy}(st) = \int_{Z_i(y^{st})}^{Z_{i-1}(y^{st})} \sigma_{y}(st) \frac{ty}{b_y} . dZ . N_{xy}(st) = 0
$$

$$
M_{xx}(st) = \int_{Z_i(x^{st})}^{Z_{i-1}(x^{st})} \sigma_{x}(st) \frac{tx}{b_x} . Z . dZ
$$

$$
M_{yy}(st) = \int_{Z_i(y^{st})}^{Z_{i-1}(y^{st})} \sigma_{y}(st) \frac{ty}{b_y} . Z . dZ . M_{xy}(st) = 0
$$

$$
P_x(st) = \int_{Z_i(x^{st})}^{Z_{i-1}(x^{st})} \sigma_{x}(st) \frac{tx}{b_x} . Z^3 . dZ
$$

$$
P_y(st) = \int_{Z_i(y^{st})}^{Z_{i-1}(y^{st})} \sigma_{y}(st) \frac{ty}{b_y} . Z^3 . dZ,
$$

Where,

$Z_i(x^{st}), Z_{i-1}(x^{st}), Z_i(y^{st}), Z_{i-1}(y^{st})$

$tx, b_x, ty, b_y$ as shown in Fig. 2 or, Eq. (13) can be written as:

$$
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
M_{xx} \\
M_{yy} \\
P_x \\
P_y
\end{bmatrix}_{st} = \begin{bmatrix}
A_{11}^{st} & 0 \\
0 & A_{22}^{st}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy}
\end{bmatrix}
$$

(14)

Where,

$$
(A_{11}^{st}, B_{22}^{st}, D_{11}^{st}, E_{11}^{st}) = \int_{Z_{i-1}(x^{st})}^{Z_i(x^{st})} \frac{tx}{b_x} . E_{xx}^{st} . (1, Z, Z^2, Z^3) . dZ
$$

(15a)

$$
(A_{11}^{st}, B_{22}^{st}, D_{11}^{st}, E_{11}^{st}) = \int_{Z_{i-1}(y^{st})}^{Z_i(y^{st})} \frac{ty}{b_y} . E_{yy}^{st} . (1, Z, Z^2, Z^3) . dZ
$$

(15b)
And \([Q]\) can be defined as in equations:

\[
\begin{align*}
Q_{yz}\) &= \int_{Z_1}^{Z_2} y_{yz} \cdot G_{xy} \cdot \frac{dy}{by} \cdot dZ \\
Q_{xx}\) &= \int_{Z_1}^{Z_2} x_{xz} \cdot G_{xz} \cdot \frac{dx}{bx} \cdot dZ \\
R_{yz}\) &= \int_{Z_1}^{Z_2} y_{yz} \cdot G_{yz} \cdot \frac{dy}{by} \cdot Z^2 \cdot dZ \\
R_{xz}\) &= \int_{Z_1}^{Z_2} x_{xz} \cdot G_{xz} \cdot \frac{dx}{bx} \cdot Z^2 \cdot dZ
\end{align*}
\] (16a)

Or, equation (16a) can be written as:

\[
\begin{bmatrix}
Q_{yz} \\
Q_{xz} \\
R_{yz} \\
R_{xz}
\end{bmatrix}_{st} = \begin{bmatrix}
A_{44}^{st} & 0 \\
0 & A_{55}^{st} \\
D_{44}^{st} & 0 \\
0 & D_{55}^{st}
\end{bmatrix} \begin{bmatrix}
y_{yz} \\
y_{xz}
\end{bmatrix}
\] (16b)

Where,

\[
A_{44}^{st} = \int_{Z_1}^{Z_2} G_{xy} \cdot y_{yz} \cdot \frac{dy}{by} \cdot dZ,
\]

\[
A_{55}^{st} = \int_{Z_1}^{Z_2} G_{xz} \cdot x_{xz} \cdot \frac{dx}{bx} \cdot dZ,
\]

\[
D_{44}^{st} = \int_{Z_1}^{Z_2} G_{yz} \cdot y_{yz} \cdot \frac{dy}{by} \cdot Z^2 \cdot dZ,
\]

\[
D_{55}^{st} = \int_{Z_1}^{Z_2} G_{xz} \cdot x_{xz} \cdot \frac{dx}{bx} \cdot Z^2 \cdot dZ.
\]

### 2.2.2 Equations of motion

The refined analysis of such a plate, the governing differential equations are expressed in terms of the displacements, \(u\), \(v\), and \(w\) of the middle surface of the plate in the directions \(x\), \(y\), and \(z\), respectively. The displacements at any point are related to those at the middle surface.

### 3. NUMERICAL ANALYSIS

#### 3.1 Element Selection and Modeling

An element called shell281 as shown in Fig. 3. Is selected for unstiffened plate which is suitable for analyzing thin to moderately thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the \(x\), \(y\), and \(z\) axes, and rotations about the \(x\), \(y\), and \(z\) axes. It may be used for layered applications for modeling composite shells. It includes the effects of transverse shear deformation.

Finite element method has been employed to analyze natural frequency and dynamic response. The model was developed in ANSYS 12.1 using the 121 quadrature elements (i.e. there are 11 elements in the axial direction and 11 along the width) which means (4416 DOF). Convergence study is the reason for choosing the particular mesh used in this study. The global \(x\) coordinate is directed along the width of the plate, while the global \(y\) coordinate is directed along the length and
the global z direction corresponds to the thickness direction and taken to be the outward normal of the plate surface. For stiffened plate, the element used is solid-shell 190. SOLSH190 is used for simulating shell structures with a wide range of thickness (from thin to moderately thick) and for stiffened plate. The element possesses the continuum solid element topology and features eight-node connectivity with three degrees of freedom at each node: translations in the nodal x, y, and z directions. And adding degree of freedom rotation on x, y and z for the present solution by command adding degree of freedom. Thus, connecting SOLSH190 with other continuum elements requires no extra efforts. A degenerate prism option is available, but should only be used as filler elements in mesh generation. The element has plasticity, creep, large deflection, and large strain capabilities. SOLSH190 can use for layered applications such as modeling laminated shells or sandwich construction. Accuracy in modeling composite shells is governed by the first-order shear-deformation theory (also known as Mindlin-Reissner shell theory).

3.2 Verification Case Studies

In the present study, Series of preselected cases are modeled to verify the accuracy of the method of analysis. The results are compared to analytical solution (Levy) and numerical solution (Finite element method). See Table 1 and Table 2 for unstiffened plate, and Table 3 for stiffened plate. From these results, it is obvious that the methods of solution gives better results for both analytical and numerical solution.

4. EXPERIMENTAL WORK

In the present work, three purposes were investigated. First, to outline the general steps to design and fabricate the rectangular test models from fiber (E-glass) and polyester resin to form laminate composite materials. Second, the manufactured models are then used to evaluate the mechanical properties ($E_1, E_2, G_{12}$) of unidirectional composite material. Third, compressive test to find buckling load of cross ply laminate plate.

4.1 Tensile Test

Each laminate was oriented in longitudinal, transverse and 45° angle relative to designated 0° direction to determine the engineering parameters $E_1$, $E_2$, $G_{12}$. Tensile test specimen includes standard geometry according to ASTM (D3039/D03039M); and the mechanical properties for glass-polyester which obtained from tensile test as shown in Table 4.

4.2 Buckling Test

In this study, buckling load of laminated plate (stiffened and un-stiffened) determined analytically, numerically and experimentally. The laminated plate length was 200 mm, the width and thickness of it are 100 mm and 4 mm respectively. For stiffened plate the same dimensions of the un-stiffened plate except with the use of (stiffeners with width of 6mm, two specimen one of them with one stiffener and the other with two). With stiffeners depth 4mm, length of 200mm and the lamination angle for all stiffeners was 0°. The mechanical properties for glass-polyester are obtained from tensile test as shown in Table 4.

The specimen was loaded in axial compression (vertical direction) using tensile test machine of 200 KN capacity as shown in Fig. 4, the specimen was simply supported by two ends and kept free at the other two ends. The specimen was loaded slowly until buckling. The experimental set up has shown in Figs. 5a-c) for three cases. (Un-stiffened and stiffened with one stiffener and with two stiffeners)
Simply supported boundary conditions were simulated along the top and bottom edges. For axial loading, the test specimen was placed between two extremely stiff machine heads of which the lower one was fixed during the test, whereas the upper head was moved downwards by servo hydraulic cylinder. The laminated plate was loaded at constant cross-head speed of 5 mm/min.

The buckling load is determined from the load-displacement curve. The vertical displacement is plotted on the x-axis in (mm) and the load was plotted on the y-axis in (KN).

5. RESULTS AND DISCUSSION
5.1 Experimental Results
The critical buckling load is shown in Tables 5 and 6 for un-stiffened and stiffened plate respectively. Maximum error between analytical and experimental is (3.925%) in case 0/90/90/0 un-stiffened plate and maximum error between numerical and experimental is 8.27% for the same case. For stiffened plate (pate with one stiffener) maximum error between analytical and experimental is (1.762%) in case 0/90/0 stiffened plate and maximum error between numerical and experimental is 7.72% for the same case. For stiffened plate (pate with two stiffeners) maximum error between analytical and experimental is (2.608%) in case 0/90 stiffened plate and maximum error between numerical and experimental is 5.47% for the same case.

5.2 Theoretical Results Contain analytical and numerical (ANSYS))
In this section the effect of different parameter on critical buckling load are discussed such as boundary conditions aspect ratio, thickness ratio, number of stiffeners and stiffener depth ratio for cross ply composite laminate plate. This result found by analytical method using HSDT with Levy method and numerically using ANSYS 12.1 program.

5.2.1 Boundary conditions
From the results listed in Table 7 it can be observed that the boundary conditions always affect on the buckling load. It's worth mentioning that the critical buckling load in SCSC and SSSC for cross ply and CSCS are higher than other cases because of B.C’S. effect.

5.2.2 Aspect ratio
For unstiffened plate, Fig. 6 shows that in SSSF and SFSF the buckling load decreases when a/b increases with high percentage reaches to 72.51% and 74.45% in SSSF and SFSF respectively. On the other hand in SCSC the buckling load decreases with small percentage reaches to 11.569% when a/b varies from 0.5 to 1.5, Then, it's increases when a/b varies from 1.5 to 2. The maximum buckling load in SCSC is at a/b=0.5. While the minimum is at a/b=1.5.

Fig. 7 shows that in un stiffened plate and plate with one stiffener the buckling load decreases when a/b increase with high percentage reaches to 67.383% and 57.878% in un stiffened plate and plate with one stiffener respectively. On the other hand in plate with two stiffeners the buckling load decreases with percentage reaches to 12.204%, It's worth mentioning the buckling load in plate with two stiffeners is higher than other cases because of the number of stiffener effect with percentages (78.4% with plate with one stiffener and 88.19% with plate without stiffener when a/b=1).
5.2.3 Thickness ratio
In Fig. 8, for unstiffened plate the buckling loads increase when thickness ratio increases. It can be observed that the buckling load is increased with high percentage when b/h varies from 10 to 20. Then, this percentage gets smaller when b/h varies from 20 to 50; it is worth mentioning that the increasing in buckling load in case (0/90/0)$_5$ is more than other case where this increase reaches to 10.81%.

5.2.4 Effect Number of Stiffeners:
From Fig. 9, it can be obtained that the buckling load increases with high percentage when the number of stiffeners increases because of the resistance of stiffener against load that causes buckling. Then, it is shown that in, SSSS, SFSF, the buckling load increases when number of stiffeners increases with percentage reaches to 57.988% and 81.619% respectively, increasing in buckling load in case SCSC is more than other case, it reaches to 59.94%.

5.2.5 Effect of stiffener depth ratio for stiffened plates:
It is shown from Fig. 10, the buckling loads decreases and increases with different percentage when d/h varies from 2 to 8 in all cases. It can be observed that the buckling load is decreased when d/h varies from 2 to 4. Then, it is increased when d/h varies from 4 to 8; it is worth mentioning that the increasing in buckling load in case of plate with two stiffeners is more than other case where this increase reaches to 17.733%. Because of the number of stiffener and its resistance against load that causes buckling.

6. CONCLUSIONS
This study considers the buckling analysis of cross-ply composite laminate plate with various B.C’s. From the present study, the following conclusions can be made:
1- The boundary conditions affect
On the critical buckling load the maximum critical buckling load occurs at clamped boundary condition of plate, while the B.C’s change from SSSS to SSCC , the percentage of increasing the buckling load is (48.86%) for anti-symmetric cross under uniaxial load and the percentage of increasing buckling load when changing B.C’s from SSFF to SSCC is (79.5%) for symmetric cross ply under uniaxial load.

2- The uniaxial buckling load decreases when a/b increase with high percentage reaches to 72.51% and 74.45% in SSSF and SFSF un-stiffened plate respectively. On the other hand in SCSC the buckling load decreases with small percentage reaches to 11.569% when a/b varies from 0.5 to 1.5, Then, it's increases when a/b varies from 1.5 to 2. For un-stiffened plate under biaxial SFSS and SF, the buckling load decreases with high percentage when a/b increase reaches to 75.133% and 77.129% in SFSS and SFSF respectively.

3- In unstiffened plate and plate with one stiffener the buckling load decreases when a/b increases with high percentage reaches to 67.383% and 57.878% in un-stiffened plate and plate with one stiffener respectively.

4- The buckling loads increases when thickness ratio increases for un-stiffened plate under uniaxial or biaxial load. It can be observed that the buckling load is increase with high percentage when b/h varies from 10 to 20. Then, this percentage gets smaller when b/h varies from 20 to 50.

5- it can be obtained that the uniaxial buckling load increases with high percentage when the number of stiffeners increased , in SSSS, SFSF, the buckling load increase when number of
stiffeners increase with percentage reaches to 57.988% and 81.619% respectively, increasing in buckling load in case SCSC is more than other case, it reaches to 59.94%. Then, for biaxial load SSSS, SCSC, the buckling load increase when number of stiffeners increase with small percentage reaches to 18.76% and 18.91% in SSSS, SCSC, respectively, increasing in buckling load in case SCSC is more than other case, it reaches to 18.91%.

6- The buckling loads decreases and increases with different percentage when d/h varies from 2 to 8 in all cases for uniaxial load. It can be observed that the buckling load is decreased when d/h varies from 2 to 4. Then, it is increase when d/h varies from 4 to 8; it is worth mentioning that the increasing in buckling load in case plate with two stiffeners is more than other case where this increase reaches to 17.733%..

REFERENCES

- Aseel Jasim Mohammed Al-Hassani , 2005 Instructional Model for Dynamic Analysis of Composite Plates Structure, PhD. Thesis Submitted to the University of Technology.
➢ Theory, analysis, and element manuals ANSYS 12.1 Program.


*Figure 1*: Distribution of normal strains for stiffened plates.

*Figure 2*: Dimensions and directions of stiffened laminated plates.

*Figure 3*: Shell281 Geometry [11]
Table 1. Dimensionless uniaxial buckling load $\bar{N} = N_{cr} \frac{a^2}{E_2 h^3}$ of anti-symmetric cross ply laminates.

<table>
<thead>
<tr>
<th>B.C.'S</th>
<th>[9] Levys method</th>
<th>Present Levys method</th>
<th>(Discrepancy %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-F-S-F</td>
<td>6.78</td>
<td>6.796</td>
<td>0.235%</td>
</tr>
<tr>
<td>S-F-S-S</td>
<td>7.05</td>
<td>7.417</td>
<td>4.94%</td>
</tr>
<tr>
<td>S-F-S-C</td>
<td>8.221</td>
<td>8.276</td>
<td>0.664%</td>
</tr>
<tr>
<td>S-S-S-S</td>
<td>12.109</td>
<td>12.722</td>
<td>4.818%</td>
</tr>
<tr>
<td>S-S-S-C</td>
<td>12.607</td>
<td>12.86</td>
<td>1.96%</td>
</tr>
<tr>
<td>S-C-S-C</td>
<td>13.254</td>
<td>13.41</td>
<td>1.12%</td>
</tr>
</tbody>
</table>

Table 2 (2) Dimensionless uniaxial buckling load $\bar{N} = N_{cr} \frac{a^2}{E_2 h^3}$ of anti-symmetric cross ply laminates $[0^\circ/90^\circ/90^\circ/0^\circ]$ $b/h = 10, G_{12} = G_{13} = 0.6 E_2, G_{23} = 0.5 E_2, v_{12} = 0.25$, $a=b$, $h=1$.

<table>
<thead>
<tr>
<th>E1/E2</th>
<th>[9] ansys</th>
<th>discrepancy %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5.114</td>
<td>2.77%</td>
</tr>
<tr>
<td>10</td>
<td>9.777</td>
<td>0.80%</td>
</tr>
<tr>
<td>20</td>
<td>15.298</td>
<td>1.77%</td>
</tr>
<tr>
<td>30</td>
<td>19.957</td>
<td>2.76%</td>
</tr>
<tr>
<td>40</td>
<td>23.34</td>
<td>1.17%</td>
</tr>
</tbody>
</table>

Table 3. Dimensionless uniaxial buckling load ($\lambda$) of anti-symmetric cross-ply laminated stiffened plate. $E_1 = 60.7 GPa$, $E_2 = 24.8 GPa, G_{12} = G_{13} = G_{31} = G_{23} = 12 GPa, v_{12} = 0.23$

$\lambda = 12 N_{cr} a^2 h (1 - v^2)/E_1 h^3$, $h/a = 0.01$, $a = 100$, Stiffener Depth ratio $(d/h) = 4$

<table>
<thead>
<tr>
<th>a/b</th>
<th>Ref.[4]</th>
<th>Present F.E.M (Discrepancy %)</th>
<th>Present Levy Method (Discrepancy %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>54</td>
<td>52.84 (2.148%)</td>
<td>53.126 (1.61%)</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
<td>74.66936 (0.44%)</td>
<td>72.865 (2.84%)</td>
</tr>
<tr>
<td>1.5</td>
<td>89</td>
<td>85.952 (3.048%)</td>
<td>87.357 (1.86%)</td>
</tr>
<tr>
<td>2</td>
<td>139</td>
<td>142.773 (2.7%)</td>
<td>140.236 (0.88%)</td>
</tr>
</tbody>
</table>
Table 4. Experimental mechanical properties of fiber glass-Polyester and young modulus for polyester.

<table>
<thead>
<tr>
<th>Mechanical properties</th>
<th>Glass-polyester</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (Mpa)</td>
<td>24800</td>
</tr>
<tr>
<td>$E_2$ (Mpa)</td>
<td>4485</td>
</tr>
<tr>
<td>$G_{12}$ (Mpa)</td>
<td>1466</td>
</tr>
<tr>
<td>$V_f$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 4. Buckling test machine.

Figure 5a. Buckling test of S-F-S-F laminates without stiffener.
**Figure 5b.** Buckling test of S-F-S-F laminates with one stiffener.

**Figure 5c.** Buckling test of S-F-S-F laminates with two stiffeners

**Table 5.** Dimensionless buckling load \( \bar{N} = N_{yy}^{0}b^2/E_2h^3 \) of SFSF laminates for unstiffened plate.

<table>
<thead>
<tr>
<th>Laminated plate</th>
<th>Levy (Error %)</th>
<th>ANSYS (Error %)</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/90</td>
<td>1.753 (3.57%)</td>
<td>1.712 (5.83%)</td>
<td>1.818</td>
</tr>
<tr>
<td>0/90/0</td>
<td>0.981 (1.3%)</td>
<td>0.972 (2.2213%)</td>
<td>0.994</td>
</tr>
<tr>
<td>0/90/0/90</td>
<td>2.487 (1.87%)</td>
<td>2.439 (3.77%)</td>
<td>2.5346</td>
</tr>
<tr>
<td>0/90/90/0</td>
<td>1.346 (3.925%)</td>
<td>1.285 (8.27%)</td>
<td>1.401</td>
</tr>
</tbody>
</table>
Table 6. Dimensionless buckling load \( \bar{N} = N_{yy}^0 * b^2 / E_z h^2 \) of SFSF laminates for stiffened plate

<table>
<thead>
<tr>
<th>Number of stiffeners</th>
<th>Laminated plate</th>
<th>Levy (Error %)</th>
<th>ANSYS (Error %)</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>One stiffener</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0/90</td>
<td>11.923 (1.217%)</td>
<td>11.645 (3.52%)</td>
<td>12.07</td>
<td></td>
</tr>
<tr>
<td>0/90/0</td>
<td>13.04 (1.762%)</td>
<td>12.249 (7.72%)</td>
<td>13.274</td>
<td></td>
</tr>
<tr>
<td>0/90/0/90</td>
<td>29.53 (1.369%)</td>
<td>28.944 (3.326%)</td>
<td>29.94</td>
<td></td>
</tr>
<tr>
<td>0/90/90/0</td>
<td>29.856 (1.747%)</td>
<td>29.26 (5.21%)</td>
<td>30.387</td>
<td></td>
</tr>
<tr>
<td>Two stiffeners</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0/90</td>
<td>13.738 (2.608%)</td>
<td>13.334 (5.47%)</td>
<td>14.106</td>
<td></td>
</tr>
<tr>
<td>0/90/0</td>
<td>15.127 (2.065%)</td>
<td>14.665 (5.05%)</td>
<td>15.446</td>
<td></td>
</tr>
<tr>
<td>0/90/0/90</td>
<td>33.691 (1.044%)</td>
<td>33.337 (2.084%)</td>
<td>34.046</td>
<td></td>
</tr>
<tr>
<td>0/90/90/0</td>
<td>43.752 (1.38%)</td>
<td>43.603 (1.719%)</td>
<td>44.366</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Effect of boundary condition on buckling load under uniaxial loading (0/90/90/0) with or without stiffeners.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Unstiffened plate Levy</th>
<th>ANSYS Error %</th>
<th>Plate with Single stiffener Levy</th>
<th>ANSYS Error %</th>
<th>Plate with two stiffeners Levy</th>
<th>ANSYS Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-C-S-C</td>
<td>50.45</td>
<td>52.33 3.59%</td>
<td>245.2 3</td>
<td>247.5 0.91%</td>
<td>260.1 8</td>
<td>264.25 1.54%</td>
</tr>
<tr>
<td>S-C-S-S</td>
<td>38.25</td>
<td>39.98 4.33%</td>
<td>242.5 3</td>
<td>245.1 1.07%</td>
<td>276.2 4</td>
<td>279.74 1.25%</td>
</tr>
<tr>
<td>S-S-S-S</td>
<td>25.97</td>
<td>28.47 8.78%</td>
<td>218.2 4</td>
<td>220.6 1.09%</td>
<td>225.7 9</td>
<td>228.33 1.11%</td>
</tr>
<tr>
<td>S-F-S-C</td>
<td>11.56</td>
<td>12.29 5.91%</td>
<td>97.14 2</td>
<td>100.8 3.69%</td>
<td>126.3 4</td>
<td>131.25 3.73%</td>
</tr>
<tr>
<td>S-F-S-S</td>
<td>2.45</td>
<td>2.59 5.4%</td>
<td>114.9 7</td>
<td>118.2 2.76%</td>
<td>130.2 5</td>
<td>133.55 2.47%</td>
</tr>
<tr>
<td>S-F-S-F</td>
<td>1.25</td>
<td>1.28 2.34%</td>
<td>7.28</td>
<td>7.43 2.01%</td>
<td>13.17 6</td>
<td>14.544 9.40%</td>
</tr>
</tbody>
</table>
Figure 6. Non-dimensional buckling load ($\lambda$) versus aspect ratio ($a/b$) of anti-symmetric cross-ply (0/90/0/90) laminates under uniaxial compressive buckling load.

Figure 7. Dimensionless uniaxial buckling load versus aspect ratio for stiffened and un stiffened plates.

Figure 8. Non-dimensional buckling load ($\lambda$) versus thickness ratio, ($a/h$).
Figure 9. Dimensionless uniaxial buckling load versus No. of stiffeners of stiffened laminated plate.

Figure 10. Dimensionless uniaxial buckling load versus stiffener depth ratio for stiffened plates.

Figure 11. SOLSH190 Geometry [11].