# Bayesian Analysis of five Exponentiated Distributions under different Priors and loss functions

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**Abstract**
The paper is concerned with posterior analysis of five exponentiated (Weibull, Exponential, Inverted Weibull, Pareto, Gumbel) distrebutions. The expressions for Bayes estimators of the shape parameters have been derived under four different prior distributions assuming four different loss functions. The posterior predictive distributions have been obtained, and the comparison between estimators made by using the mean squared errors through generated different sample sizes by using simulation technique. In general, the performance of estimators under Chi-square prior using squared error loss function is the best.

**Keyword:** Bayesian Analysis, Exponentiated (Weibull, Exponential, Inverted Weibull, Pareto, Gumbel) distrebutions**,** simulation.

## التحليل البيزي لخمسة من التوزيعات الأسيه تحت دوال لاحقه ودوال خساره مختلفه

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**الخلاصة**
يدرس البحث ا لتحليل اللاحق لخمسه من التوزيعات التي تم جعلها اسيه (ويبل، اسي، معكوس ويبل ، باريتو ، غامبل). تم اشتقاق مقدرات بيز لمعلمة الشكل تحت اربع توزيعات سابقه مختلفه بافتراض اربع دوال خساره مختلفه. تم ايجاد التوزيعات اللاحقه ، وتمت المقارنه بين المقدرات من خلال دراسة محاكاه عند حجوم عينات وقيم معالم مختلفه باستخدام متوسط مربعات الخطأ . تم التوصل، بشكل عام، الى ان المقدرات تحت التوزيع السابق مربع–كاي باستخدام مربع الخطأ كدالة خساره هي الأفضل انجازا.

## 1. Introduction.

The exponential distribution has been widely used in failure time data analysis and is preferred for situations where hazard rate is constant, in case of monotonic hazard rate, a number of distributions have been suggested but weibull and Gamma distributions are mostly used . [1]

Exponentated distributions can be obtained by three techniques:

1. Powering a positive real number $\alpha$ to the cumulative distribution function (CDF), i.e. if we have CDF F(x) of any random variable X, then the function: $G(z) = [F(z)]^{\alpha}$ , $\alpha > 0$, Is called an exponentiated distribution.
2. Using the formula $G(z) = 1 - [1 - F(z)]^{\alpha}$ , $\alpha > 0$.
3. Using the transformation $z = \log(x)$ , where x is non-negative random variables.

For the same distribution , we can use more than one technique from previous techniques. [2]

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The exponentiated exponentiatial distribution (EE) was used during the first half of the nineteenth century as a particular case of Gompertz-Verhulst distribution function , the genesis of the model, several properties and different estimation procedures are discussed by Gupta and Kundu, it was observed that many properties of this new family are quite similar to those of a Weibull or a Gamma family; therefore it can be used as a possible alternative to a Weibull or a Gamma distribution [3 - 5]

Showky and Bakoban discussed the Exponentiated Gamma (EG) distribution as an important model of life time models and derived Bayesian and non- Bayesian estimators of the shape parameter, reliability and failure rate functions in the case of complete and type-II censored samples [6], they addressed the estimation problem of the finite mixture of two components [7]. Singh et al. proposed Bayes estimators of the parameter of the (EG) distribution and associated reliability function under General Entropy loss function for a censored sample [8]. Raja and Mir proposed extension of four different exponentiated distributions , two parameter Exponentiated Weibull (EW) was found to be suitable to fit unimodels, monotonic and risk functions unlike weibull model [9]. Raqab and Madi considered the Bayesian estimation and prediction for the Exponentiated Rayleigh (ER) model using informative and non-informative priors [10].

Flaih , et al., Considered the standard Exponentiated Inverted Weibull distribution (EIW) that generalizes the standard inverted weibull distribution [11].

Person and Ryden discussed the estimation of T-year return values for significant wave height in a case study using Exponentiated Gumbel (EGu) or generalized extreme value distribution. [12]

Hassan and Basheikh deals with the Bayesian and non- Bayesian estimation of reliability of an s-out of-k system with non-identical component strengths using Exponentiated Pareto (EP) distribution[13]. Abdul-Moniem and Abdel- Hamed generalized the Lomax distribution by powering a positive real number to the CDF, this known as Exponentiated Lomax (EL) distribution, some properties are discussed and the parameter estimation handeled [2].

In this paper, we compare the Bayesian estimators of the shape exponentiated parameters of five different exponentiated distributions (EW, EE, EIW, EP and EGu) using four different prior distributions (Non-informative, Exponential, Chi-square and Gamma) distributions under four different loss functions (Squared error, Quadratic, Modified linear exponential and Non- linear exponential), the performance of the obtained estimators are compared by using the mean square error, through generated many sample sizes by using simulation technique.

## 2. The Exponentiated Distributions.

### 2-1. The Exponentiated Weibull Distribution.

The comulative density function (CDF) corresponding to random variable X with EW distribution is given as :[9]

$$F(x;\beta,\lambda,\sigma) = \left[1-\exp(-\sigma x)^{\lambda}\right]^{\beta} \qquad x>0$$

where $\lambda > 0$ , $\beta > 0$ are shape parameters and $\sigma > 0$ is a scale parameter.

It will be a weibull distribution if $\beta = 1$ and the exponential distribution if $\lambda = 1$ and $\beta = 1$. The great flexibility of this model is in fitting survival data. Now if $\lambda = 1$ then

$$F(x;\beta,\sigma) = [1-e^{-\sigma x}]^{\beta} \qquad x>0\ ;\ \sigma,\beta,>o$$

and the PDF is: $f(x;\beta,\sigma) = \sigma\beta\, e^{-\sigma x}[1-e^{-\sigma x}]^{\beta-1}$ … (1)

The random number X has been generated by inverse function method, which is for uniform random U: $X = \frac{-1}{\sigma} ln\left(1-U^{\frac{1}{\beta}}\right)$

### 2-2. The Exponentiated Exponential Distribution.

The EE distribution has the CDF as: [5]

$$F(x;\alpha,\lambda) = (1-e^{-\lambda x})^{\alpha} \qquad x>0\ ;\ \alpha,\lambda>0$$

where $\alpha$ is the shape parameter and $\lambda$ is the scale parameter, if α=1 then it will be exponential distribution with scale parameter $\lambda$. Then if $\lambda = 1$ the CDF will be as:

$$F(x;\alpha) = (1-e^{-x})^{\alpha}$$

and then the PDF as : $f(x;\alpha) = \alpha e^{-x}(1-e^{-x})^{\alpha-1} \quad x>0 \; ; \; \alpha>0$ …(2)

The random number X has been generated by inverse function method, which is for uniform random U: $X = -\ln\left(1-U^{\frac{1}{\alpha}}\right)$

**2-3. The Exponentiated Inverted Weibull Distribution.**

The random variable X has a standard EIW distribution if its CDF takes the following from: [11]

$F(x;k,\lambda) = \left[exp\left(-x^{-\lambda}\right)\right]^{k} \quad x>0 \; ; \; k,\lambda>0$

Which is simply the k-th power of the CDF of the standard inverted Weibuul distribution. Here , k and $\lambda$ are the shape parameters. For k=1, it represents the standard IWD, and for $\lambda = 1$ it represents the Exponentiated Inverted Exponential distribution(EIED). Thus, the EIWD is a generalization of the EIED as well as the IWD. Then if $\lambda = 1$

$F(x,k) = [exp(-x^{-1})]^{k} = \exp[-k\,x^{-1}] \quad x>0 \; ; \; k>0$

and the PDF is: $f(x,k) = k\,x^{-2}\left(exp(-k\,x^{-1})\right) \quad x>0 \; ; \; k>0$ …(3)

The EIWD also has a physical interpretation as for parallel systems. And the random number X has been generated by inverse function method, which is for uniform random U: $X = -\left[\ln\left(U^{\frac{1}{k}}\right)\right]^{-1}$

**2-4. The Exponentiated Pareto Distribution.**

A random variable X has EPD if the CDF is:[14]

$F(x;\theta,\lambda) = \left[1-(1+x)^{-\lambda}\right]^{\theta} \quad x>0; \; \theta,\lambda>0$

where $\theta$ and $\lambda$ are two shape parameters. When $\theta = 1$, the above distribution corresponds to the standard Pareto distribution of the second kind. And when $\lambda = 1$ it will be as:

$F(x;\theta) = [1-(1+x)^{-1}]^{\theta}$

Therefore the PDF is: $f(x;\theta) = \theta[1-(1+x)^{-1}]^{\theta-1}\,(1+x)^{-2}$

$$= \frac{\theta}{(1+x)^2}\left[\frac{x}{1+x}\right]^{\theta-1} = \frac{\theta\,x^{\theta-1}}{(1+x)^{\theta+1}} \quad x>0; \; \theta>0 \quad \ldots(4)$$

and the random number X has been generated by inverse function method, which is for uniform random U: $X = U^{\frac{1}{\theta}}\left[1-U^{\frac{1}{\theta}}\right]^{-1}$

**2-5. The Exponentiated Gumbel Distribution.**

The CDF of the EGu distribution is:[15]

$F(x,Q,\lambda) = \left[\exp(-e^{\frac{-x}{\lambda}})\right]^{Q} \quad -\infty<x<\infty; \; Q,\lambda>0$

which is simply the $Q^{th}$ power of CDF of the Gumbel distribution. Where Q and $\lambda$ are shape and scale parameters respectively. Then if $\lambda = 1$ the CDF will be:

$F(x;Q) = [\exp(-e^{-x})]^{Q}$

and the PDF is: $f(x;Q) = Qe^{-x}(\exp(-e^{-x}))^{Q-1} \quad -\infty<x<\infty; Q>0$ …(5)

The random number X has been generated by inverse function method, which is for uniform random U: $X = -\ln\left[-\ln U^{\frac{1}{Q}}\right]$

**3. Different Bayesian Estimators of the shape parameters.**

In this section Bayes estimators of the shape parameter for four different prior functions and four different loss functions has been determined.

**3-1. Posterior distributions with different priors.**

For Bayesian estimation, we specify four different prior distributions for the shape parameter, as:

1- The **Non – informative** prior , for any parameter A, with PDF as: $g(A) = \frac{1}{A} \quad A>0$

2- The **Exponential** prior, for any parameter A, as: $g(A) = be^{-bA} \quad A > 0; b > 0$

3- The **Chi-square** prior is assumed to be: $g(A) = \frac{1}{\Gamma(\frac{r}{2}) 2^{\frac{r}{2}}} \; A^{\frac{r}{2}-1} \; e^{\frac{-A}{2}} \quad A > 0 ; r > 0$

4- The **Gamma** prior , assumed to be: $g(A) = \frac{b^a}{\Gamma(a)} \; A^{a-1} \; e^{-bA} \quad A > 0 ; a, b > 0$

where r, a and b are hyper parameters.

Then the posterior density function of the shape parameters for the given random sample X is well known as:

$$f(A,x) = \frac{f(x_i/A)\, g(A)}{\int_A f(x_i/A)\, g(A)\, d(A)}$$

Now the posterior distributions under the assumption of **Non-informative** prior are:

3-1-1. **For EWD**, then by eq, (1):

$$g(\beta) \prod_{i=1}^{n} f(x_i/\beta) = \frac{1}{\beta} \sigma^n \beta^n \prod_{i=1}^{n} (1 - e^{-\sigma x_i})^{\beta-1} \quad e^{-\sigma \sum_{i=1}^{n} x_i}$$

$$= \sigma^n \beta^{n-1}\, exp[-\beta(-\textstyle\sum_{i=1}^{n} \ln(1 - e^{-\sigma x_i}))]$$

$$.\, exp[-\textstyle\sum_{i=1}^{n} \ln(1 - e^{-\sigma x_i}) - \sigma \sum_{i=1}^{n} x_i]$$

then $f(\beta/\underline{x}) = \frac{[-\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i})]^n}{\Gamma(n)} \quad \beta^{n-1}\, e^{-\beta(-\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i}))}$

which implies that: $f(\beta/x) \sim Gam\left(n, \; -\sum_{i=1}^{n} \ln(1 - e^{-\sigma x_i})\right)$

3-1-2. **For EED** , by equ. (2):

$$g(\alpha) \prod_{i=1}^{n} f(x_i/\alpha) = \frac{1}{\alpha} \alpha^n \; e^{\sum_{i=1}^{n} x_i} \; e^{(\alpha-1)\sum_{i=1}^{n} \ln(1-e^{-x_i})}$$

$$= \alpha^{n-1} exp[-\alpha(-\textstyle\sum_{i=1}^{n} \ln(1 - e^{-x_i}))]$$

$$.\, exp[-\textstyle\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \ln(1 - e^{-x_i})]$$

Then $f(\alpha/\underline{x}) = \frac{[-\sum_{i=1}^{n} \ln(1-e^{-x_i})]^n}{\Gamma(n)} \; \alpha^{n-1} e^{-\alpha(-\sum_{i=1}^{n} \ln(1-e^{-x_i}))}$

which implies that: $f(\alpha/\underline{x}) \sim Gam\left(n, \; -\sum_{i=1}^{n} \ln(1 - e^{-x_i})\right)$

3-1-3. **For EIWD** , and by eq. (3):

$$g(k) \prod_{i=1}^{n} f(x_i/k) = \frac{1}{k} k^n \prod_{i=1}^{n} x_i^{-2} \; e^{-k\sum_{i=1}^{n} x_i^{-1}}$$

then $f(k/\underline{x}) = \frac{[\sum_{i=1}^{n} x_i^{-1}]^n}{\Gamma(n)} \; k^{n-1}\, e^{-k\sum_{i=1}^{n} x_i^{-1}}$

which implies that: $f(k/\underline{x}) \sim Gam\left(n, \; \sum_{i=1}^{n} x_i^{-1}\right)$

3-1-4. **For EPD** ,and by eq. (4):

$$g(\theta) \prod_{i=1}^{n} f(x_i/\theta) = \frac{1}{\theta} \theta^n \; \frac{e^{(\theta-1)\sum_{i=1}^{n} \ln x_i}}{e^{(\theta+1)\sum_{i=1}^{n} \ln(1+x_i)}}$$

$$= \theta^{n-1} \; e^{-\theta(\sum_{i=1}^{n} \ln(1+x_i) - \sum_{i=1}^{n} \ln x_i)} \; e^{-\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln(1+x_i)}$$

Then $f(\theta/\underline{x}) = \frac{[\sum_{i=1}^{n} \ln(1+x_i) - \sum_{i=1}^{n} \ln x_i]}{\Gamma(n)} \; \theta^{n-1} \; e^{-\theta(\sum_{i=1}^{n} \ln(1+x_i) - \sum_{i=1}^{n} \ln x_i)}$

implies that : $f(\theta/\underline{x}) \sim Gam\left(n, \; \sum_{i=1}^{n} \ln(1 + x_i) - \sum_{i=1}^{n} \ln x_i\right)$

3-1-5. **For EGuD**, and by eq. (5):

$$g(Q) \prod_{i=1}^{n} f(Q/x_i) = \frac{1}{Q} Q^n \, e^{-\sum_{i=1}^{n} x_i} \, e^{Q\sum_{i=1}^{n} \ln(\exp(-e^{-x_i}))} = Q^{n-1} \; e^{-\sum_{i=1}^{n} x_i} \, e^{-Q\sum_{i=1}^{n} e^{-x_i}}$$

then $f(Q/\underline{x}) = \frac{(\sum_{i=1}^{n} e^{-x_i})^n}{\Gamma(n)} \; Q^{n-1} \; e^{-Q\sum_{i=1}^{n} e^{-x_i}}$

Which implies that: $f(\theta/\underline{x}) \sim Gam(n, \sum_{i=1}^{n} e^{-x_i})$

By the same procedure, we can drive the posterior distributions under the assumption of exponential, Chi-Square, Gamma prior distributions as recorded in table (1).

**3-2. Bayes Estimators of the shape parameter for different Loss functions.**

The Bayes estimator and associated posterior risks derived under the assumption of prior distributions using the following four different loss functions:

**1. Squared Error loss function (SELF).**

Here we have determined Bayes estimator of any parameter A for SELF defined as:[16]

$$L(\hat{A},A) = (\hat{A}-A)^2$$

For SELF the Bayes estimators for the shape parameters of the exponentiated distributions, $\ddot{A}_{BS}$ , are the mean of posterior density functions.

**2. Quadratic Loss function (QLF).**

Now suppose the loss function is quadratic , which is defined as:[16] $L(\hat{A},A) = \left(\frac{\hat{A}-A}{A}\right)^2$

Under QLF the Bayes estimators for the shape parameters are obtained by solving:

$$\frac{\partial}{\partial t}\int_A \left(\frac{\hat{A}-A}{A}\right)^2 \; f(A/\underline{x}) \; dA = 0$$

which implies that: $\hat{A}_{BQ} = \int_A A^{-1} f(A/\underline{x}) \; dA / \int_A A^{-2} f(A/\underline{x}) \; dA$

**3. Modified liner exponential loss function (MLINEXLF).**

Let us consider the MLINEX loss function defined as:[16]

$$L(\hat{A};A) = w\left[\left(\frac{\hat{A}}{A}\right)^c - c\,\ln\left(\frac{\hat{A}}{A}\right) - 1\right] \quad w > 0 \;, c \neq 0$$

For this loss function Bayes estimator of the shape parameters are obtained from:

$$\hat{A}_{BM} = [E(A^{-c}/\underline{x})]^{\frac{-1}{c}} \qquad \text{where} \qquad E(A^{-c}/\underline{x}) = \int_A A^{-c} \;\; f(A/\underline{x}) \;\; dA$$

**4. Non-Linear exponential loss function (NLINEXF).**

Let us consider the following NLINEX loss function of the form:[16]

$$\mathrm{L(D) = J[exp(cD) + cD^2 - cD - 1]} \qquad \mathrm{J} > 0 \;\; , c > 0$$

where D represents the estimation error i.e. D=$\hat{A} - A$.

For NLINEX loss function the Bayes estimators of the shape parameters are:

$$\hat{A}_{BM} = \frac{-1}{c+2}[ln\, E_A(e^{-cA}/\underline{x}) - 2E_A(A/\underline{x})]$$

Therefore; the Bayes estimator of the shape parameter of the exponentiated distribution under assumption of the Non-informative and Gamma prior distributions are derived and recorded in tables-(2, 3).

Since the exponential and the Chi-square prior distributions are special cases from Gamma prior distribution, then the Bayes estimators can be derived as in table (3) by a=1 for Exponential prior distribution, and by a=r/2 and b=1/2 for Chi-square prior distributi

**Table 1-** posterior distributions under the four prior distributions.

| | **posterior distributions under Non-informative prior distribution.** |
|---|---|
| EWD | $f(\beta/x) \sim Gam\left(n,\ -\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i})\right)$ |
| EED | $f(\alpha/\underline{x}) \sim Gam\left(n,\ -\sum_{i=1}^{n} \ln(1-e^{-x_i})\right)$ |
| EIWD | $f(k/\underline{x}) \sim Gam\left(n,\ \sum_{i=1}^{n} x_i^{-1}\right)$ |
| EPD | $f(\theta/\underline{x}) \sim Gam\left(n,\ \sum_{i=1}^{n} \ln(1+x_i) - \sum_{i=1}^{n} \ln x_i\right)$ |
| EGuD | $f(\theta/\underline{x}) \sim Gam\left(n,\ \sum_{i=1}^{n} e^{-x_i}\right)$ |
| | **posterior distributions under Exponential prior distribution.** |
| EWD | $f(\beta/\underline{x}) \sim Gam\left(n+1,\ b-\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i})\right)$ |
| EED | $f(\alpha/\underline{x}) \sim Gam\left(n+1,\ b-\sum_{i=1}^{n} \ln(1-e^{-x_i})\right)$ |
| EIWD | $f(k/\underline{x}) \sim Gam\left(n+1,\ b+\sum_{i=1}^{n} x_i^{-1}\right)$ |
| EPD | $f(\theta/\underline{x}) \sim Gam\left(n+1,\ b-\sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} \ln(1+x_i)\right)$ |
| EGuD | $f(\theta/\underline{x}) \sim Gam\left(n+1,\ b+\sum_{i=1}^{n} e^{-x_i}\right)$ |
| | **posterior distributions under Chi-Square prior distribution.** |
| EWD | $f(\beta/\underline{x}) \sim Gam\left(n+\frac{r}{2},\ \frac{1}{2}-\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i})\right)$ |
| EED | $f(\alpha/\underline{x}) \sim Gam\left(n+\frac{r}{2},\ \frac{1}{2}-\sum_{i=1}^{n} \ln(1-e^{-x_i})\right)$ |
| EIWD | $f(k/\underline{x}) \sim Gam\left(n+\frac{r}{2},\ \frac{1}{2}+\sum_{i=1}^{n} x_i^{-1}\right)$ |
| EPD | $f(\theta/\underline{x}) \sim Gam\left(n+\frac{r}{2},\ \frac{1}{2}-\sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} \ln(1+x_i)\right)$ |
| EGuD | $f(\theta/\underline{x}) \sim Gam\left(n+\frac{r}{2},\ \frac{1}{2}+\sum_{i=1}^{n} e^{-x_i}\right)$ |
| | **posterior distributions under Gamma prior distribution.** |
| EWD | $f(\beta/\underline{x}) \sim Gam\left(n+a,\ b-\sum_{i=1}^{n} \ln(1-e^{-\sigma x_i})\right)$ |
| EED | $f(\alpha/\underline{x}) \sim Gam\left(n+a\ ,b-\sum_{i=1}^{n} \ln(1-e^{-x_i})\right)$ |
| EIWD | $f(k/\underline{x}) \sim Gam\left(n+a, b+\sum_{i=1}^{n} x_i^{-1}\right)$ |
| EPD | $f(\theta/\underline{x}) \sim Gam\left(n+a,\ b-\sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} \ln(1+x_i)\right)$ |
| E Gu D | $f(\theta/\underline{x}) \sim Gam\left(n+a,\ \ b+\sum_{i=1}^{n} e^{-x_i}\right)$ |

**Table 2-** the Bayes estimator under Non-informative prior assumption.

| **Dist.s** | | **BS** | **BQ** |
|---|---|---|---|
| EWD | $\hat{\beta}$ | $\frac{n}{-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}$ | $\frac{n-2}{-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}$ |
| EED | $\hat{\alpha}$ | $\frac{n}{-\sum_{i=1}^{n}\ln(1-e^{-x_i})}$ | $\frac{n-2}{-\sum_{i=1}^{n}\ln(1-e^{-x_i})}$ |
| EIWD | $\hat{k}$ | $\frac{n}{\sum_{i=1}^{n}{x_i}^{-1}}$ | $\frac{n-2}{\left(\sum_{i=1}^{n}{x_i}^{-1}\right)}$ |
| EPD | $\hat{\theta}$ | $\frac{n}{\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i}$ | $\frac{n-2}{\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i}$ |
| EGuD | $\hat{Q}$ | $\frac{n}{\sum_{i=1}^{n}e^{-x_i}}$ | $\frac{n-2}{\sum_{i=1}^{n}e^{-x_i}}$ |
| | | **BM** | **BN** |
| EWD | $\hat{\beta}$ | $\left[\frac{\Gamma(n-c)}{\Gamma n}\right]^{\frac{-1}{c}}\left(-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})\right)^{-1}$ | $\frac{n}{c+2}\left[\ln\left(1+\frac{c}{-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}\right)+\frac{}{-}\right.$ |
| EED | $\hat{\alpha}$ | $\left[\frac{\Gamma(n-c)}{\Gamma n}\right]^{\frac{-1}{c}}\left(-\sum_{i=1}^{n}\ln(1-e^{-x_i})\right)^{-1}$ | $\frac{n}{c+2}\left[\ln\left(1+\frac{c}{-\sum_{i=1}^{n}\ln(1-e^{-x_i})}\right)+\frac{}{-\sum}\right.$ |
| EIWD | $\hat{k}$ | $\left[\frac{\Gamma(n-c)}{\Gamma n}\right]^{\frac{-1}{c}}\left(\sum_{i=1}^{n}{x_i}^{-1}\right)^{-1}$ | $\frac{n}{c+2}\left[\ln\left(1+\frac{c}{\left(-\sum_{i=1}^{n}{x_i}^{-1}\right)}\right)+\frac{2}{\left(-\sum_{i=1}^{n}x\right.}\right.$ |
| EPD | $\hat{\theta}$ | $\left[\frac{\Gamma(n-c)}{\Gamma n}\right]^{\frac{-1}{c}}\left(\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i\right)$ | $\frac{n}{c+2}\left[\ln\left(1+\frac{c}{\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i}\right.\right.$ |
| EGuD | $\hat{Q}$ | $\left[\frac{\Gamma(n-c)}{\Gamma n}\right]^{\frac{-1}{c}}\left(\sum_{i=1}^{n}e^{-x_i}\right)^{-1}$ | $\frac{n}{c+2}\left[\ln\left(1+\frac{c}{\sum_{i=1}^{n}e^{-x_i}}\right)+\frac{2}{\sum_{i=1}^{n}e^{-x_i}}\right]$ |

**Table 3-** the Bayes estimators under Gamma prior distribution.

| **Dist.s** | | **BS** | **BQ** |
|---|---|---|---|
| EWD | $\hat{\beta}$ | $\frac{n+a}{b-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}$ | $\frac{n+a-2}{b-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}$ |
| EED | $\hat{\alpha}$ | $\frac{n+a}{b-\sum_{i=1}^{n}\ln(1-e^{-x_i})}$ | $\frac{n+a-2}{b-\sum_{i=1}^{n}\ln(1-e^{-x_i})}$ |
| EIWD | $\hat{k}$ | $\frac{n+a}{b+\sum_{i=1}^{n}{x_i}^{-1}}$ | $\frac{n+a-2}{\left(b+\sum_{i=1}^{n}{x_i}^{-1}\right)}$ |
| EPD | $\hat{\theta}$ | $\frac{n+a}{b+\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i}$ | $\frac{n+a-2}{b+\sum_{i=1}^{n}\ln(1+x_i)-\sum_{i=1}^{n}\ln x_i}$ |
| EGuD | $\hat{Q}$ | $\frac{n+a}{b+\sum_{i=1}^{n}e^{-x_i}}$ | $\frac{n+a-2}{b+\sum_{i=1}^{n}e^{-x_i}}$ |
| | | **BM** | **BN** |
| EWD | $\hat{\beta}$ | $\left[\frac{\Gamma(n+a-c)}{\Gamma(n+a)}\right]^{\frac{-1}{c}}\left(b-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})\right.$ | $\frac{n+a}{c+2}\left[\ln\left(1+\frac{c}{b-\sum_{i=1}^{n}\ln(1-e^{-\sigma x_i})}\right)+\right.$ |
| EED | $\hat{\alpha}$ | $\left[\frac{\Gamma(n+a-c)}{\Gamma(n+a)}\right]^{\frac{-1}{c}}\left(b-\sum_{i=1}^{n}\ln(1-e^{-x_i})\right)$ | $\frac{n+a}{c+2}\left[\ln\left(1+\frac{c}{b-\sum_{i=1}^{n}\ln(1-e^{-x_i})}\right)+\right.$ |
| EIWD | $\hat{k}$ | $\left[\frac{\Gamma(n+a-c)}{\Gamma(n+a)}\right]^{\frac{-1}{c}}\left(b+\sum_{i=1}^{n}{x_i}^{-1}\right)^{-1}$ | $\frac{n+a}{c+2}\left[\ln\left(1+\frac{c}{b+\sum_{i=1}^{n}{x_i}^{-1}}\right)+\frac{2}{b+\sum_{i=1}^{n}}\right.$ |
| EPD | $\hat{\theta}$ | $\left[\frac{\Gamma(n+a-c)}{\Gamma(n+a)}\right]^{\frac{-1}{c}}\left(b+\sum_{i=1}^{n}\ln(1+x_i)-\sum\right.$ | $\frac{n+a}{c+2}\left[\ln\left(1+\frac{c}{b+\sum_{i=1}^{n}ln(1+x_i)-\sum_{i=1}^{n}}\right.\right.$ |
| EGuD | $\hat{Q}$ | $\left[\frac{\Gamma(n+a-c)}{\Gamma(n+a)}\right]^{\frac{-1}{c}}\left(b+\sum_{i=1}^{n}e^{-x_i}\right)^{-1}$ | $\frac{n+a}{c+2}\left[\ln\left(1+\frac{c}{b+\sum_{i=1}^{n}e^{-x_i}}\right)+\frac{2}{b+\sum_{i=1}^{n}}\right.$ |

**4- Simulation results and Conclusions.**

In this section, the results presented of some numerical experiments to compare the performance of the different Bayes estimators for the shape parameters of five considered distributions proposed in the previous sections, applying a Monte Carlo simulations to comper the performance of different estimators, mainly with respect to their mean squared errors(MSEs) for different sample sizes (n=5, 10, 15, 20 and 30) and for two values of the shape parameters(2 and 2.4).The results of MSEs are computed over (1000) replications for two different cases( case 1: a=0.5, b=1, c=2, r=2 and case 2: a=1.5, b=2, c=3, r=3), and recorded in tables from (4) to (8).

**Table 4-** the MSE for Bayes estimators for EWD .

| n | prior | σ=1.5, a=1.5, b=2, c=3, r=3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | β=2 | | | | β=2.4 | | | |
| | | **BS** | **BQ** | **BM** | **BN** | **BS** | **BQ** | **BM** | **BN** |
| 5 | **Non** | 0.7008 | 1.6449 | 1.7147 | 0.9895 | 4.7947 | 5.1702 | 5.1923 | 4.8270 |
| | **Exp** | 1.0860 | 1.8431 | 1.8801 | 1.3141 | 4.6969 | 5.0392 | 5.0392 | 4.7297 |
| | **Chi** | 0.4475 | 1.1031 | 1.1354 | 0.6841 | 4.5464 | 4.9046 | 4.9046 | 4.5857 |
| | **Gam** | 0.9302 | 1.6338 | 1.6644 | 1.1585 | 4.6132 | 4.9525 | 4.9525 | 4.6484 |
| 10 | **Non** | 0.7279 | 0.4588 | 0.4581 | 0.3585 | 3.8747 | 4.2215 | 4.2289 | 3.9354 |
| | **Exp** | 0.2551 | 0.4990 | 0.5054 | 0.3566 | 3.8533 | 4.1714 | 4.1775 | 3.9104 |
| | **Chi** | 0.6115 | 0.3602 | 0.5389 | 0.2951 | 3.6638 | 3.9943 | 4.0003 | 3.7295 |
| | **Gam** | 0.2245 | 0.4199 | 0.4250 | 0.3008 | 3.7757 | 4.0907 | 4.0964 | 3.8349 |
| 15 | **Non** | 0.4093 | 0.3098 | 0.3097 | 0.2562 | 3.0288 | 3.3424 | 3.3465 | 3.1132 |
| | **Exp** | 0.2033 | 0.3244 | 0.3265 | 0.2552 | 3.0745 | 3.3642 | 3.3678 | 3.1515 |
| | **Chi** | 0.3789 | 0.2686 | 0.2682 | 0.2277 | 2.8565 | 3.1544 | 3.1579 | 2.9429 |
| | **Gam** | 0.1888 | 0.2847 | 0.2847 | 0.2250 | 3.0041 | 3.2906 | 3.2939 | 3.0826 |
| 20 | **Non** | 0.2471 | 0.1998 | 0.1998 | 0.1720 | 2.2852 | 2.5610 | 2.5636 | 2.3851 |
| | **Exp** | 0.1475 | 0.2199 | 0.2208 | 0.1809 | 2.3827 | 2.6406 | 2.6429 | 2.4734 |
| | **Chi** | 0.2396 | 0.1807 | 0.1805 | 0.1585 | 2.1493 | 2.4108 | 2.4132 | 2.2491 |
| | **Gam** | 0.1391 | 0.1960 | 0.1968 | 0.1620 | 2.3203 | 2.5748 | 2.5771 | 2.4120 |
| 30 | **Non** | 0.1585 | 0.1373 | 0.1373 | 0.1231 | 1.0664 | 1.2603 | 1.2615 | 1.1741 |
| | **Exp** | 0.1094 | 0.1423 | 0.1425 | 0.1239 | 1.2201 | 1.4099 | 1.4111 | 1.3209 |
| | **Chi** | 0.1563 | 0.1287 | 0.1286 | 0.1167 | 0.9938 | 1.1768 | 1.1778 | 1.0983 |
| | **Gam** | 0.1058 | 0.1312 | 0.1314 | 0.1148 | 1.1749 | 1.3612 | 1.3622 | 1.2753 |
| | | **σ=1.5, a=0.5, b=1, c=2, r=2** | | | | | | | |
| 5 | **Non** | 2.9272 | 3.3362 | 3.2389 | 2.9539 | 4.6891 | 5.1042 | 5.0063 | 4.7116 |
| | **Exp** | 2.7960 | 3.1734 | 3.0821 | 2.8241 | 4.5418 | 4.9318 | 4.8383 | 4.5661 |
| | **Chi** | 2.7652 | 3.1515 | 3.0580 | 2.7947 | 4.5156 | 4.9135 | 4.8181 | 4.5409 |
| | **Gam** | 2.8881 | 3.2714 | 3.1795 | 2.9143 | 4.6378 | 5.0318 | 4.9380 | 4.6603 |
| 10 | **Non** | 1.9248 | 2.2790 | 2.1903 | 1.9734 | 3.6987 | 4.0745 | 3.9816 | 3.7401 |
| | **Exp** | 1.8640 | 2.1918 | 2.1095 | 1.9109 | 3.6080 | 3.9619 | 3.8742 | 3.6490 |
| | **Chi** | 1.8127 | 2.1461 | 2.0623 | 1.8616 | 3.5638 | 3.9239 | 3.8346 | 3.6064 |
| | **Gam** | 1.9434 | 2.2779 | 2.1942 | 1.9891 | 3.6949 | 4.0529 | 3.9644 | 3.7345 |
| 15 | **Non** | 1.1068 | 1.3859 | 1.3144 | 1.1655 | 2.7486 | 3.0859 | 3.0013 | 2.8061 |
| | **Exp** | 1.1013 | 1.3624 | 1.2955 | 1.1566 | 2.7090 | 3.0277 | 2.9477 | 2.7645 |
| | **Chi** | 1.0413 | 1.3034 | 1.2362 | 1.0984 | 2.6493 | 2.9723 | 2.8912 | 2.7067 |
| | **Gam** | 1.1638 | 1.4322 | 1.3636 | 1.2190 | 2.7870 | 3.1102 | 3.0291 | 2.8415 |
| 20 | **Non** | 0.6368 | 0.8368 | 0.7851 | 0.6898 | 1.1361 | 1.4329 | 1.3563 | 1.2236 |
| | **Exp** | 0.6549 | 0.8461 | 0.7969 | 0.7055 | 1.1810 | 1.4642 | 1.3914 | 1.2639 |
| | **Chi** | 0.6014 | 0.7905 | 0.7412 | 0.6524 | 1.0921 | 1.3734 | 1.3007 | 1.1765 |
| | **Gam** | 0.7003 | 0.8988 | 0.8474 | 0.7516 | 1.2488 | 1.5401 | 1.4652 | 1.3321 |
| 30 | **Non** | 0.2282 | 0.3227 | 0.2972 | 0.2588 | 0.3975 | 0.5466 | 0.5069 | 0.4546 |
| | **Exp** | 0.2484 | 0.3443 | 0.3186 | 0.2795 | 0.4472 | 0.5988 | 0.5587 | 0.5048 |
| | **Chi** | 0.2178 | 0.3078 | 0.2835 | 0.2472 | 0.3891 | 0.5327 | 0.4945 | 0.4445 |
| | **Gam** | 0.2705 | 0.3714 | 0.3445 | 0.3029 | 0.4826 | 0.6408 | 0.5991 | 0.5420 |

**Table 5-** the MSE for Bayes estimators for EED .

| n | prior | a=1.5, b=2, c=3, r=3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | α=2 | | | | α=2.4 | | | |
| | | **BS** | **BQ** | **BM** | **BN** | **BS** | **BQ** | **BM** | **BN** |
| 5 | **Non** | 1.7363 | 2.5267 | 2.5770 | 1.9159 | 3.5931 | 4.3982 | 4.4472 | 3.7402 |
| | **Exp** | 1.8447 | 2.4700 | 2.4986 | 1.9866 | 3.5978 | 4.2619 | 4.2914 | 3.7229 |
| | **Chi** | 1.3730 | 2.0308 | 2.0582 | 1.5571 | 3.1545 | 3.8725 | 3.9010 | 3.3182 |
| | **Gam** | 1.7030 | 2.3049 | 2.3294 | 1.8508 | 3.4406 | 4.0905 | 4.1161 | 3.5733 |
| 10 | **Non** | 0.6862 | 1.1113 | 1.1215 | 0.8557 | 1.8483 | 2.4524 | 2.4660 | 2.0746 |
| | **Exp** | 0.9085 | 1.3003 | 1.3083 | 1.0562 | 2.1123 | 2.6397 | 2.6501 | 2.2987 |
| | **Chi** | 0.5385 | 0.8911 | 0.8982 | 0.6918 | 1.5985 | 2.1316 | 2.1417 | 1.8185 |
| | **Gam** | 0.8223 | 1.1952 | 1.2025 | 0.9690 | 1.9899 | 2.5022 | 2.5118 | 2.1791 |
| 15 | **Non** | 0.3869 | 0.2825 | 0.2823 | 0.2346 | 0.5203 | 0.7637 | 0.7663 | 0.6705 |
| | **Exp** | 0.1852 | 0.3033 | 0.3054 | 0.2370 | 0.7688 | 1.0325 | 1.0350 | 0.9221 |
| | **Chi** | 0.3623 | 0.2461 | 0.2457 | 0.2103 | 0.4536 | 0.6693 | 0.6714 | 0.5913 |
| | **Gam** | 0.1717 | 0.2642 | 0.2659 | 0.2075 | 0.7098 | 0.9624 | 0.9648 | 0.8599 |
| 20 | **Non** | 0.2525 | 0.2032 | 0.2032 | 0.1757 | 0.2252 | 0.1954 | 0.1953 | 0.1717 |
| | **Exp** | 0.1502 | 0.2215 | 0.2224 | 0.1827 | 0.1625 | 0.2253 | 0.2257 | 0.2043 |
| | **Chi** | 0.2454 | 0.1846 | 0.1844 | 0.1625 | 0.2124 | 0.1794 | 0.1794 | 0.1583 |
| | **Gam** | 0.1420 | 0.1979 | 0.1986 | 0.1640 | 0.1533 | 0.2057 | 0.2016 | 0.1866 |
| 30 | **Non** | 0.1634 | 0.1423 | 0.1423 | 0.1276 | 0.9431 | 1.2240 | 1.2279 | 1.0538 |
| | **Exp** | 0.1133 | 0.1461 | 0.1466 | 0.1277 | 1.0710 | 1.3326 | 1.3359 | 1.1696 |
| | **Chi** | 0.1607 | 0.1333 | 0.1332 | 0.1208 | 0.8232 | 1.0766 | 1.0797 | 0.9296 |
| | **Gam** | 0.1097 | 0.1352 | 0.1355 | 0.1185 | 1.0102 | 1.2644 | 1.2675 | 1.1089 |
| | | **a=0.5, b=1, c=2, r=2** | | | | | | | |
| 5 | **Non** | 2.8998 | 3.3186 | 3.2189 | 2.9278 | 4.4145 | 4.9312 | 4.8087 | 4.4495 |
| | **Exp** | 2.7674 | 3.1530 | 3.0597 | 2.7967 | 4.2501 | 4.7279 | 4.6128 | 4.2869 |
| | **Chi** | 2.7351 | 3.1300 | 3.0343 | 2.7659 | 4.2096 | 4.6994 | 4.5813 | 4.2483 |
| | **Gam** | 2.8614 | 3.2533 | 3.1592 | 2.8888 | 4.3672 | 4.8513 | 4.7356 | 4.4013 |
| 10 | **Non** | 1.9871 | 3.3333 | 2.2467 | 2.0331 | 2.8366 | 3.3383 | 3.2128 | 2.9162 |
| | **Exp** | 1.9239 | 2.2450 | 2.1645 | 1.9684 | 2.7742 | 3.2359 | 3.1202 | 2.8499 |
| | **Chi** | 1.8754 | 2.2019 | 2.1200 | 1.9218 | 2.6900 | 3.1613 | 3.0430 | 2.7697 |
| | **Gam** | 2.0018 | 2.3291 | 2.2473 | 2.0452 | 2.8862 | 3.3570 | 3.2393 | 2.9600 |
| 15 | **Non** | 1.1688 | 1.4463 | 1.3754 | 1.2256 | 1.8327 | 2.2271 | 2.1267 | 1.9241 |
| | **Exp** | 1.1596 | 1.4194 | 1.3530 | 1.2133 | 1.8420 | 2.2106 | 2.1169 | 1.9279 |
| | **Chi** | 1.1015 | 1.3626 | 1.2958 | 1.1569 | 1.7481 | 2.1199 | 2.0251 | 1.8371 |
| | **Gam** | 1.2220 | 1.4886 | 1.4206 | 1.2755 | 1.9309 | 2.3082 | 2.2123 | 2.0161 |
| 20 | **Non** | 0.2775 | 0.4307 | 0.3884 | 0.3226 | 0.8555 | 1.4107 | 1.0661 | 0.9465 |
| | **Exp** | 0.3050 | 0.4590 | 0.4171 | 0.3509 | 0.9151 | 1.1899 | 1.1184 | 1.0020 |
| | **Chi** | 0.2597 | 0.4023 | 0.3630 | 0.3024 | 0.8226 | 1.0920 | 1.0216 | 0.1900 |
| | **Gam** | 0.3396 | 0.5040 | 0.4596 | 0.3880 | 0.9801 | 1.2647 | 1.1909 | 1.0681 |
| 30 | **Non** | 0.1411 | 0.1258 | 0.1259 | 0.1207 | 0.2356 | 0.2025 | 0.2054 | 0.1934 |
| | **Exp** | 0.1112 | 0.1168 | 0.1123 | 0.1035 | 0.1722 | 0.1822 | 0.1752 | 0.1606 |
| | **Chi** | 0.1305 | 0.1173 | 0.1172 | 0.1124 | 0.2080 | 0.1852 | 0.1860 | 0.1748 |
| | **Gam** | 0.1094 | 0.1094 | 0.1169 | 0.1058 | 0.1702 | 0.1923 | 0.1823 | 0.1654 |

**Table 6-** the MSE for Bayes estimators for EIWD .

| n | prior | a=1.5, b=2, c=3, r=3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | k=2 | | | | k=2.4 | | | |
| | | BS | BQ | BM | BN | BS | BQ | BM | BN |
| 5 | Non | 1.7990 | 2.5727 | 2.6217 | 1.9703 | 3.9780 | 4.6511 | 4.6916 | 4.0808 |
| | Exp | 1.8922 | 2.5068 | 2.5349 | 2.0287 | 3.9265 | 4.4986 | 4.5238 | 4.0188 |
| | Chi | 1.4358 | 2.0844 | 2.1113 | 1.6129 | 3.5855 | 4.1996 | 4.2237 | 3.7034 |
| | Gam | 1.7523 | 2.3449 | 2.3690 | 1.8947 | 3.7896 | 4.3519 | 4.3739 | 3.8879 |
| 10 | Non | 0.3305 | 0.6499 | 0.6588 | 0.4626 | 2.6449 | 3.1708 | 3.1824 | 2.7996 |
| | Exp | 0.5479 | 0.9169 | 0.9248 | 0.7057 | 2.7673 | 3.2299 | 3.2389 | 2.8996 |
| | Chi | 0.2493 | 0.4849 | 0.4906 | 0.3510 | 2.3837 | 2.8641 | 2.8730 | 2.5409 |
| | Gam | 0.4737 | 0.8139 | 0.8209 | 0.6243 | 2.6573 | 3.1109 | 3.1192 | 2.9290 |
| 15 | Non | 0.3492 | 0.2771 | 0.2773 | 0.2262 | 1.6317 | 2.0329 | 2.0383 | 1.8038 |
| | Exp | 0.1918 | 0.3218 | 0.3240 | 0.2514 | 1.8461 | 2.2146 | 2.2192 | 1.9958 |
| | Chi | 0.3238 | 0.2364 | 0.2362 | 0.1987 | 1.4659 | 1.8320 | 1.8365 | 1.6332 |
| | Gam | 0.1748 | 0.2800 | 0.2818 | 0.2195 | 1.7594 | 2.1193 | 2.1236 | 1.9100 |
| 20 | Non | 0.2466 | 0.1970 | 0.1976 | 0.1702 | 0.8200 | 1.1027 | 1.1055 | 0.9807 |
| | Exp | 0.1453 | 0.2169 | 0.2178 | 0.1782 | 1.0644 | 1.3450 | 1.3476 | 1.2138 |
| | Chi | 0.2395 | 0.1789 | 0.1788 | 0.1572 | 0.7288 | 0.9849 | 0.9872 | 0.8801 |
| | Gam | 0.1371 | 0.1932 | 0.1939 | 0.1595 | 0.9998 | 1.2715 | 1.2740 | 1.1478 |
| 30 | Non | 0.1679 | 0.1441 | 0.1441 | 0.1298 | 0.2391 | 0.2050 | 0.2049 | 0.1807 |
| | Exp | 0.1141 | 0.1454 | 0.1457 | 0.1272 | 0.1682 | 0.2285 | 0.2290 | 0.2076 |
| | Chi | 0.1657 | 0.1357 | 0.1357 | 0.1234 | 0.2261 | 0.1892 | 0.1891 | 0.1673 |
| | Gam | 0.1110 | 0.1348 | 0.1350 | 0.1184 | 0.1596 | 0.2095 | 0.2099 | 0.1903 |
| | | a=0.5, b=1, c=2, r=2 | | | | | | | |
| 5 | Non | 2.6973 | 3.1875 | 3.0701 | 2.7360 | 4.2942 | 4.8546 | 4.7215 | 4.3354 |
| | Exp | 2.5588 | 3.0034 | 2.8952 | 2.5984 | 4.1248 | 4.6395 | 4.5153 | 4.1676 |
| | Chi | 2.5147 | 2.9714 | 2.8601 | 2.5565 | 4.0773 | 4.6058 | 4.4781 | 4.1225 |
| | Gam | 2.6666 | 3.1201 | 3.0106 | 2.7036 | 4.2506 | 4.7729 | 4.6478 | 4.2905 |
| 10 | Non | 1.6889 | 2.0712 | 1.9748 | 1.7479 | 2.9118 | 3.4037 | 3.2808 | 2.9877 |
| | Exp | 1.6394 | 1.9904 | 1.9018 | 1.6955 | 2.8453 | 3.2989 | 3.1854 | 2.9178 |
| | Chi | 1.5775 | 1.9343 | 1.8440 | 1.6362 | 2.7650 | 3.2279 | 3.1119 | 2.8412 |
| | Gam | 1.7239 | 2.0836 | 1.9930 | 1.7788 | 2.9555 | 3.4176 | 3.3022 | 3.0260 |
| 15 | Non | 0.8343 | 1.1139 | 1.0413 | 0.9000 | 1.7521 | 2.1496 | 2.0482 | 1.8465 |
| | Exp | 0.8455 | 1.1074 | 1.0395 | 0.9073 | 1.7668 | 2.1381 | 2.0435 | 1.8553 |
| | Chi | 0.7788 | 1.0395 | 0.9714 | 0.8419 | 1.6699 | 2.0440 | 1.9485 | 1.7616 |
| | Gam | 0.9074 | 1.1787 | 1.1086 | 0.9696 | 1.8562 | 2.2367 | 2.1399 | 1.9441 |
| 20 | Non | 0.4434 | 0.6274 | 0.5782 | 0.4950 | 0.7411 | 1.0175 | 0.9446 | 0.8320 |
| | Exp | 0.4687 | 0.6478 | 0.6005 | 0.5196 | 0.8058 | 1.0740 | 1.0038 | 0.8932 |
| | Chi | 0.4158 | 0.5890 | 0.5430 | 0.4660 | 0.7132 | 0.9741 | 0.9054 | 0.8004 |
| | Gam | 0.5104 | 0.6977 | 0.6484 | 0.5626 | 0.8688 | 1.1478 | 1.0750 | 0.9578 |
| 30 | Non | 0.1403 | 0.1242 | 0.1246 | 0.1197 | 0.2165 | 0.1921 | 0.1929 | 0.1813 |
| | Exp | 0.1103 | 0.1153 | 0.1109 | 0.1024 | 0.1632 | 0.1795 | 0.1711 | 0.1561 |
| | Chi | 0.1298 | 0.1160 | 0.1160 | 0.1115 | 0.1926 | 0.1774 | 0.1764 | 0.1652 |
| | Gam | 0.1084 | 0.1219 | 0.1154 | 0.1046 | 0.1628 | 0.1911 | 0.1796 | 0.1623 |

**Table 7-** the MSE for Bayes estimators for EPD .

| n | prior | a=1.5, b=2, c=3, r=3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | θ=2 | | | | θ=2.4 | | | |
| | | BS | BQ | BM | BN | BS | BQ | BM | BN |
| 5 | Non | 0.1406 | 0.1242 | 0.1242 | 0.1116 | 4.2189 | 4.8065 | 4.8416 | 4.2973 |
| | Exp | 0.1024 | 0.1379 | 0.1382 | 0.1199 | 4.1429 | 4.6523 | 4.6746 | 4.2158 |
| | Chi | 0.1393 | 0.1161 | 0.1160 | 0.1055 | 3.8625 | 4.4059 | 4.4271 | 3.9540 |
| | Gam | 0.0982 | 0.1263 | 0.1265 | 0.1102 | 4.0202 | 4.5221 | 4.5416 | 4.0980 |
| 10 | Non | 0.6749 | 0.4582 | 0.4584 | 0.3488 | 2.5711 | 3.1059 | 3.1177 | 2.7323 |
| | Exp | 0.2638 | 0.5200 | 0.5265 | 0.3721 | 2.7053 | 3.1750 | 3.1841 | 2.8425 |
| | Chi | 0.5577 | 0.3506 | 0.3500 | 0.2806 | 2.3093 | 2.7965 | 2.8055 | 2.4726 |
| | Gam | 0.2291 | 0.4383 | 0.4437 | 0.3139 | 2.5938 | 3.0540 | 3.0624 | 2.7344 |
| 15 | Non | 0.3138 | 0.2481 | 0.2484 | 0.2035 | 1.3232 | 1.7272 | 1.7327 | 1.5126 |
| | Exp | 0.1783 | 0.3091 | 0.3112 | 0.2400 | 1.5801 | 1.9560 | 1.9607 | 1.7455 |
| | Chi | 0.2966 | 0.2128 | 0.2127 | 0.1800 | 1.1712 | 1.5353 | 1.5398 | 1.3523 |
| | Gam | 0.1611 | 0.2671 | 0.2689 | 0.2076 | 1.4927 | 1.8581 | 1.8626 | 1.6584 |
| 20 | Non | 0.2483 | 0.2084 | 0.2084 | 0.1785 | 0.6466 | 0.9127 | 0.9154 | 0.8057 |
| | Exp | 0.1561 | 0.2323 | 0.2323 | 0.1914 | 0.8994 | 1.1731 | 1.1756 | 1.0525 |
| | Chi | 0.2389 | 0.1870 | 0.1869 | 0.1631 | 0.5681 | 0.8066 | 0.8089 | 0.7160 |
| | Gam | 0.1466 | 0.2075 | 0.2083 | 0.1718 | 0.8372 | 1.1009 | 1.1033 | 0.9881 |
| 30 | Non | 0.1502 | 0.1315 | 0.1315 | 0.1182 | 0.2204 | 0.1925 | 0.1925 | 0.1691 |
| | Exp | 0.1068 | 0.1410 | 0.1413 | 0.1228 | 0.1611 | 0.2245 | 0.2250 | 0.2036 |
| | Chi | 0.1485 | 0.1232 | 0.1231 | 0.1119 | 0.2083 | 0.1766 | 0.1765 | 0.1557 |
| | Gam | 0.1029 | 0.1297 | 0.1299 | 0.1134 | 0.1516 | 0.2048 | 0.2052 | 0.1857 |
| | | a=0.5, b=1, c=2, r=2 | | | | | | | |
| 5 | Non | 2.7122 | 3.1972 | 3.0810 | 2.7500 | 4.3090 | 4.8641 | 4.7323 | 4.3494 |
| | Exp | 2.5739 | 3.0143 | 2.9072 | 2.6127 | 4.1402 | 4.6504 | 4.5272 | 4.1822 |
| | Chi | 2.5307 | 2.9830 | 2.8728 | 2.5717 | 4.0935 | 4.6174 | 4.4908 | 4.1379 |
| | Gam | 2.6807 | 3.1298 | 3.0215 | 2.7170 | 4.2649 | 4.7826 | 4.6586 | 4.3014 |
| 10 | Non | 1.7443 | 2.1204 | 2.0257 | 1.8008 | 2.5121 | 3.0537 | 2.9174 | 2.6083 |
| | Exp | 1.6919 | 2.0377 | 1.9505 | 1.7458 | 2.4696 | 2.9644 | 2.8397 | 2.5598 |
| | Chi | 1.6325 | 1.9841 | 1.8953 | 1.6889 | 2.3685 | 2.8732 | 2.7458 | 2.4636 |
| | Gam | 1.7753 | 2.1293 | 2.0403 | 1.8280 | 2.5890 | 3.0953 | 2.9680 | 2.6772 |
| 15 | Non | 1.0664 | 1.3462 | 1.2743 | 1.1262 | 1.4528 | 1.8571 | 1.7530 | 1.5576 |
| | Exp | 1.0632 | 1.3249 | 1.2578 | 1.1196 | 1.4885 | 1.8662 | 1.7693 | 1.5863 |
| | Chi | 1.0022 | 1.2646 | 1.1971 | 1.0602 | 1.3812 | 1.7597 | 1.6622 | 1.4822 |
| | Gam | 1.1258 | 1.3951 | 1.3262 | 1.1822 | 1.5788 | 1.9676 | 1.8680 | 1.6764 |
| 20 | Non | 0.6210 | 0.8201 | 0.7682 | 0.6739 | 0.6885 | 0.9593 | 0.8876 | 0.7788 |
| | Exp | 0.6398 | 0.8304 | 0.7808 | 0.6904 | 0.7550 | 1.0191 | 0.9497 | 0.8422 |
| | Chi | 0.5864 | 0.7740 | 0.7250 | 0.6373 | 0.6630 | 0.9185 | 0.8510 | 0.6495 |
| | Gam | 0.6849 | 0.8823 | 0.8311 | 0.7362 | 0.8168 | 1.0921 | 1.0200 | 0.9059 |
| 30 | Non | 0.1589 | 0.1360 | 0.1380 | 0.1335 | 0.2266 | 0.1980 | 0.1998 | 0.1877 |
| | Exp | 0.1224 | 0.1222 | 0.1190 | 0.1111 | 0.1678 | 0.1811 | 0.1734 | 0.1581 |
| | Chi | 0.1464 | 0.1265 | 0.1280 | 0.1239 | 0.2005 | 0.1816 | 0.1815 | 0.1701 |
| | Gam | 0.1191 | 0.1275 | 0.1222 | 0.1121 | 0.1666 | 0.1920 | 0.1812 | 0.1640 |

**Table 8-** the MSE for Bayes estimators for EGuD .

| n | prior | a=1.5, b=2, c=3, r=3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Q=2 | | | | Q=2.4 | | | |
| | | BS | BQ | BM | BN | BS | BQ | BM | BN |
| 5 | Non | 2.7846 | 3.2443 | 3.2719 | 2.8434 | 4.3410 | 4.8845 | 4.9169 | 4.4082 |
| | Exp | 2.7197 | 3.1190 | 3.1366 | 2.7747 | 4.2560 | 4.7320 | 4.7528 | 4.3196 |
| | Chi | 2.5067 | 2.9290 | 2.9455 | 2.5749 | 4.0050 | 4.5110 | 4.5307 | 4.3196 |
| | Gam | 2.6241 | 3.0166 | 3.0319 | 2.6827 | 4.1409 | 4.6106 | 4.6288 | 4.2089 |
| 10 | Non | 1.7797 | 2.1517 | 2.1598 | 1.8753 | 3.0898 | 3.5373 | 3.5674 | 3.2070 |
| | Exp | 1.8334 | 2.1645 | 2.1709 | 1.9174 | 3.1481 | 3.5643 | 3.5723 | 3.2513 |
| | Chi | 1.5859 | 1.9257 | 1.9320 | 1.6835 | 2.8381 | 3.2721 | 3.2800 | 2.9598 |
| | Gam | 1.7550 | 2.0791 | 2.0850 | 1.8409 | 3.0481 | 3.4578 | 3.4653 | 3.1543 |
| 15 | Non | 0.8582 | 1.1385 | 1.1424 | 0.9725 | 1.8661 | 2.2592 | 2.2645 | 2.0234 |
| | Exp | 0.9957 | 1.2583 | 1.2616 | 1.0979 | 2.0481 | 2.4079 | 2.4124 | 2.1854 |
| | Chi | 0.7426 | 0.9938 | 0.9909 | 0.8516 | 1.6934 | 2.0552 | 2.0596 | 1.8479 |
| | Gam | 0.9351 | 1.1896 | 1.1927 | 1.0371 | 1.9628 | 2.3151 | 2.3194 | 2.1014 |
| 20 | Non | 0.4090 | 0.5884 | 0.2902 | 0.5009 | 1.0525 | 1.3472 | 1.3501 | 1.2089 |
| | Exp | 0.5423 | 0.7270 | 0.7288 | 0.6327 | 1.2778 | 1.5623 | 1.5649 | 1.4199 |
| | Chi | 0.3481 | 0.5066 | 0.5081 | 0.4324 | 0.9476 | 1.2174 | 1.2199 | 1.0969 |
| | Gam | 0.5009 | 0.6780 | 0.6796 | 0.5894 | 1.2113 | 1.4884 | 1.4909 | 1.3530 |
| 30 | Non | 0.1503 | 0.1343 | 0.1343 | 0.1204 | 0.2240 | 0.1930 | 0.1930 | 0.1701 |
| | Exp | 0.1100 | 0.1460 | 0.1463 | 0.1272 | 0.1609 | 0.2229 | 0.2233 | 0.2023 |
| | Chi | 0.1479 | 0.1252 | 0.1252 | 0.1134 | 0.2119 | 0.1778 | 0.1777 | 0.1572 |
| | Gam | 0.1057 | 0.1342 | 0.1345 | 0.1173 | 0.1519 | 0.2035 | 0.2039 | 0.1847 |
| | | **a=0.5, b=1, c=2, r=2** | | | | | | | |
| 5 | Non | 4.2847 | 4.8486 | 4.7146 | 4.3264 | 1.9574 | 0.8481 | 0.8767 | 0.8818 |
| | Exp | 4.1150 | 4.6326 | 4.5076 | 4.1583 | 0.3594 | 0.6847 | 0.5278 | 0.3240 |
| | Chi | 4.0669 | 4.5985 | 4.4700 | 4.1127 | 0.8851 | 0.5512 | 0.5063 | 0.4599 |
| | Gam | 4.2415 | 4.7668 | 4.6409 | 4.2818 | 0.3574 | 0.9048 | 0.6968 | 0.3969 |
| 10 | Non | 2.5724 | 3.1071 | 2.9726 | 2.6655 | 0.6695 | 0.4695 | 0.4783 | 0.4275 |
| | Exp | 2.5258 | 3.0149 | 2.8918 | 2.6132 | 0.2918 | 0.3643 | 0.3220 | 0.2544 |
| | Chi | 2.4278 | 2.9269 | 2.8010 | 2.5201 | 0.4703 | 0.3617 | 0.3574 | 0.3195 |
| | Gam | 2.6439 | 3.1440 | 3.0184 | 2.7294 | 0.2849 | 0.4242 | 0.3657 | 0.2761 |
| 15 | Non | 1.6476 | 2.0479 | 1.9455 | 1.7457 | 0.3425 | 0.2725 | 0.2738 | 0.2534 |
| | Exp | 1.6696 | 2.0435 | 1.9480 | 1.7614 | 0.2072 | 0.2356 | 0.2169 | 0.1849 |
| | Chi | 1.5689 | 1.9450 | 1.8487 | 1.6640 | 0.2842 | 0.2348 | 0.2333 | 0.2157 |
| | Gam | 1.7594 | 2.1431 | 2.0453 | 1.8507 | 0.2024 | 0.2627 | 0.2361 | 0.1946 |
| 20 | Non | 0.8743 | 1.1598 | 1.0852 | 0.9649 | 0.2611 | 0.2127 | 0.2159 | 0.2048 |
| | Exp | 0.9330 | 1.2080 | 1.1365 | 1.0194 | 0.1776 | 0.1842 | 0.1756 | 0.1577 |
| | Chi | 0.8408 | 1.1106 | 1.0401 | 0.9278 | 0.2288 | 0.1902 | 0.1919 | 0.1821 |
| | Gam | 0.9981 | 1.2828 | 1.2089 | 1.0857 | 0.1722 | 0.1976 | 0.1844 | 0.1613 |
| 30 | Non | 0.2148 | 0.1872 | 0.1887 | 0.1777 | 0.1468 | 0.1291 | 0.1299 | 0.1250 |
| | Exp | 0.1593 | 0.1732 | 0.1653 | 0.1509 | 0.1151 | 0.1189 | 0.1148 | 0.1064 |
| | Chi | 0.1904 | 0.1724 | 0.1720 | 0.1614 | 0.1358 | 0.1205 | 0.1209 | 0.1164 |
| | Gam | 0.1583 | 0.1841 | 0.1733 | 0.1566 | 0.1129 | 0.1252 | 0.1190 | 0.1083 |

The respective best performances as in the following tables:

**Table 9-** the best performances for EWD .

| prior | σ=1.5, a=1.5, b=2, c=3, r=3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | β=2 | | | | | β=2.4 | | | | |
| | n=5 | n=10 | n=15 | n=20 | n=30 | n=5 | n=10 | n=15 | n=20 | n=30 |
| Non | BS | BN | BN | BN | BN | BS | BS | BS | BS | BS |
| Exp | BS | BS | BS | BS | BS | BS | BS | BN | BN | BN |
| Chi | BS | BN | BN | BN | BN | BS | BS | BS | BS | BS |
| Gam | BS | BS | BS | BS | BS | BS | BS | BN | BN | BN |
| LF | | | | | | | | | | |
| BS | Chi | Chi | Gam | Gam | Gam | Chi | Chi | Chi | Gam | Chi |
| BQ | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BM | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BN | Chi | Chi | Gam | Chi | Gam | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss fuc.s:** BS, BN, BQ, BM **Performance of Priors:** Chi, Gam, Non, Exp | | | | | | | | | | |
| prior | a=0.5, b=1, c=2, r=2 | | | | | | | | | |
| | β=2 | | | | | β=2.4 | | | | |
| Non | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| Exp | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| Chi | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| Gam | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| LF | | | | | | | | | | |
| BS | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BQ | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BM | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BN | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss fuc.s:** BS, BN, BM, BQ **Performance of Priors:** Chi, Exp, Non, Gam | | | | | | | | | | |

From table (4) it is observed that the values of MSE decreasing convergently with increasing in the sample size for any type of the four prior distributions and loss functions used in this study as the values of β indexed in the table. the achievement as is evident in table (9), the performance of the prior distribution Chi was best for the rest of the distributions, which is no secret that it is a special case of the Gamma distribution, as well as the completion of BS loss function.

**Table 10-** the best performances for EED.

| prior | a=1.5, b=2, c=3, r=3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | α=2 | | | | | α =2.4 | | | | |
| | n=5 | n=10 | n=15 | n=20 | n=30 | n=5 | n=10 | n=15 | n=20 | n=30 |
| Non | BS | BS | BN | BN | BN | BS | BS | BS | BN | BS |
| Exp | BS | BN | BS | BS | BS | BS | BS | BS | BS | BS |
| Chi | BS | BS | BN | BN | BN | BS | BS | BS | BN | BS |
| Gam | BS | BS | BS | BS | BS | BS | BS | BN | BS | BS |
| LF | | | | | | | | | | |
| BS | Chi | Chi | Gam | Gam | Gam | Chi | Chi | Chi | Gam | Chi |
| BQ | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BM | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| BN | Chi | Chi | Gam | Chi | Gam | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss F.:** BS, BN, BM, BQ **Performance of Priors:** Chi, Gam, Non, Exp | | | | | | | | | | |

| **prior** | **a=0.5, b=1, c=2, r=2** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **α =2** | | | | | **α =2.4** | | | | |
| **Non** | BN | BS | BN | BN | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BN |
| **Chi** | BN | BS | BN | BN | BN | BS | BS | BS | BN | BN |
| **Gam** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BN |
| **LF** | | | | | | | | | | |
| **BS** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Exp |
| **BM** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Exp |
| **BN** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Exp |
| **Performance of Loss F.:** BS, BN, BQ, BM **Performance of Priors:** Chi, Exp, Non, Gam | | | | | | | | | | |

In Table (5) illustrated the decreasing values of criterion MSE to the bayse estimators in the study with the increasing the values of sample sizes, and so was the preference of the prior Chi distribution with the advent of all of distrebutions Gam and Exp in second place when changing the value of α, and as is clear in Table (10).

**Table 11-** the best performances for EIWD.

| **prior** | **a=1.5, b=2, c=3, r=3** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **k=2** | | | | | **k =2.4** | | | | |
| | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** |
| **Non** | BS | BS | BN | BN | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **Chi** | BS | BS | BN | BN | BN | BS | BS | BS | BS | BN |
| **Gam** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **LF** | | | | | | | | | | |
| **BS** | Chi | Chi | Gam | Gam | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Chi |
| **BM** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Chi |
| **BN** | Chi | Chi | Gam | Chi | Gam | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss F.:** BS, BN, BQ, BM **Performance of Priors:** Chi, Non, Gam, Exp | | | | | | | | | | |
| **prior** | **a=0.5, b=1, c=2, r=2** | | | | | | | | | |
| | **k =2** | | | | | **k =2.4** | | | | |
| **Non** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **Chi** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **Gam** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **LF** | | | | | | | | | | |
| **BS** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Chi |
| **BM** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Exp |
| **BN** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Exp |
| **Performance of Loss F.:** BS, BN, BM, BQ **Performance of Priors:** Chi, Exp, Non, Gam | | | | | | | | | | |

In table (6) also decreases the values of Ms. for this distribution as the increasing of sample sizs, with a Shi prior ranks first in the achievement and the appearance of Non and Exp in second places, as well as stay BS and BN in the first and second places as the best achievement for loss functions. Order achievement has been arranged of this distribution in Table (11).

**Table 12-** the best performances for EPD.

| **prior** | **a=1.5, b=2, c=3, r=3** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **θ=2** | | | | | **θ =2.4** | | | | |
| | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** |
| **Non** | BN | BN | BN | BN | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **Chi** | BN | BN | BN | BN | BN | BS | BS | BS | BS | BN |
| **Gam** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **LF** | | | | | | | | | | |
| **BS** | Gam | Gam | Gam | Gam | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| **BM** | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| **BN** | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss F.:** BS, BN, BQ, BM **Performance of Priors:** Chi, Non, Gam, Exp | | | | | | | | | | |
| **prior** | **a=0.5, b=1, c=2, r=2** | | | | | | | | | |
| | **θ =2** | | | | | **θ =2.4** | | | | |
| **Non** | BS | BS | BS | BN | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BS |
| **Chi** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **Gam** | BS | BS | BS | BS | BN | BS | BS | BS | BS | BN |
| **LF** | | | | | | | | | | |
| **BS** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Exp | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Chi |
| **BM** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Chi |
| **BN** | Chi | Chi | Chi | Chi | Exp | Chi | Chi | Chi | Chi | Chi |
| **Performance of Loss F.:** BS, BN, BM, BQ **Performance of Priors:** Chi, Non, Exp, Gam | | | | | | | | | | |

Similarly, in Table (7), the values of MSE also convergently decreases with increasing of sample sizs, and the preference for priors Chi and Non in the first and second order, and stay BS and BN ranked in the same orders with respect to the loss functions. As is evident in Table (12).

**Table 13-** the best performances for EGuD.

| **prior** | **a=1.5, b=2, c=3, r=3** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Q=2** | | | | | **Q=2.4** | | | | |
| | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** | **n=5** | **n=10** | **n=15** | **n=20** | **n=30** |
| **Non** | BS | BS | BS | BN | BN | BS | BS | BS | BS | BN |
| **Exp** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **Chi** | BS | BS | BS | BN | BN | BS | BS | BS | BS | BN |
| **Gam** | BS | BS | BS | BS | BS | BS | BS | BS | BS | BS |
| **LF** | | | | | | | | | | |
| **BS** | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi |
| **BM** | Chi | Chi | Chi | Non | Chi | Chi | Chi | Chi | Chi | Chi |
| **BN** | Chi | Chi | Chi | Chi | Chi | Gam | Chi | Chi | Chi | Chi |
| **Performance of Loss F.:** BS, BN, BQ, BM **Performance of Priors:** Chi, Gam, Non, Exp | | | | | | | | | | |
| **prior** | **a=0.5, b=1, c=2, r=2** | | | | | | | | | |
| | **Q =2** | | | | | **Q =2.4** | | | | |
| **Non** | BS | BS | BS | BN | BN | BQ | BN | BN | BN | BN |
| **Exp** | BS | BS | BS | BS | BN | BN | BN | BN | BN | BN |
| **Chi** | BS | BS | BS | BS | BN | BN | BN | BN | BN | BN |
| **Gam** | BS | BS | BS | BS | BN | BS | BN | BN | BN | BN |
| **LF** | | | | | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **BS** | Chi | Chi | Chi | Chi | Gam | Gam | Gam | Gam | Gam | Gam |
| **BQ** | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Chi | Exp | Exp |
| **BM** | Chi | Chi | Chi | Chi | Exp | Chi | Exp | Exp | Exp | Exp |
| **BN** | Chi | Chi | Chi | Chi | Exp | Exp | Exp | Exp | Exp | Exp |
| **Performance of Loss F.:** BS, BN, BM, BQ **Performance of Priors:** Chi, Exp, Non, Gam | | | | | | | | | | |

Finally, in Table (8), the amount of decreasing is the same with respect to sample sizs. As for the achievement, has remained a priority distribution of Chi in first place while Gam and Exp in the second order for different values of parameters, as in Table (13).

Generally, its observed that the estimates using Chi-square prior under squared error loss function is significantly better than others for all distributions in this study as well as with the other loss functions, and the MSEs of the estimators are decrease as sample sizes are increases.

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