LQR Controller for Kufasat

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Abstract
In this paper, Linear Quadratic Regulator (LQR) controller is applied to the attitude stabilization control of Kufasat. Using the linearized equations of motion for a rigid body in space, the linearized stability, effectiveness and robustness of a linear quadratic regulator (LQR) control design were compared with that of a Proportional-Integral-Derivative (PID) control design. The detailed design procedure of the LQR controller is presented. Simulation results show that precise attitude control is accomplished and the time of satellite maneuver is shortened in spite of the uncertainty in the system.

Key words: Linear Quadratic Regulator , Kufasat

الخلاصة

النظام السيطرة التربيعي الخطي للقمر الصناعي (LQR) للقمر الصناعي كوفت سات

محمد جساب مهدي محمد جعفر البيرماني

تم استخدام نظام السيطرة التربيعي الخطي للقمر الصناعي كوفت سات ل,stabilization control of Kufasat. Using the linearized equations of motion for a rigid body in space, the linearized stability, effectiveness and robustness of a linear quadratic regulator (LQR) control design were compared with that of a Proportional-Integral-Derivative (PID) control design. The detailed design procedure of the LQR controller is presented. Simulation results show that precise attitude control is accomplished and the time of satellite maneuver is shortened in spite of the uncertainty in the system.

كلمات مفتاحية: نظام السيطرة التربيعي الخطي للقمر الصناعي Kufasat
1- Introduction

The Iraqi student satellite project kufasat was started at 2012. The launch of the satellite is planned for late 2016. The main tasks for kufasat will be to perform scientific measurements. The project is sponsored by the University of Kufa and it will be the first Iraqi satellite. Kufasat is a nano-satellite based on the cubesat concept. This means that its mass is restricted to 1 kg, and its size is restricted to a cube measuring 10×10×10 cm. It also contains 1.5m long gravity boom, which will be used for passive attitude stabilization. The satellite attitude control problem includes attitude stabilization and attitude maneuver. Attitude stabilization is the process of keeping original attitude and the attitude maneuver is the re-orientation process of changing one attitude to another [1]. In general, attitude stabilization systems are classified as active or passive. The simplicity and low cost of active magnetic control makes it an attractive option for small satellites in Low Earth Orbit (LEO).

A gravity gradient stabilized satellite has limited stability and pointing capabilities so, magnetic coils are added to improve both the three axis stabilization and the pointing properties. Magnetic coils around the satellite’s XYZ axes can be fed with a constant current-switched in two directions- to generate a magnetic dipole moment M which will interact with the geomagnetic field vector B to generate a satellite torque N by taking the cross product[2]:

\[ N = M \times B \]  

This torque is used to control the rotation of the satellite. The magnetic coils are controlled using LQR controller. Gravity gradient stabilization has been used in attitude control since the early sixties [3], but accurate three-axis control has not been achieved using gravity gradient stabilization alone. Gravity gradient stabilization combined with magnetic torquing, has gained increased attention as an attractive attitude control system (ACS) for small cheap satellites and is also proposed used in this satellite [4].

A problem is that both the direction and the strength of the geomagnetic field change and magnetic control become non-linear and time dependent. Attitude control with high accuracy cannot be achieved because the magnetic torques are constrained on a plane perpendicular to the local magnetic field. In this paper, a comparison between two attitude control laws that have been suggested used for kufasat. This paper proposes PID and LQR controller.

2- Dynamic model

The mathematical model of a satellite is described by the dynamic equations and kinematic equations of motion [5]. The dynamic equation of motion for a satellite in low earth orbit is

\[ \frac{d}{dt} \dot{\omega}_b = I \dot{\omega}_b + \sum bT_b / I \]  

where \( bT_b \) is the angular velocity of body frame relative to an inertial frame. \( I \) is the moment of inertia matrix refer to body frame, \( I = \text{diag}[I_x, I_y, I_z] \). \( bT_b \) is total torque acting on satellite expressed in body frame components, which is consist of gravity gradient torque, magnetic torque and disturbance torque.

\[ bT = bT_C + bT_m + bT_D \]  

Equation (1) can be expanded in components; we have three dynamic equations for the roll, pitch, yaw axes respectively as follows:

\[ T_x = \omega_x I_x + (I_z - I_y)\omega_y \omega_z \]  
\[ T_y = \omega_y I_y + (I_x - I_z)\omega_z \omega_x \]  
\[ T_z = \omega_z I_z + (I_y - I_x)\omega_x \omega_y \]
Where $\omega_x, \omega_y, \omega_z$ are angular velocities of body frame and $I_x, I_y, I_z$ are the moment of inertia in body frame and $T_x, T_y, T_z$ are the torques expressed in body frame components. These three equations are known as Euler’s equations of motion for a rigid body [6]. If the Euler angles $\phi, \theta, \psi$ are small in magnitude, the relationship between body angular velocities and Euler angular velocities may be approximated[7] as,

$$\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix} = 
\begin{bmatrix}
\dot{\phi} + \omega_y \omega_z \\
\dot{\theta} - \omega_z \omega_x \\
\dot{\psi} - \omega_x \omega_y
\end{bmatrix}$$

(5)

**Gravity Gradient Torque:**

The gravity gradient torque, using a small Euler angle approximation and taking principal axes as reference axis is given [3] by:

$$T_{Gr} = 3\omega_0^2 (I_z - I_y) \theta$$

(6)

$$T_{Gy} = 3\omega_0^2 (I_z - I_x) \theta$$

$$T_{Gz} = 0$$

Where $T_{Gr}, T_{Gy}, T_{Gz}$ are the gravity gradient torque about the Roll, Pitch, Yaw axis, respectively.

**Magnetic Field Torque:**

The magnetic coil produces a magnetic dipole when currents flow through its windings, which is proportional to the ampere-turns and the area enclosed by the coil. The torque generated by the magnetic coils can be modeled as:

$$T_m^b = m^b \times B^b$$

(7)

Where $m^b$ is the generated magnetic moment inside the body and $B^b = [B_{x}^b, B_{y}^b, B_{z}^b]^T$ is the local geomagnetic field vector

$$m^b = m_x^b + m_y^b + m_z^b = \begin{bmatrix} N_k^b A_x \\ N_y^b A_y \\ N_z^b A_z \end{bmatrix} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

where $N_k$ is number of windings in the magnetic coil, $i_k$ is the coil current and $A_k$ is the span area of the coil. The magnetic torque can be represented as:

$$\begin{bmatrix}
T_{mx} \\
T_{my} \\
T_{mz}
\end{bmatrix} = \begin{bmatrix}
m_y B_\psi - m_z B_\theta \\
m_z B_\phi - m_x B_\psi \\
m_x B_\theta - m_y B_\phi
\end{bmatrix}$$

(8)

Where $T_{mx}, T_{my}, T_{mz}$ are the magnetic torque about the Roll, Pitch, Yaw axes, respectively, and $m_x, m_y, m_z$ are the corresponding of the magnetic moments and $B_\phi, B_\theta, B_\psi$ is the earth’s magnetic field affects the Roll, Pitch and Yaw axis respectively. After adding equation (6) and equation (8)to equation (4) the final form of linearized attitude dynamic model of the satellite including gravity gradient torque and magnetic coil torque written in body frame components becomes

$$\begin{align*}
\dot{\phi} &= -\frac{4\omega_0^2 (I_y - I_z)}{I_x} \phi + \frac{\omega_0 (I_x - I_y + I_z)}{I_x} \psi + \\
& \quad \left(\frac{m_y B_\psi - m_z B_\theta}{I_x}\right) \psi + \\
& \quad \left(\frac{m_x B_\theta - m_y B_\phi}{I_y}\right) \phi + \\
& \quad \left(\frac{m_z B_\phi - m_x B_\psi}{I_z}\right) \theta \\
\dot{\theta} &= -\frac{3\omega_0^2 (I_x - I_z)}{I_y} \theta + \left(\frac{m_z B_\phi}{I_y}\right) \psi + \\
& \quad \left(\frac{m_y B_\psi}{I_z}\right) \theta + \\
& \quad \left(\frac{m_x B_\theta}{I_x}\right) \phi \\
\dot{\psi} &= -\frac{\omega_0^2 (I_x - I_z)}{I_z} \phi + \frac{\omega_0 (I_x - I_y + I_z)}{I_z} \psi + \\
& \quad \left(\frac{m_z B_\phi}{I_x}\right) \theta + \\
& \quad \left(\frac{m_y B_\psi}{I_y}\right) \phi + \\
& \quad \left(\frac{m_x B_\theta}{I_z}\right) \psi
\end{align*}$$

(9)

If the states are given as:

$$\begin{bmatrix}
x_1 = [\phi] & x_4 = [\dot{\phi}] \\
[\theta] & x_5 = [\dot{\theta}] \\
[\psi] & x_6 = [\dot{\psi}]
\end{bmatrix}$$

, then the linear system can be expressed as a state space model taking the form:

$$\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}$$

(10, 11)
PID controller design

A Proportional-Integral-Derivative controller (PID controller) is the most widely used controller with feedback mechanism. It is one of the simplest control algorithms, and in the absence of knowledge of the underlying process, PID controller is often the best choice.

A typical structure of a PID control system is shown in Figure 1, where $K_p$ is the proportional gain, $K_i$ is the derivative gain, and $K_d$ is the integral gain. By appropriately adjusting theses gains, the desired output can be achieved. It can be seen that in a PID controller, the error signal $e(t)$ is used to generate the proportional, integral, and derivative actions, with the resulting signals weighted and summed to form the control signal $u(t)$ applied to the plant model.

The PID controller is tuned by selecting parameters $K_p$, $K_i$, and $K_d$, that give an acceptable closed-loop response. A desirable response is often characterized by the measures of settling time, oscillation period, and overshoot, to mention a few. Many PID tuning methods have been proposed over the years, ranging from the simple, but most famous Ziegler-Nichols tuning method, to the more modern simple internal model control (SIMC) tuning rules by Skogestad. In this work all gains of PID controller tuned automatically in Simulink environment. The block diagram of the Simulink set up for the attitude control using PID controller is shown in Figure 2.

3- PID controller design

4- LQR controller design

The Linear Quadratic Regulator (LQR) is a powerful technique for designing controllers for complex systems that have stringent performance requirements. The standard theory of the optimal control is presented in [8,9,10]. Under the assumption that all state variables are available for feedback, the
LQR design method starts with a defined set of states which are to be controlled. In general, the system model can be written in state space equation as in equation (10)

\[
\dot{x} = Ax + Bu
\]  

(10)

Where: \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) denote the state variable, and control input vector, respectively. \( A \) is the state matrix of order \( n \times n \); \( B \) is the control matrix of order \( m \times m \).

**Controllability:**

The conditions of controllability may govern the existence of a complete solution to the control system design problem. The solution to this problem may not exist if the system considered is not controllable [8]. The system described by Equation (10) is said to be state controllable at \( t = t_0 \) if it is possible to construct an unconstrained control signal that will transfer an initial state to any final state in a finite time interval \( [t_0, t_1] \). If every state is controllable, then the system is said to be completely state controllable. The system given by equation (10) is completely state controllable if and only if the vectors \( B, AB, \ldots, A^{n-1}B \) are linearly independent, or the \( n \times n \) matrix \([B, AB, \ldots, A^{n-1}B]\) is of rank \( n \) [8].

**Weighting matrices \( Q \) and \( R \) determination:**

The weighting matrices \( Q \) and \( R \) are important components of an LQR optimization process. The compositions of \( Q \) and \( R \) elements have great influences of system performance. The designer is free to choose the matrices \( Q \) and \( R \), but the selection of matrices \( Q \) and \( R \) is normally based on an iterative procedure using experience and physical understanding of the problems involved. Commonly, a trial and error method has been used to construct the matrices \( Q \) and \( R \) elements. This method is very simple and very familiar in LQR application. However, it takes long time to choose the best values for matrices \( Q \) and \( R \). The number of matrices \( Q \) and \( R \) elements are dependent on the number of state variable \( (n) \) and the number of input variable \( (m) \), respectively. The block diagram of the Simulink set up for the attitude control using LQR controller is shown in Figure (3).

**Fig (3) SIMULINK diagram of satellite model with LQR controller**

### 5- Simulation

In this paper, several simulations of the proposed controller have been done. The parameters values used for kufasat are listed in Table (1):

**Table (1) kufasat parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite height</td>
<td>600 km</td>
</tr>
<tr>
<td>Weight</td>
<td>1 kg</td>
</tr>
<tr>
<td>Size</td>
<td>10 x 10 x 10 cm</td>
</tr>
<tr>
<td>Moments of inertia</td>
<td>Ix = 0.1043, Iy = 0.1020, Iz = 0.0031 kgm²</td>
</tr>
<tr>
<td>Boom length</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Orbit angular velocity</td>
<td>1.083 x 10⁻³</td>
</tr>
<tr>
<td>Maximum magnetic moment</td>
<td>0.1 Am²</td>
</tr>
<tr>
<td>Magneto-torquer</td>
<td>3 perpendicular magnetic coils</td>
</tr>
<tr>
<td>Desired Euler values</td>
<td>[ϕ 0 θ ψ]</td>
</tr>
</tbody>
</table>
A- Stabilization test

Fig (5) Attitude response for (1) rad step input with PID controller

Fig (6) Attitude response for (1) rad step input with LQR controller

A- In this section PID and LQR controllers are tested to achieve different orientations. Figure (7, 8, 9) illustrate kufasat attitude response to a small and a large ACM with PID and LQR controllers.

Fig (7) Response to a small ACM from [0° 10° 10°] to [20° 20° 20°] with PID
Fig (8) Response to a small ACM from $[0^\circ - 10^\circ 10^\circ]$ to $[20^\circ 20^\circ 20^\circ]$ with LQR

Fig (9) Response to a large ACM from $[0^\circ - 170^\circ -170^\circ]$ to $[50^\circ 50^\circ 50^\circ]$ with PID

Fig (10) Response to a large ACM from $[0^\circ - 170^\circ -170^\circ]$ to $[50^\circ 50^\circ 50^\circ]$ with LQR
6- Conclusion

In this paper, LQR controller for attitude control of kufasat is developed and its performance compared with the conventional PID controller. From the analysis it is observed that

1- The LQR controller was able to meet the design goals, minimum overshoot, minimum rise time and minimum steady state error.

2- The LQR has better performance in terms of percentage overshoot and rise time. It is observed that LQR is controllable and more stable than PID controller when the system is under effect of AMC. In addition to the time of satellite maneuver is shortened.

3- Even though, the PID controller produces the response with lower delay time and rise time, but it offers very high settling time due to the oscillatory behavior in transient period. It has severe oscillations with a very high peak overshoot which causes the damage in the system performance. The proposed LQR controller can effectively eliminate these dangerous oscillations and provides smooth operation in transient period.

4- Due to an onboard power limitation only one magneto-torquer coil can be switched on at a time. A control algorithm must be modified to allow for the choice of the coil that will achieve the best results, given the local geomagnetic field vector.

References


