

An Enhanced Solution for the Axial Current Using Electromagnetic Wire Scattering Analysis

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Abstract

this study presents a solution of electromagnetic scattering from an arbitrary oriented thin-wire excited by an incident electromagnetic plane wave by using Method of Moment (MoM). The solution for the axial current is obtained. The application of the Method of Moment (MoM) solution to the thin wire integral equation is obtained by enforcing the boundary condition on the electric field; this method inherently produces a full and dense matrix. The numerical results are performed for validation of the efficiency and accuracy of the proposed method.

Keywords: Method of Moments (MoM); arbitrary oriented thin wire; magnetic vector potential; electric scalar potential .

الحل المحسن لل (Axial Current) باستخدام تحليل الاستطارة الكهرومغناطيسية للسلك

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الخلاصة :

في هذه الدراسة، نقدم حلاً للاستطارة الكهرومغناطيسية من سلك رقيق عشوائي الاتجاه مهيج بواسطة موجة كهرومغناطيسية ساقط بصورة مستوية باستخدام طريقة (MoM) (Method of Moment). وتم الحصول على حل للتيار المحوري. أن تطبيق حلول الطريقة أعلاه على المعادلة التكاملية للسلك الرقيق كان ممكناً من خلال فرض شروط حدودية على المجال الكهربائي. هذه الطريقة تنتج بطبيعتها مصفوفة كثيفة. تم تنفيذ الطريقة عددياً للتأكد من صحة وكفاءة ودقة الطريقة المقترحة.

Introduction

The analysis of electromagnetic scattering from arbitrarily shaped conducting and/or dielectric bodied has been of considerable interest. The integral Equations based on Electric and Magnetic Field are termed the (EFIE) and (MFIE) ^[1, 2, 3, 4], respectively. For these kinds of problems the Method of Moments (MoM) ^[5, 6] is the most suitable tool due to its accuracy and

versatility. By means of this technique, an approximate included current is obtained. For simple antennas such as a wire of constant radius or a circular loop, special analytical methods have been developed [7, 8]. In the integral equation approach, the most popular and highly accurate method of solving electromagnetic field problems is the familiar Method of Moments (MoM). Many numerical schemes for studying a wire antenna of arbitrary shape are based on the Method of Moments (MoM) [2] [9] [10]. Attempts have been made to enhance the accuracy by replacing the reduce kernel with the exact kernel the satisfactory results were observed [11-13]. The coal of this paper, the Moment of Method (MoM) is applied to conventional electromagnetic scattering problems, this problem included currents on arbitrarily oriented thin wires due to an incident electromagnetic field.

This paper is organized as follows, in the next Section; we develop the thin wire analysis with comprehensive treatments and an efficient analysis of arbitrary wire by using a moment method and develop an efficient solution by solving the matrix to obtain the induced currents. The electric field integral equation is used with the moment method to solve the currents on the antenna, and the triangle basis and pulse testing functions exactly follow of the arbitrary. Section three shows the numerical results for the currents from the arbitrary wire structures. Finally, some conclusions based on this work are givened.

1. Theory and Formulations

On conducting wire surface, $I_s(\mathbf{r})$ is the current along the axis of an arbitrary shape thin wire, we can write the scattering field \mathbf{E}_s in terms of the magnetic vector potential \bar{A} , and the electric scalar potential Φ as [1]

$$\mathbf{E}^s = -j\omega \bar{A} - \nabla\Phi \dots\dots\dots(1)$$

where

$$\bar{A} = \mu \int_{sw} I_s(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') \hat{a}_s ds' \dots\dots\dots (2)$$

$$\Phi = \frac{j}{4\pi\epsilon_0} \int_{sw} \frac{dI_s(\mathbf{r}')}{ds'} G(\mathbf{r}, \mathbf{r}') ds' \dots\dots\dots (3)$$

and

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}}}{4\pi\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}} \dots\dots\dots (4)$$

where m and e are the permeability and permittivity of the space, \hat{a}_s is the unite vector along the surface at r , a is the radius of the wire ($a \ll l$), and r and r' (seen in **Figure. 1**) are the position vectors to the observation and source points, from the coordinate origin, respectively. After applying the boundary condition the total tangential electric field on the surface of the wire (along the axis) is zero, which implies that

$$(\overline{E^i} + \overline{E^s}) \cdot \hat{a}_s = 0 \text{ For } r \in s_w, \text{ to obtain the integral equation given by}$$

$$jkh \int_{sw} I_s(r') G(r, r') (\hat{a}_s \cdot \hat{a}_s) ds' + \frac{jh}{k} \frac{\partial}{\partial s} \int_{sw} \frac{dI_s(r')}{ds'} G(r, r') ds' = \overline{E^i}(r) \cdot \hat{a}_s \quad \dots\dots\dots(5)$$

for $r \in s_w$ and

$$\overline{E^i}(r) = E_0 e^{-jk \cdot r} \quad \dots\dots\dots (6)$$

in Equation (5) and equation (6), $k = \frac{2p}{l}$ is the wavenumber, h is the wave impedance of the medium. In order to solve for the induced axial current on the wire using Equation (5), by using the moment method solution procedure with triangle expansion function, which can be written in form as

$$I_s(r) \approx \sum_{n=1}^{N-1} I_n P_n(r) \quad \dots\dots\dots (7)$$

Where the I_n are the unknown coefficient, now by substituting equation (7) into equation (5) and defining

$$h_1(r) = \int_{r_{n-1}}^{r_{n+1}} P_n(r') G(r, r') (\hat{a}_s \cdot \hat{a}_s) ds' \quad \dots\dots\dots (8)$$

And

$$h_2(r) = \int_{r_{n-1}}^{r_{n+1}} \frac{dP_n(r')}{ds'} G(r, r') ds' \quad \dots\dots\dots (9)$$

Get

$$\sum_{n=1}^{N-1} I_n \left[jkh h_1(r) + \frac{jh}{k} \frac{\partial}{\partial s} h_2(r) \right] = \overline{E^i}(r) \cdot \hat{a}_s \quad \dots\dots\dots (10)$$

Next, we apply the testing procedure. Here, we used pulse function as testing functions, which defined as

$$Q_m(\mathbf{r}) = \begin{cases} 1, r_m - \frac{\Delta s_m}{2} \leq r \leq r_m + \frac{\Delta s_{m+1}}{2} \\ 0, otherwise \end{cases} \dots\dots\dots (11)$$

for $m = 1, 2, \dots, N - 1$. By defining the symmetric product as

$$\langle a(\mathbf{r}), b(\mathbf{r}) \rangle = \int_{s_w} a(\mathbf{r})b(\mathbf{r})ds \dots\dots\dots(12)$$

Get

$$\left\langle \sum_{n=1}^{N-1} I_n \left[jkh h_1(\mathbf{r}) + \frac{jh}{k} \frac{\partial}{\partial s} h_2(\mathbf{r}) \right], Q_m(\mathbf{r}) \right\rangle = \left\langle \bar{E}^i(\mathbf{r}) \cdot \hat{a}_s, Q_m(\mathbf{r}) \right\rangle \dots\dots\dots(13)$$

for $m = 1, 2, \dots, N - 1$. To simplify the testing procedure, evaluate $h_1(\mathbf{r})$ and $\bar{E}^i(\mathbf{r}) \cdot \hat{a}_s$, in Equation (15) at center of the pulse, and multiply by Δ_m where $\Delta_m = \left(\frac{\Delta s_m + \Delta s_{m+1}}{2} \right)$. The term

$\frac{\partial}{\partial s} h_2(\mathbf{r})$ in Equation (15) may be evaluated as

$$\begin{aligned} \left\langle \frac{\partial}{\partial s} h_2(\mathbf{r}), Q_m \right\rangle &= \int_{r_m - \frac{\Delta s_m}{2}}^{r_m + \frac{\Delta s_{m+1}}{2}} \frac{\partial}{\partial s} h_2(\mathbf{r}) ds \\ &= \left[h_2(\mathbf{r}) \right]_{r_m - \frac{\Delta s_m}{2}}^{r_m + \frac{\Delta s_{m+1}}{2}} \\ &= h_2\left(r_m + \frac{\Delta s_{m+1}}{2}\right) - h_2\left(r_m - \frac{\Delta s_m}{2}\right) \dots\dots\dots(14) \end{aligned}$$

Thus, we have

$$\begin{aligned} \sum_{n=1}^{N-1} I_n \left[jkh \Delta_m h_1(\mathbf{r}) + \frac{jh}{k} h_2\left(r_m + \frac{\Delta s_{m+1}}{2}\right) - \frac{jh}{k} h_2\left(r_m - \frac{\Delta s_m}{2}\right) \right] \\ = \Delta_m \bar{E}(\mathbf{r}_m) \cdot \hat{a}_{s_m} \dots\dots\dots(15) \end{aligned}$$

for $m = 1, 2, \dots, N - 1$, which is a system of equations and be written in the matrix form as

$$\begin{pmatrix} Z_{1,1} & Z_{1,2} \mathbf{L} & Z_{1,N-1} \\ Z_{2,1} & Z_{2,2} & Z_{2,N-1} \\ \mathbf{M} & \mathbf{O} & \mathbf{M} \\ Z_{N-1,1} & \mathbf{L} & Z_{N-1,N-1} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \mathbf{M} \\ I_{N-1} \end{pmatrix} = \begin{pmatrix} \Delta_1 \mathbf{E}^{\mathbf{u}, \mathbf{v}}(r) \cdot \hat{a}_{s_1} \\ \Delta_2 \mathbf{E}^{\mathbf{u}, \mathbf{v}}(r) \cdot \hat{a}_{s_1} \\ \mathbf{M} \\ \Delta_{N-1} \mathbf{E}^{\mathbf{u}, \mathbf{v}}(r_{N-1}) \cdot \hat{a}_{s_{N-1}} \end{pmatrix} \dots\dots\dots (16)$$

Each of the impedance elements of Equation (16) are given by

$$Z_{m,n} = jk h \Delta_m \int_{r_{n-1}}^{r_{n+1}} P_n^{\mathbf{v}}(r) G(r_m, r) (\hat{a}_{s_n} \cdot \hat{a}_{s_m}) ds + \frac{jk}{k} \int_{r_{n-1}}^{r_{n+1}} \frac{dP_n^{\mathbf{v}}(r)}{ds} G(r_m + \frac{\Delta_{s_{m+1}}}{2}, r) ds - \frac{jh}{k} \int_{r_{n-1}}^{r_{n+1}} \frac{dP_n^{\mathbf{v}}(r)}{ds} G(r_m - \frac{\Delta_{s_m}}{2}, r) ds \dots\dots\dots (17)$$

Lastly, the elements of the excitation vector are given by

$$\Delta_m \overline{E}^i(r_m) \cdot \hat{a}_{s_m} = \Delta_m \overline{E} e^{-jk \cdot r_m} \cdot \hat{a}_{s_m} \dots\dots\dots (18)$$

Equation (16) can now be solved to obtain the current induced along the axis of the wire due to the incident electromagnetic field. Using this standard approach, we note that this method generate a full/dense complex impedance matrix. For electrically large wires, the storage and solution of this matrix can be prohibitive.

2. Numerical Results and Discussion

For comparison with the previous references, we present case study to test the validity of the moment of method to create a banded matrix for arbitrary wire problem. For the case presented in this section, we choose the wire radius equal to 0.001λ . We consider a 20λ long straight wire illuminated by an axially polarized, normally incident plane wave. This incident field is shown in **(Figure 2)**. For the numerical solution, the wire divided into 200 equal divisions, which resulted in 199 unknowns because the current at the ends of the wire is assumed to be zero. In **Figure 2 (a, b)**, the results for the Moment Method solution are presented and the magnitude of the current as well as the real and imaginary parts. In **(Figure 3)** shows the incident field we consider the circumference of the circular wire is 40λ long and the wire is divided into 400 linear divisions. The result for the moment method also plotted in **(Figure 3)**.

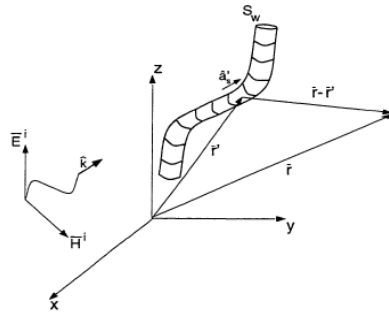


Fig .(1) Arbitrary shaped thin wire illumination by an electromagnetic plane wave.

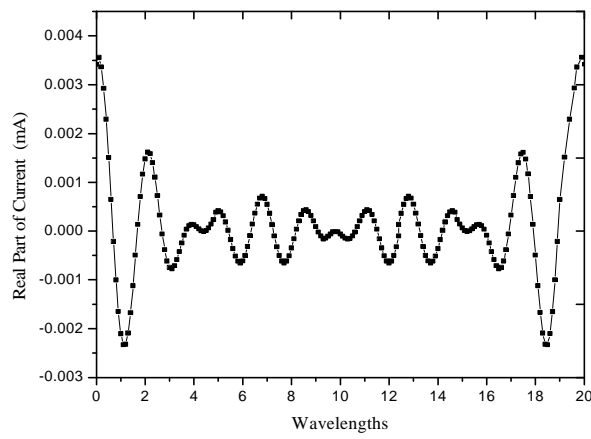


Fig .(2-a) Induced currents (Real Part) on 20λ long straight wire due to an axially polarized incident field

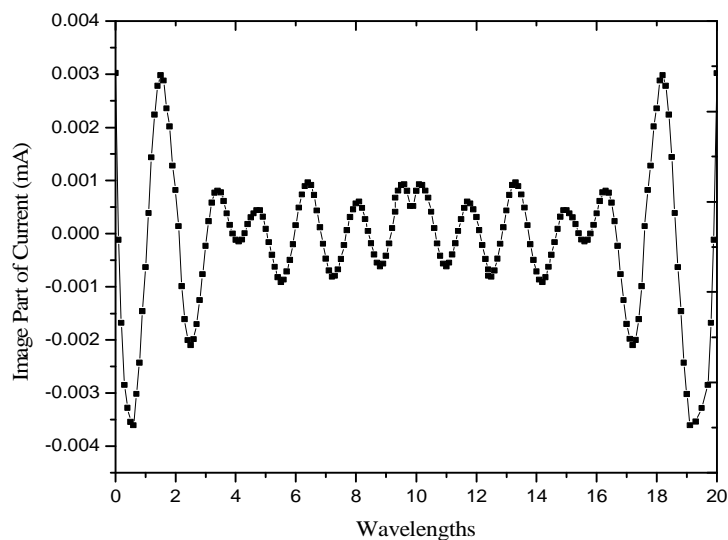


Fig . (2-b) Induced currents (Imaginary Part) on 20λ long straight wire due to an axially polarized incident field.

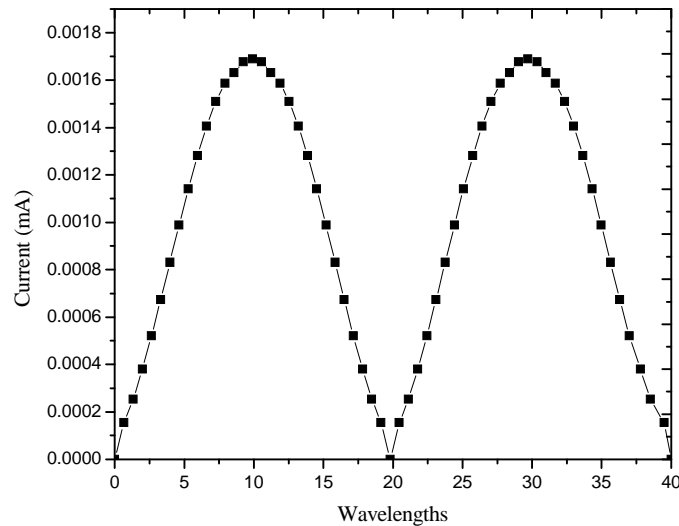


Fig .(3) Induced currents on 40λ long straight wire due to an axially polarized incident field incident on the face of the loop.

3. Conclusion

This paper has been presented an accurate solution method to analysis a wire scattering by using the Moment of Method (MoM) and we develop an efficient solution by solving the banded matrix to obtain the induced current. The application of the Method of Moment solution procedure to the thin wire integral equation is obtained by enforcing the boundary condition on the electric field. The details of the moment method are outlined and associated illustrative examples are described. The arbitrarily thin wire approximation is used throughout the analysis and the wire is assumed to be oriented.

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