

## Shell Model Calculations of Some Titanium Isotopes

### حسابات أنموذج القشرة لبعض نظائر التيتانيوم

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#### Abstract

The nuclear shell model has been employed to compute the binding energies , low-lying excitation states electric quadrupole transition rates of titanium isotopes in *fp*-shell region. The model space includes all configurations of nucleons in the  $0f_{7/2}$   $1p_{3/2}$   $0f_{5/2}$  and  $1p_{1/2}$  orbits, as well as the shell model describes well the energies of the intruder states. Calculations have been performed with effective interactions GXFP1 , FPD6 and KB3G in full *fp* space and the shell model code OXSASH for Windows were employed . The computed binding energies, the low-lying states and electric quadrupole transition rates were in reasonably agreement with experimental data for the isotopes <sup>42,44,46,48</sup>Ti .

#### الخلاصة:

تم استخدام أنموذج القشرة النووي لحساب طاقات الربط وحالات التهيج ومعدلات الانتقال رباعي القطب الكهربائي لنظائر التيتانيوم في منطقة القشرة *fp* . الترتيبات المتضمنة في أنموذج الفضاء تشمل المدارات (  $0f_{7/2}$   $1p_{3/2}$   $0f_{5/2}$   $1p_{1/2}$  ) ، يصف أنموذج القشرة بشكل جيد الطاقات للحالات المتداخلة. الحسابات أنجزت مع التفاعلات المؤثرة ( GXFP1 , FPD6 , KB3G ) في أنموذج قشرة OXSASH . طاقات الحالات الأرضية والتهيجة المحسوبة ومعدلات الانتقال رباعي القطب الكهربائي في توافق مقبول مع النتائج العملية.

**Key word:** excitation energies, shell model nuclear, <sup>42,44,46,48</sup>Ti .

### 1. Introduction

Nuclear shell model is one of the most powerful tools for giving a quantitative interpretation to the experimental data. The two main ingredients of any shell model calculations are the N-N interaction and the configuration space for valence particles. In principle one can either perform shell model calculations with realistic N-N interaction in unlimited configuration space or with renormalized effective interaction

limited configuration space [1].

The neutron rich nuclei in the *fp* shell region are at the focus of attention of the nuclear physics community at present [1]. The nuclei in the  $1p$   $0f$  region have been the focus of attention for various theoretical investigations based on the practical assumption of a <sup>40</sup>Ca inert core within the domain of the shell model [2]. Nuclei with valence nucleons in the *fp* shell have been intensively studied and discussed in recent years following the striking progress in the development and refinement of large-scale shell model calculations [3].

The spectroscopy of nuclei, in the *fp*-shell region, has been well described within the shell model framework. Extensive shell model calculations have been performed in this mass region, using several model spaces and two-body interactions, the most remarkable work of Brown and *et al.* [4].

In the present work, The calculations have been carried out using the code OXSASH for windows [5] in the *fp* model space which comprised of the  $1p_{3/2}$ ,  $1p_{1/2}$ ,  $0f_{7/2}$  and  $0f_{5/2}$  valence orbits outside the <sup>40</sup>Ca and FPD6 [6] GXFP1[7] and KB3G [8] interactions.

**2.Shell Model Calculations**

**2.1. Binding Energy**

Binding energies are important to nuclear astrophysicists when determining Q-values of proton capture reactions and beta decays[9]. To compare our shell model results with the experimental binding energies relative to binding energies of core. we use following formula [9]:

$$B=B(\text{core}) - \langle H \rangle \dots\dots\dots(1)$$

The experimental and calculated binding energies for titanium isotopes are shown in Fig.1. We see the theoretical calculations are deviate on experimental binding energies in the interactions GXFP1[6] , FPD6[7] and KB3G[8]. With neutron increase the deviation is increasing.

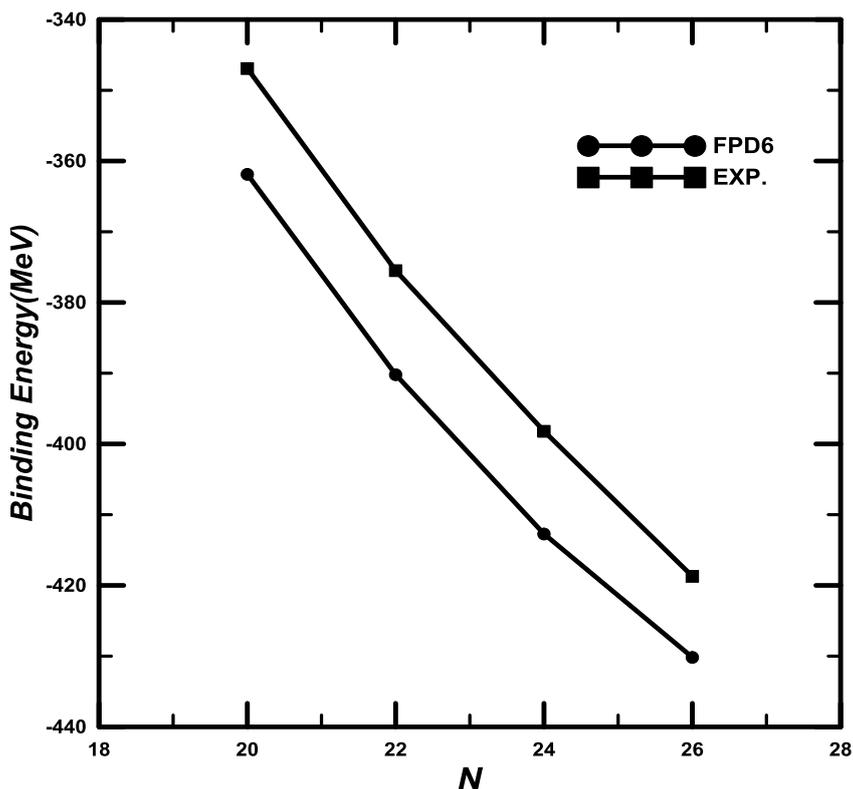


Fig.(1). Experimental [10] and calculated binding energies for Ti isotopes from A=42 to 48

**2.2 Energy Levels**

Besides ground states, low -lying excitation states and reduced transition probabilities in most nuclei have also extensively studied. since the number of excitation states that can be investigated in detail is quite large- tens of level in some nuclei. In this work, nuclear shell model has been using for study the nuclear structure of <sup>42</sup>Ti, <sup>44</sup>Ti, <sup>46</sup>Ti and <sup>48</sup>Ti nuclei by adopting effective interactions FPD6[6], GXFP1[7] and KB3G[8] to calculation excitations levels in *fp* shell region. The code OXBASH was adopted.

The matrix elements of the Hamiltonian operator H between the many-particle basis wave functions can be obtained by using the second-quantized operators a<sup>+</sup> and a. Thus, H can be written as [11].

$$H = H_{core} + \sum \epsilon_i a_i^+ a_i + \sum_{i>j=1, k>l=1} v_{ijkl} a_i^+ a_j^+ a_l a_k \dots\dots\dots(2)$$

where  $\epsilon_i$  is the energy of a single-particle state, and

$$v_{ijkl} = \langle ij | V | kl \rangle \dots\dots\dots(3)$$

and  $|ij\rangle$  is an antisymmetrized two-particle m-scheme state. The results of  $a_i^+ a_i$  and  $a_i^+ a_j^+ , a_k a_l$  Creation and Annihilation operations, and the matrix elements  $\langle ij | V | kl \rangle$  can easily be derived from the conventional J and T coupled two-body matrix elements

**3.Results and discussion**

The energy levels have been calculated of *fp*-Shell region for Ti nuclei for the isovector T=1,0 positive parity states. In nucleus  $^{44}\text{Ti}$  have isovector part T=1 and isoscalar T=0 but in  $^{42,46}\text{Ti}$  have isovector T=1. In our calculations the isovector T=1 for two nuclei ( $^{42,46}\text{Ti}$ ) and the isoscalar T=0 for  $^{44}\text{Ti}$  nucleus are taken.

The low-lying positive parity, T = 1 spectra of  $^{42}\text{Ti}$  are shown in Fig.2. This figure show that the comparison between our calculations and experimental data [12]. The first excitation level  $2^+_1$  (1.639MeV) at GXPF1 respect to that obtained with the KBG3 and FPD6 is nearest to the experimental value (1.556MeV). Furthermore, the excitation state  $\text{Ex}(4^+_1)$  at the value (2.709MeV) in interaction FPD6 agreement the experimental value (2.679MeV). As spins that obtain for high spectra  $6^+_1, 2^+_2, 4^+_2$  are in good agreement with exdata for two interactions FPD6 and KB3G . In this the comparison, we can notice that the use of these two body interactions has been of considerable interest to many theoretical attempts in *fp* shell for the description of energy levels.

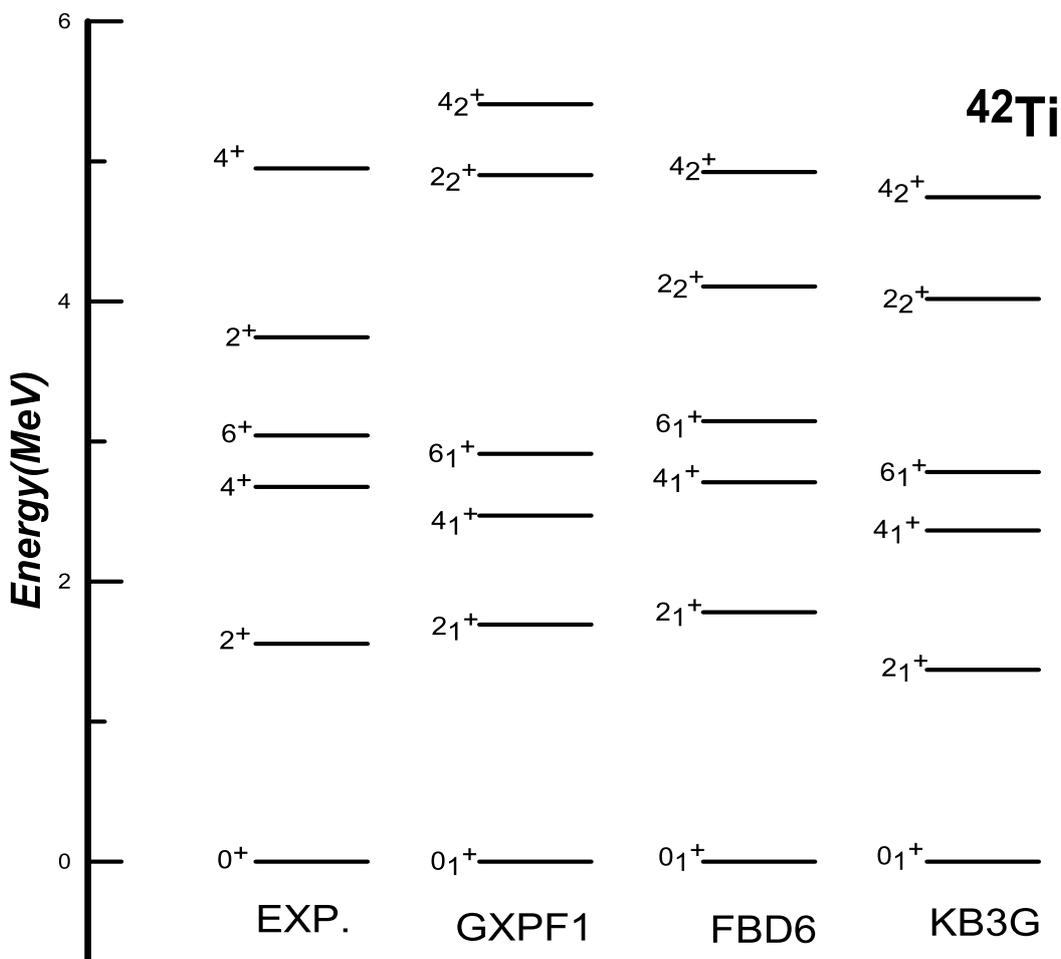


Fig.2. A comparison between theoretical energy levels for three interaction and the experimental data [12].

Fig.3. shows that the energy spectrum of  $^{44}\text{Ti}$  nucleus . When compared our calculations with data[13], we can see that rotational energy levels are represented by  $(0^+, 2^+, 4^+, 6^+, 8^+)$  of theory results in agreement of employed three interactions KB3G, FPD6 and GXPF1 with experimental data. As well as the excitation energies computed in interaction FPD6 an excellent agreement with data, this comparison proves unequivocally that interaction FPD6 is a better from KB3G and GXPF1 interactions.

This is due to model space is not restricted, one should use an effective operator in order to take account of included all model space.

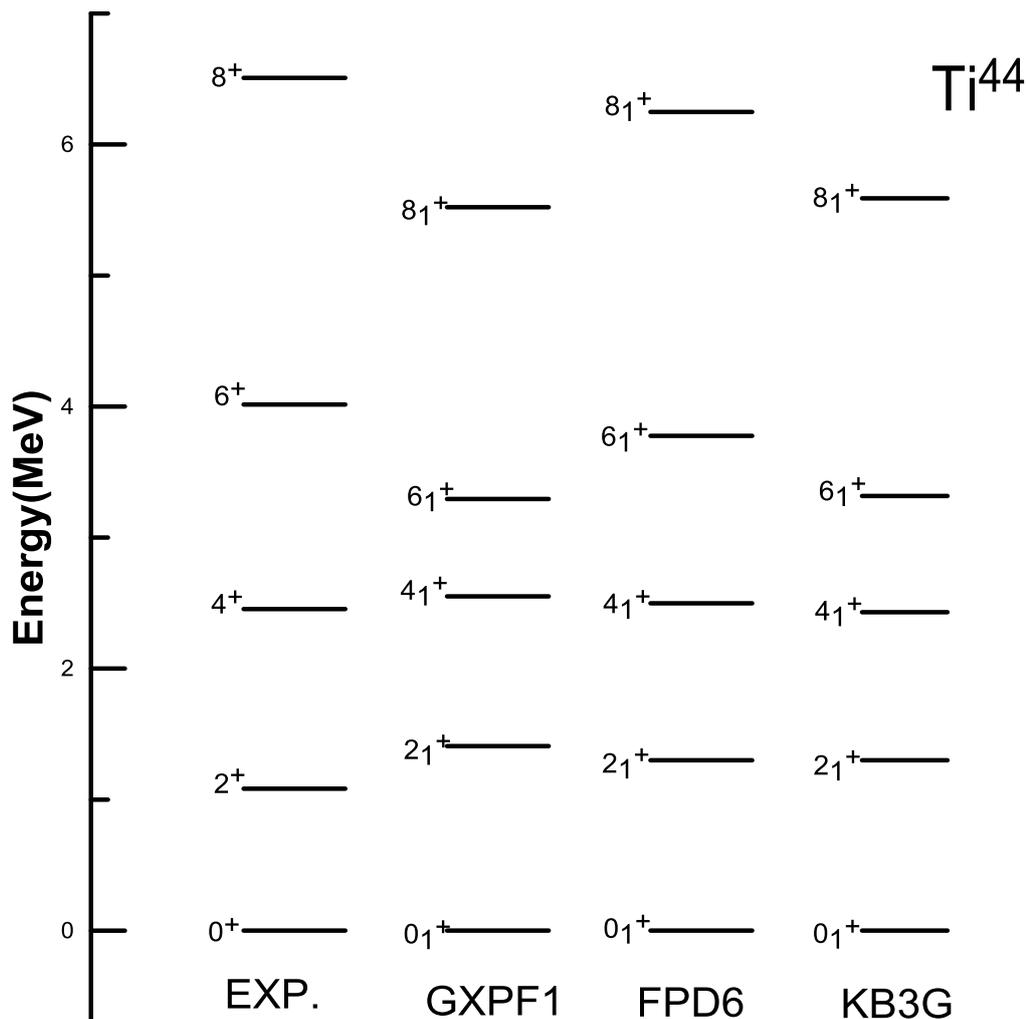


Fig.3. A comparison between theoretical energy levels for three interactions and the experimental data [13].

The adopted values for  $^{46}\text{Ti}$  are shown graphically in Fig .4. and it has been compared with the experimental data[14]. The prediction values of ground band by adopted FPD6 and KB3G in an excellent agreement with date, but except Ex(  $2^+_1$ ) at 1.117MeV is a larger from experimental value 0.889MeV, where the absolute difference between the two values is 0.228MeV in GXPF1. In these calculations, we can see the  $\beta$  and  $\gamma$  bands are obtained in the framework FPD6 are very close to data . We notice with two different configurations has been successfully evolved to study these anomalies where the role of mixing between  $fp$  shell orbits is found to be crucial.

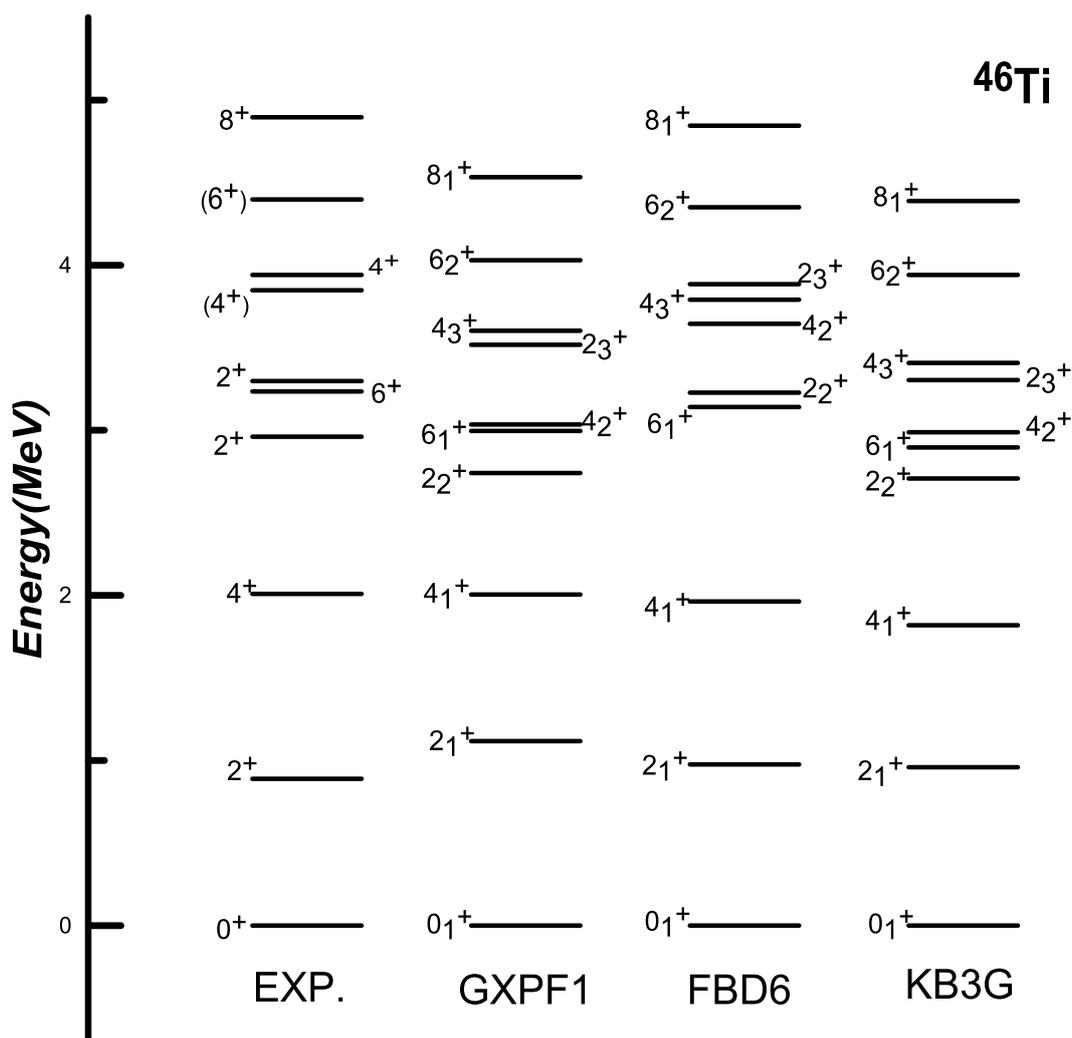


Fig.4. A comparison between theoretical energy levels for three interaction and the experimental data[14].

In Fig.5. The calculated energy levels for positive parities are compared with the experimental data [15]. The first state  $Ex(2_1^+)$  in KB3G interaction is in a good agreement with experimental data, but in GXPF1 and FPD6 are up slightly of experimental data. It has been predicted the parity for angular momenta (3,10). From comparison between three interactions and experimental data, we found the FPD6 interaction is better from the GXPF1 and KB3G interactions for low and high spins.

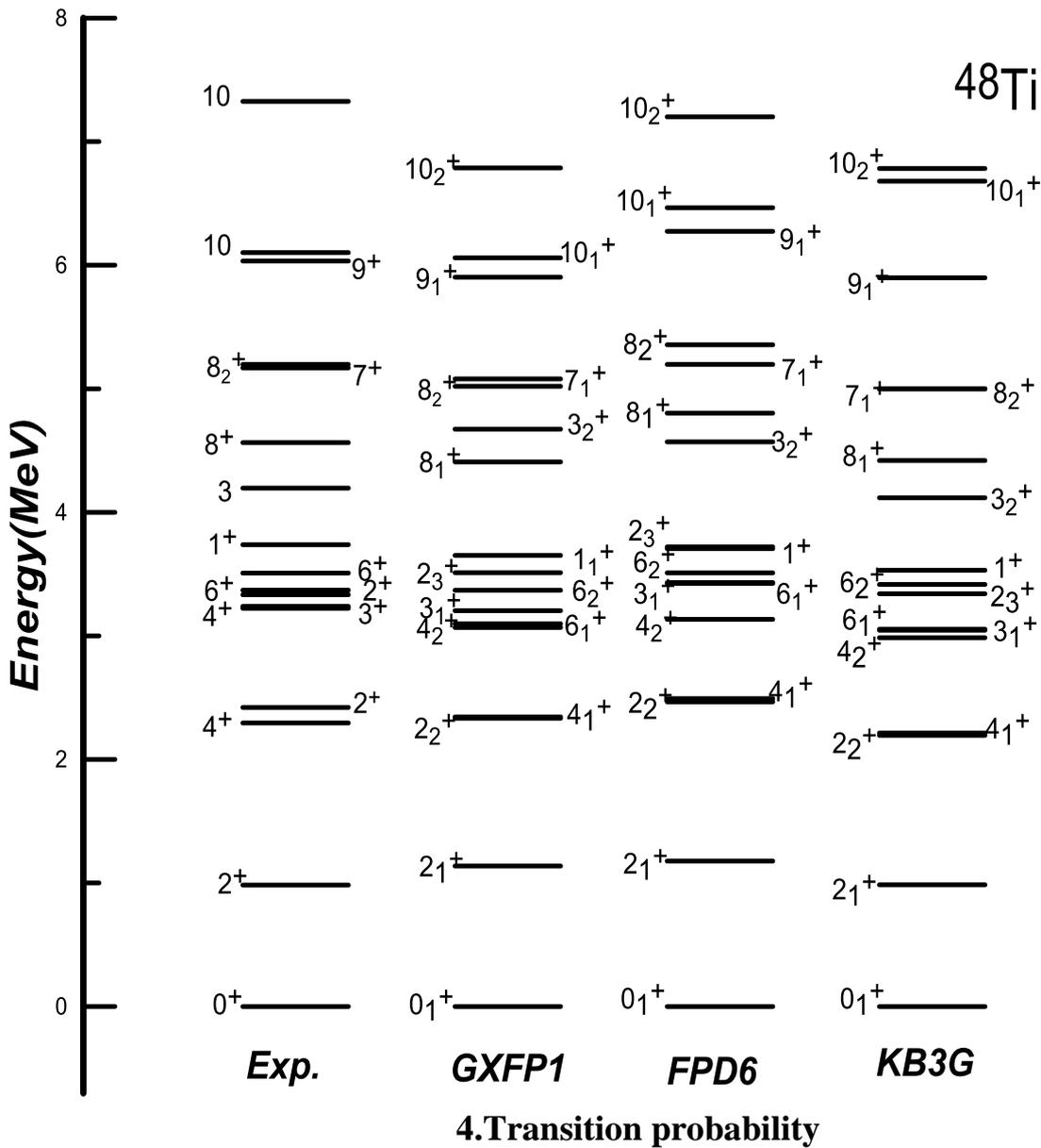


Fig.5. A comparison between theoretical energy levels for three interaction and the experimental data [15].

In Fig.6. The reduced electric quadrupole transition probabilities  $B(E2; 0_1 \rightarrow 2_1^+) e^2 \cdot fm^4$  values are plotted calculated for  $fp$  model space for Ti isotopes in chain  $A=42-48$ . Transition strengths were calculated in this study using FPD6 with Skyrme- Hartree- Fock potentials as residual interactions. From the Fig.6, we can see that our calculations are reasonably agreement with experimental data.

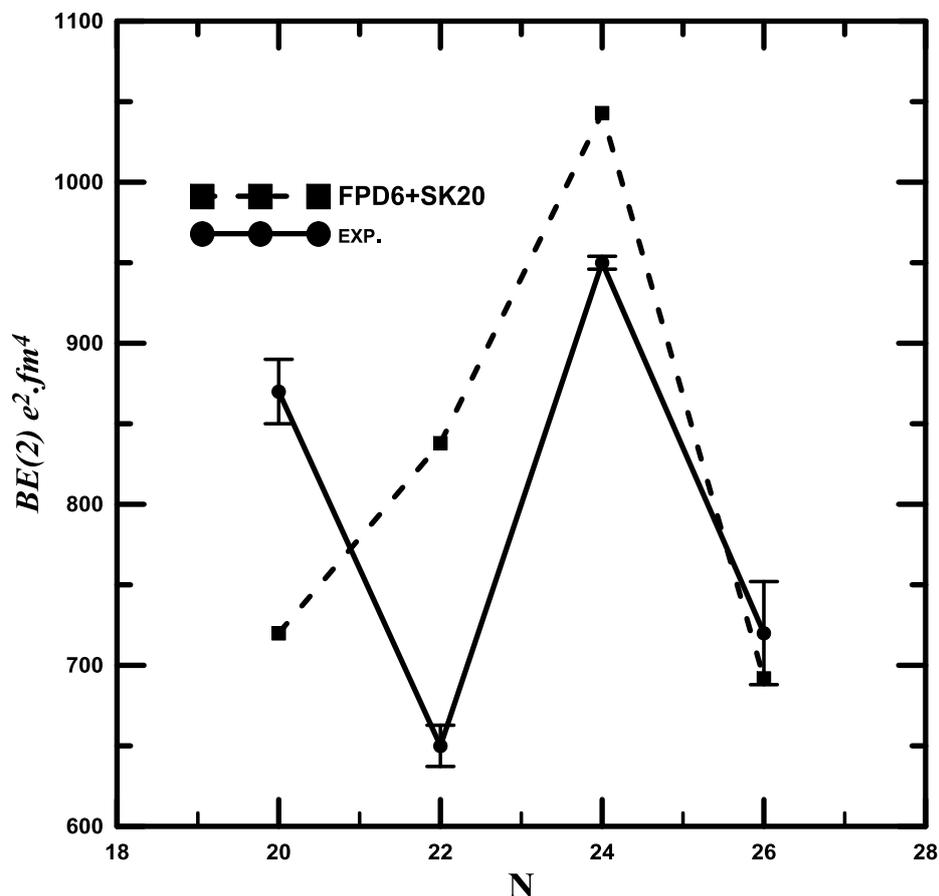


Fig.(6). B(E2) transitions probabilities in  $e^2 \cdot fm^4$  for Ti isotopes. Experimental data (closed circles) are compared with present work (dashed line), Experimental data are taken from Ref.[16].

## 5.Conclusions

The excitation energies with binding energies and transition rates have been computed within framework nuclear shell model calculations were performed in full  $fp$  space . The shell model code OXSBASH for Windows with three interactions (GXPF1, FPD6, KB3G) was employed using nucleus  $^{40}\text{Ca}$  as close core for  $^{42}\text{Ti}$ ,  $^{44}\text{Ti}$ ,  $^{46}\text{Ti}$  and  $^{48}\text{Ti}$ . The results of shell model calculations are in agreement with experimental data. We conclude the shell model configuration mixing in this model space are very successful.

**REFERENCES**

- [1] P.C. Srivastava, I. Mehrotra , nucl-th.,0907.4050,V1,( 2009).
- [2] Y. K. Gambhir, S. Haq, and J. K. Suri Phys. Rev.,**C 25**, No. 1. ( 1982).
- [3] K. H. Speidel, *et al*, Prog. Part. , *Nucl. Phys.*, 49 , 91-154 (2002).
- [4] D. C. Dinca, *et al.*, *Phys. Rev.*, **C71**, 041302( R), (2005).
- [5] B. A. Brown, A. Etchegoyen, N.S. Godwin, W. D. M.Rae, W.A.Richter,E.K.Warburton, J.S.Winfield, L.Zaho, C.H.Zimmerman, Oxbash for windows,MSUNSCL, report number 1289 (2004).
- [6] W. A. Richter, M. G. van der Merwe, R. E. Julies, B. A. Brown, *Nucl. Phys.*, **A523, 325** (1991).
- [7] M. Honma, T.Otsuka, B. A. Brown, T. Mizusaki, Phys. Rev. **C 69**, 034335,(2004).
- [8] A. Poves, J. Sánchez-Solano, E. Caurier, F. Nowacki, Nucl. Phys., **A 694**, 157 (2001).
- [9] F. I.Sarrad., A. A.Okhunov, H. Y.Abdullah and H. Abu Kassim Rom. Journ. Phys., **Vol. 58**, Nos. 1–2, P. 99–105,(2013).
- [10] . G.Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. **A 729**, 337, (2003).
- [11] M. Honma, B. A. Brown, T. Mizusaki and T. Otsuka, Nucl. Phys. **A704**, 134c (2002).
- [12] Balraj Singh and John. A. Cameron, Nuclear Data Sheets,**92**,1,(2001).
- [13] Jun Chen, Balraj Singh and John A. Cameron, Nuclear Data Sheets **112**, 2357 (2011).
- [14] S. -c. WU, Nuclear Data Sheets ,**91**, 1 (2000).
- [15] T.W.Burrow, Nuclear Data Sheets, **107**,1747 (2006).
- [16] S.Raman, C.W.Nestor, P.Tikkanen ,At.Data Nucl.Data Tables **78**, 1, (2001).