



Design of On-Line Nonlinear Kinematic Trajectory Tracking Controller for Mobile Robot based on Optimal Back-Stepping Technique

Asst. Prof. Dr. Ahmed Sabah Al-Araji

Control and Systems Engineering Department, University of Technology, Baghdad-Iraq

e-mail: dr.ahmed_alaraje@uotechnology.edu.iq

Received: 5/2/2014

Accepted: 4/6/2014

Abstract – **T**his paper presents an on-line nonlinear trajectory tracking control algorithm for differential wheeled mobile robot using optimal back-stepping technique based particle swarm optimization while following a pre-defined continuous path. The aim of the proposed feedback nonlinear kinematic controller is to find the optimal velocity control action for the real mobile robot. The particle swarm optimization algorithm is used to find the on-line optimal parameters for the proposed controller based on the Lyapunov criterion in order to check the stability of the control system. Simulation results (Matlab) and experimental work (LabVIEW) show the effectiveness and robustness of the proposed on-line nonlinear kinematic control algorithm. This is demonstrated by minimizing tracking error and obtaining smoothness of the optimal velocity control signal, especially with regards to the external disturbance attenuation problem.

Keywords:- Mobile Robots, Nonlinear Kinematic Controller, Back-Stepping Technique, Particle Swarm Optimization, Trajectory Tracking, Matlab package, LabVIEW package.

1. Introduction

During the past few years, there has been an increasing amount of research on the subject of wheel-based mobile robots which have attracted considerable attention in various industrial and service applications. For example, room cleaning, lawn mowers, factory automation, transportation, nuclear-waste cleaning, etc. [1], [2]. These applications require mobile robots to have the ability to track a specified path stably.

Several studies have been published for solving the control problems of the trajectory tracking for mobile robot in terms of designing and implementing the driving control that the mobile robot must track to follow a desired path accurately and minimize the tracking error [3].

Tracking errors of mobile robot causes collisions with obstacles due to deviations from the planned path and also causes the robot to fail in accomplishing the mission successfully. It also causes an increase of the traveling time, as well as the travel distance, thus the additional adjustments will be needed to satisfy the driving sates.

There are three major reasons for increasing tracking error for a mobile robot:

The first major reason of tracking error is the discontinuity of the rotation radius on the path of the differential driving mobile robot. The rotation radius changes at the connecting point of the straight line route and curve route, or at a point of inflection.

At these points it can be easy for a differential driving mobile robot to secede from its determined orbit due to the rapid change of direction [3].

Therefore, in order to decrease tracking error, the trajectory of the mobile robot must be planned so that the rotation radius is maintained as constant as possible.

The second major reason of increasing tracking error is that the small rotation

radius interferes with the accurate driving of the mobile robot. The path of the mobile robot can be divided into curved and straight-line segments. While tracking error is not generated in the straight-line segment, significant error is produced in the curved segment due to centrifugal and centripetal forces, which cause the robot to slide over the surface [3].

The Third major reason for increasing tracking error is that the rotation radius is not constant such as the complex curvature or randomly curvature, that is, the points of inflection exist at several locations. This means that the mobile robot wheels velocity needs to be changed whenever the rotational radius and traveling direction are changed [4].

The fundamental essence of the motivation of this work is to reduce the tracking error for the differential wheeled mobile robot in order to track the desired continuous trajectory by designing optimal on-line nonlinear kinematic controller.

The traditional control methods for trajectory tracking of the mobile robot have used linear or non-linear feedback control while artificial intelligent controller were carried out using neural networks or fuzzy inference [5] - [7].

There are other techniques for trajectory tracking controllers such as predictive control technique [8]. In addition, an adaptive trajectory-tracking controller based on the robot dynamics was proposed in [9] – [11] and its stability property was proved using the Lyapunov theory. The back-stepping method and virtual feedback parameter are calculated using the signum function that has been used as a variable structure tracking control algorithm for the mobile robot as explained in [12] , [13].

The contributions of this paper can be understood by considering the following points.

- The analytically derived control law which has significantly high computational accuracy to obtain the best control action and lead to minimum tracking error of the mobile robot based on back-stepping method and particle swarm optimization.
- Investigation of the kinematic controller robustness and adaptation performance through adding unknown boundary disturbances.
- Verification of the proposed on-line controller capability of tracking continuous trajectory is done by an experimental work using Eddie mobile robot.

Simulation results and experimental work show that the proposed controller is robust and effective in terms of minimum tracking error and in generating an optimal velocity control action despite of the presence of bounded external disturbances.

The remainder of the paper is organized as follows: Section two is a description of the kinematics model of the differential wheeled mobile robot model and checking the controllability for the mobile robot. In section three, the proposed nonlinear kinematic controller is derived based on back-stepping method and particle swarm optimization. The simulation results and experimental work of the proposed controller are presented in section four and the conclusions are drawn in section five.

2. Model of Differential Wheeled Mobile Robot

The schematic of the differential wheeled mobile robot model shown in Fig. 1 consists of a cart with two driving

wheels mounted on the same axis and an omni-directional castor in the front of the cart. The castor carries the mechanical structure and keeps the platform more stable [13].

Two independent analogous DC motors are the actuators of left and right wheels for motion and orientation. The two wheels have the same radius denoted by r , and L is the distance between the two wheels. The centre of mass of the mobile robot is located at point c , centre of axis of wheels.

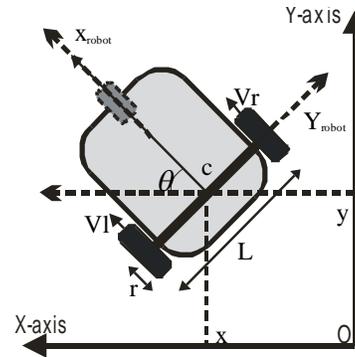


Figure 1 Mobile robot platform.

The pose of the mobile robot in the global coordinate frame $[o, X, Y]$ and the pose vector in the surface is defined as:

$$q = (x, y, \theta)^T \quad (1)$$

where $q(t) \in \mathbb{R}^{3 \times 1}$,

x and y are coordinates of point c and θ is the robotic orientation angle measured with respect to the X -axis. These three generalized coordinates can describe the configuration of the mobile robot.

It is assumed that the mobile robot wheels are ideally installed in such a way that they have ideal rolling without skidding [14], as shown in (2):

$$-\dot{x}(t) \sin \theta(t) + \dot{y}(t) \cos \theta(t) = 0 \quad (2)$$

Therefore, the kinematics equations in the world frame can be represented as follows [15]:

$$\dot{x}(t) = V_l(t) \cos \theta(t) \quad (3)$$

$$\dot{y}(t) = V_l(t) \sin \theta(t) \quad (4)$$

$$\dot{\theta}(t) = V_w(t) \quad (5)$$

where V_l and V_w are the linear and angular velocities.

In the computer simulation, the currently form of the pose equations is as follows:

$$x(k) = 0.5[V_R(k) + V_L(k)]\cos\theta(k)\Delta t + x(k-1) \quad (6)$$

$$y(k) = 0.5[V_R(k) + V_L(k)]\sin\theta(k)\Delta t + y(k-1) \quad (7)$$

$$\theta(k) = \frac{1}{L}[V_L(k) - V_R(k)]\Delta t + \theta(k-1) \quad (8)$$

where $x(k), y(k), \theta(k)$ are the components of the pose at the k step of the movement and Δt is the sampling period between two sampling times.

Using Jacobi-Lie-Bracket to check controllability of the nonlinear MIMO kinematic mobile robot system in (3, 4, 5), the accessibility rank condition is globally satisfied and is implied controllability. The mobile robot kinematics can be described by the left and right velocities as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0.5\cos\theta(t) & 0.5\cos\theta(t) \\ 0.5\sin\theta(t) & 0.5\sin\theta(t) \\ 1/L & -1/L \end{bmatrix} \begin{bmatrix} V_R(t) \\ V_L(t) \end{bmatrix} \quad (9)$$

$$\dot{q} = [f]V_R(t) + [g]V_L(t) \quad (10)$$

f and g can be defined as two vectors with components as:

$$f = \begin{bmatrix} 0.5\cos\theta(t) \\ 0.5\sin\theta(t) \\ 1/L \end{bmatrix} \quad \text{and} \quad g = \begin{bmatrix} 0.5\cos\theta(t) \\ 0.5\sin\theta(t) \\ -1/L \end{bmatrix} \quad (11)$$

$$[f, g] = \begin{bmatrix} [f, g]^1 \\ [f, g]^2 \\ [f, g]^3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}\sin\theta(t) \\ \frac{1}{L}\cos\theta(t) \\ 0 \end{bmatrix} \quad (12)$$

$$\text{rank}\{f, g, [f, g]\} = \text{rank} \begin{bmatrix} 0.5\cos\theta(t) & 0.5\cos\theta(t) & -\frac{1}{L}\sin\theta(t) \\ 0.5\sin\theta(t) & 0.5\sin\theta(t) & \frac{1}{L}\cos\theta(t) \\ 1/L & -1/L & 0 \end{bmatrix} \quad (13)$$

The detertment of the matrix in (13) is equal to $(1/L^2) \neq 0$, then the full rank of matrix is equal to 3, therefore, the system in (3, 4 and 5) is controllable.

3. On-Line Adaptive Kinematic Control Methodology

The feedback nonlinear kinematic controller is essential to stabilize the tracking error of the mobile robot system when the trajectory of the robot drifts from the desired trajectory during transient state.

The proposed structure of the on-line nonlinear kinematic trajectory tracking controller for mobile robot system can be given in the form of block diagram, as shown in Fig. 2.

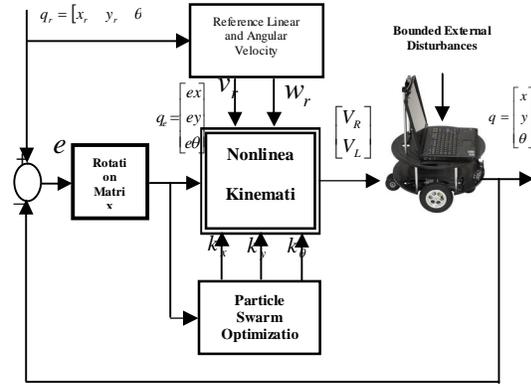


Figure 2 The proposed structure of on-line nonlinear kinematic controller for the mobile robot system.

The objective of the nonlinear kinematic controller is to design a controller for the transformed kinematics model of the mobile robot that forces the actual Cartesian position and orientation of the robot to a constant desired position and orientation.

The proposed on-line control algorithm will generate the optimal parameters for the nonlinear kinematic controller in order to obtain best velocity control signal that will minimize the tracking error of the mobile robot in the presence of external disturbance.

The structure of the nonlinear kinematic feedback controller consists of the nonlinear feedback velocity control equation based on back-stepping

technique and particle swarm optimization, as shown in Fig.3.

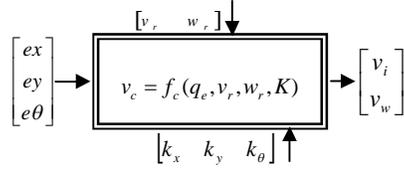


Figure 3 The nonlinear kinematic feedback controller structure.

The reference linear velocity and the reference angular velocity are given by (14) and (15) respectively [16].

$$v_r = \sqrt{(\dot{x}_r)^2 + (\dot{y}_r)^2} \quad (14)$$

$$w_r = \frac{\dot{y}_r \dot{x}_r - \dot{x}_r \dot{y}_r}{(\dot{x}_r)^2 + (\dot{y}_r)^2} \quad (15)$$

with $v_r > 0$ and $w_r > 0$, for all t , determine a smooth velocity control law $v_c = f_c(q_e, v_r, w_r, K)$ such that $\lim_{t \rightarrow \infty} (q_r - q) = 0$ is asymptotically stable.

The reference posture of the mobile robot $q_r = [x_r, y_r, \theta_r]^T$ in the world frame can be described by a virtual reference velocity of the mobile robot as follows [16]:

$$\dot{x}_r = v_r \cos \theta_r, \dot{y}_r = v_r \sin \theta_r, \dot{\theta}_r = w_r \quad (16)$$

The configuration error can be represented by using a transformation matrix as:

$$\begin{bmatrix} ex \\ ey \\ e\theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (17)$$

After taking the time derivative of (17), the configuration error for the mobile robot can be expressed as follows:

$$\begin{bmatrix} \dot{ex} \\ \dot{ey} \\ \dot{e\theta} \end{bmatrix} = \begin{bmatrix} v_w ey - v_i + v_r \cos e\theta \\ -v_w ex + v_r \sin e\theta \\ w_r - v_w \end{bmatrix} \quad (18)$$

Based on back-stepping method, the proposed nonlinear feedback velocity time-varying control law can be described as (19):

$$\begin{bmatrix} v_i \\ v_w \end{bmatrix} = \begin{bmatrix} v_r \cos e\theta + k_x ex \\ w_r + k_y v_r ey + k_\theta v_r \sin e\theta \end{bmatrix} \quad (19)$$

$k_x, k_y, k_\theta > 0$ are design control gain parameters that will be found by using particle swarm optimization technique.

The proposed control law is proved by using the Lyapunov criterion which represents the simple and successful method in finding the kinematics stability. Thus the constructive Lyapunov functions criterion considered is based on [17], as follows:

$$V = \frac{1}{2}(ex^2 + ey^2) + \frac{1}{k_y}(1 - \cos e\theta) \quad (20)$$

The time derivative of (20) becomes:

$$\dot{V} = ex\dot{ex} + ey\dot{ey} + \frac{1}{k_y}\dot{\theta} \sin \theta \quad (21)$$

Then

$$\dot{V} = ((w_r + v_r(k_y ey + k_\theta \sin e\theta))ey - k_x ex)ex \quad (22)$$

$$+ (-v_r + k_y v_r ey + k_\theta v_r \sin e\theta)ex + v_r \sin e\theta ey$$

$$+ \frac{1}{k_y} \sin e\theta (-v_r(k_y ey + k_\theta \sin e\theta))$$

$$\dot{V} = -k_x ex^2 - v_r \frac{k_\theta}{k_y} \sin^2 e\theta \leq 0 \quad (23)$$

Clearly, $V \geq 0$. If $q_e = 0$, $V = 0$. If $q_e \neq 0, V > 0$. and $\dot{V} \leq 0$. If $q_e = 0$, $\dot{V} = 0$. If $q_e \neq 0, \dot{V} < 0$.

Then, V becomes a Lyapunov function so that the closed loop system is globally asymptotically stable. The controller gains (k_x, k_y, k_θ) are determined by two stages, as follows:

$$k_x = (k_x^* + \Delta k_x) > 0, k_y = (k_y^* + \Delta k_y) > 0, k_\theta = (k_\theta^* + \Delta k_\theta) > 0 \quad (24)$$

where $(k_x^*, k_y^*, k_\theta^*)$ are determined by comparing the actual and the desired characteristic polynomial equations. While $(\Delta k_x, \Delta k_y, \Delta k_\theta)$ are determined by using the particle swarm optimization method in order to adjust the parameters of the nonlinear feedback kinematic controller.

The desired characteristic polynomial takes the following form:

$$(Z + z_1)(Z + z_2)(Z + z_3) = 0 \quad (25)$$

where

$$z_1 = -e^{-2\zeta\omega_n T_s} \text{ and } z_{2,3} = -e^{-\zeta\omega_n T_s} \pm j\omega_n \sqrt{1 - \zeta^2} T_s$$

The desired damping coefficient $\xi \in (0,1)$ and the characteristic frequency $w_n \gg |w_{rMax}|$ are selected, where w_{rMax} is the maximum allowed mobile robot angular velocity.

Substituting (19) in (18) then linearizing the derivative state vector error and comparing coefficients at the same power of Z in (25) the $(k_x^*, k_y^*, k_\theta^*)$ gains are obtained as follows:

$$k_x^* = k_\theta^* = \frac{(z_1 + z_2 + z_3 + 3)}{T_s(1 + v_r)} \quad (26)$$

$$k_y^* = \frac{z_1 z_2 + z_1 z_3 + z_2 z_3 - 3 - T_s^2 w_r^2 + 2(z_1 + z_2 + z_3 + 3) - T_s v_r (\frac{z_1 + z_2 + z_3 + 3}{1 + v_r})^2}{T_s^2 v_r^2} \quad (27)$$

or

$$k_y^* = \frac{z_1 z_2 z_3 + 4 + z_1 + z_2 + z_3 + v_r (\frac{z_1 + z_2 + z_3 + 3}{1 + v_r})^2 + T_s^2 w_r^2 - T_s v_r (\frac{z_1 + z_2 + z_3 + 3}{1 + v_r})}{T_s^2 v_r^2 (\frac{z_1 + z_2 + z_3 + 3}{1 + v_r}) - T_s^2 v_r^2} \quad (28)$$

When v_r is close to zero or T_s is very small sampling time, k_y^* goes to infinity and we will lose the stability of the mobile robot system, therefore, $v_r > 0$, as we have proved in the Lyapunov function and to avoid small sampling time, k_y^* can be chosen as gain scheduling in [18], as (29).

$$k_y^* = \lambda \times |v_r| \quad (29)$$

where λ is constant gain.

So the closed loop system is globally asymptotically stable. The control parameters (k_x, k_y, k_θ) of nonlinear feedback kinematic controller are adjusted by using the particle swarm optimization method.

Particle Swarm Optimization (PSO) is a kind of algorithm to search for the best solution by simulating the movement and flocking of birds. PSO algorithms use a population of individuals (called particles) “flies” over the solution space to search for the optimal solution.

Each particle has its own position and velocity to move around the search space. The particles are evaluated using a fitness

function to see how close they are to the optimal solution [19] – [21].

The previous best value is called as *pbest*. Thus, *pbest* is related only to a particular particle. It also has another value called *gbest*, which is the best value of all the particles *pbest* in the swarm.

The nonlinear kinematic controller with three weights parameters is rewritten as an array to form a particle. Particles are then initialized randomly and updated afterwards according to (30, 31, 32, 33, 34 and 35) in order to tune the control gains:

$$\Delta Kx_m^{k+1} = \Delta Kx_m^k + c_1 r_1 (pbest_m^k - Kx_m^k) + c_2 r_2 (gbest^k - Kx_m^k) \quad (30)$$

$$Kx_m^{k+1} = Kx_m^k + \Delta Kx_m^{k+1} \quad (31)$$

$$\Delta Ky_m^{k+1} = \Delta Ky_m^k + c_1 r_1 (pbest_m^k - Ky_m^k) + c_2 r_2 (gbest^k - Ky_m^k) \quad (32)$$

$$Ky_m^{k+1} = Ky_m^k + \Delta Ky_m^{k+1} \quad (33)$$

$$\Delta K\theta_m^{k+1} = \Delta K\theta_m^k + c_1 r_1 (pbest_m^k - K\theta_m^k) + c_2 r_2 (gbest^k - K\theta_m^k) \quad (34)$$

$$K\theta_m^{k+1} = K\theta_m^k + \Delta K\theta_m^{k+1} \quad (35)$$

$m = 1, 2, 3, \dots, pop$

where

pop is the number of particles.

$K_{xy\theta_m}^k$ is the weight of particle m at k iteration.

c_1 and c_2 are the acceleration constants with positive values equal to 1.25.

r_1 and r_2 are random numbers between 0 and 1.

$pbest_m$ is best previous weight of m^{th} particle.

$gbest$ is best particle among all the particle in the population.

The number of dimensions in particle swarm optimization is equal to three because there are only three parameters of the nonlinear kinematic controller. The mean square error function is chosen as criterion for estimating the model performance as in (36):

$$J = \frac{1}{2} \sum_{k=1}^N (x_r(k+1) - x(k+1))^2 + (y_r(k+1) - y(k+1))^2 + (\theta_r(k+1) - \theta(k+1))^2 \quad (36)$$

The steps of PSO for nonlinear kinematic controller can be described as follows:

Step1 Initial searching points $Kx_1^0, Ky_1^0, K\theta_1^0, \Delta Kx_1^0, \Delta Ky_1^0$ and $\Delta K\theta_1^0$ of each particle are usually generated randomly within the allowable range. Note that the dimension of search space consists of all the parameters used in the proposed controller, as shown in Fig. 2. The current searching point is set to $pbest$ for each particle. The best-evaluated value of $pbest$ is set to $gbest$ and the particle number with the best value is stored.

Step2 The objective function value is calculated for each particle by using (36). If the value is better than the current $pbest$ of the particle, the $pbest$ value is replaced by the current value. If the best value of $pbest$ is better than the current $gbest$, $gbest$ is replaced by the best value and the particle number with the best value is stored.

Step3 The current searching point of each particle is updated using (30, 31, 32, 33, 34 and 35).

Step4 If the current iteration number reaches the predetermined maximum iteration number, then exit. Otherwise, go to step 2.

4. Simulation Results

The kinematic model of the differential wheeled mobile robot described in section 2 is used and the proposed controller is verified by means of computer simulation using MATLAB package. The simulation is carried out by tracking a desired position (x, y) and orientation angle (θ) with continuous trajectory in the tracking control of the Eddie mobile robot. The parameter values of the Eddie robot model are taken from [22]: L=0.452 m, r=0.076 m and sampling time is equal to 0.5 second. The proposed nonlinear kinematic controller scheme as in Fig. 2 is applied to the mobile robot model.

The values of the parameters (k_x, k_y, k_θ) of the proposed controller are found using (26) and (29) and tuned by using particle swarm optimization algorithm steps with the following parameters of operation:

The population of particle is equal to 22 and number of iteration is equal to 100.

The number of weight in each particle is 3 because there are three parameters of the nonlinear kinematic controller.

The desired damping coefficient is equal to $\xi = 0.2$ for non-oscillation and fast response of mobile robot. The characteristic frequency w_n is equal to 5rad/sec with constant scheduling gain equal to $\sigma = 10$.

The desired path which has explicitly continuous gradient with rotational radius changes can be described by the following equations:

$$x_r(t) = -0.5 \times \sin\left(\frac{2\pi}{30}\right) \tag{37}$$

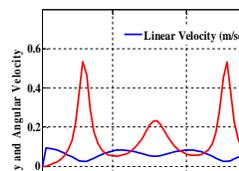
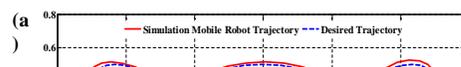
$$y_r(t) = 0.5 \times \sin\left(\frac{2\pi}{20}\right) \tag{38}$$

$$\theta_r(t) = 2 \tan^{-1}\left(\frac{\Delta y_r(t)}{\sqrt{(\Delta x_r(t))^2 - (\Delta y_r(t))^2} + \Delta x_r(t)}\right) \tag{39}$$

The mobile robot model starts from the initial posture $q(0) = [0.1, 0, 0.5\pi]$ as its initial conditions.

A disturbance term $\bar{w}t = [0.0 \sin 2t \ 0.0 \sin 2t]^T$ [23] is added to the mobile robot system as unmodelled kinematics disturbances in order to prove the adaptation and robustness ability of the proposed controller. The mobile robot trajectory tracking obtained by the proposed nonlinear kinematic controller is shown in Fig. 4.

The adaptive learning and robustness of nonlinear kinematic controller based on back-stepping with on-line PSO show excellent position and orientation tracking performance and small effect of the disturbances because the PSO algorithm has capability to obtain smooth values of



the nonlinear kinematic controller's parameters with smoothness convergence behaviour in the parameters values that depend on previous values.

model motion at 100 iteration can be shown in Fig. 7.

The simulation results demonstrated the effectiveness of the proposed controller by showing its ability to generate small smooth values of the control input linear velocity and angular velocity without sharp spikes therefore, smaller power is required to drive the DC motors of the mobile robot model.

The mean linear velocity of the Eddie mobile robot is equal to 0.1137 m/sec, and the maximum peak of the angular velocity is equal to ± 0.546 rad/sec as shown in Fig. 5.

Figure 6 shows the effectiveness of the proposed control algorithm based on back-stepping and PSO is clear by showing the convergence of the pose trajectory and orientation errors for the robot model motion and the performance index MSE for pose trajectory and orientation errors for the mobile robot

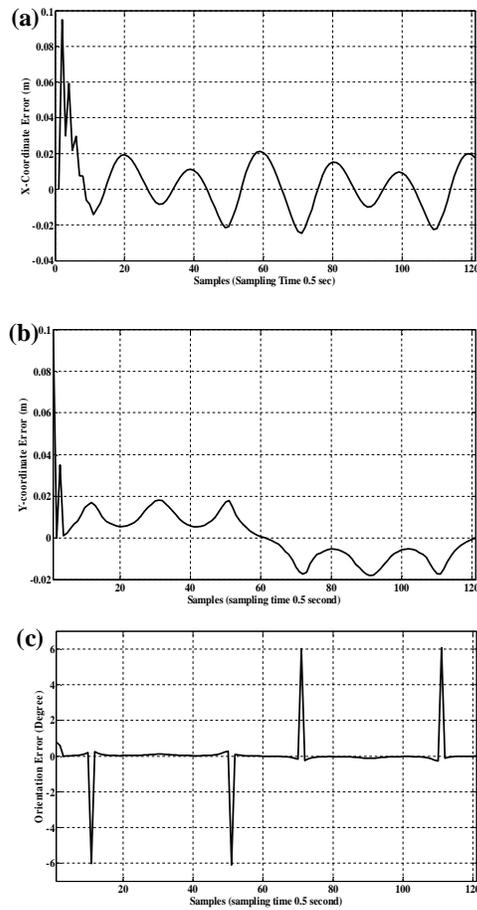


Figure 6 Position tracking error: (a) in X-coordinate (b) in Y-coordinate (c) Orientation tracking error. The instantaneous parameters of the proposed controller, (k_1, k_2) , have been tuned by using particle swarm optimization which was demonstrated, as shown in Fig. 8.

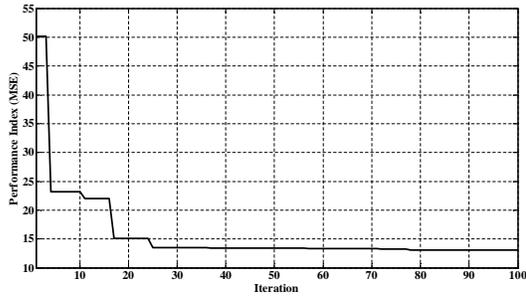


Figure 7. The performance index (MSE).

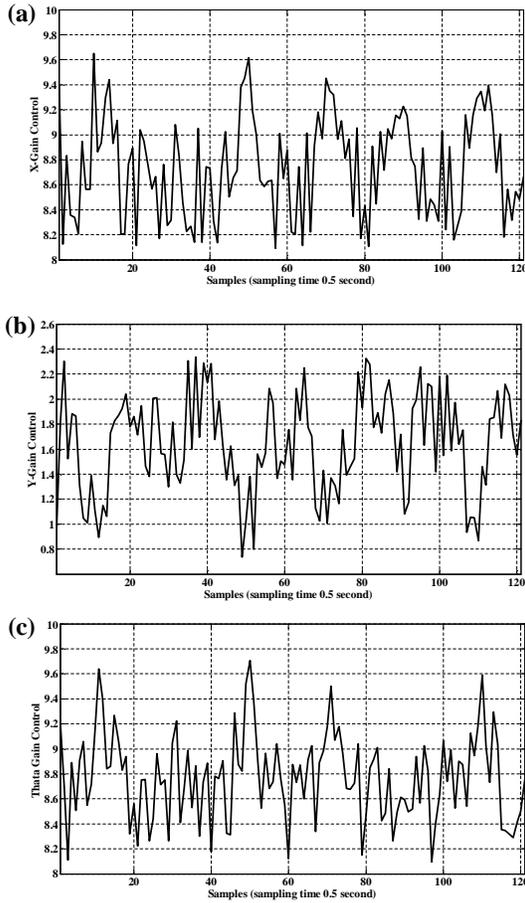


Figure 8 Gains control (a) k_x ; (b) k_y ; (c) k_θ .

In order to investigate the applicability of the proposed optimal on-line nonlinear kinematic control methodology, experiments have been executed by using mobile robot from Parallax Company type Eddie mobile robot, as shown in Fig. 9, which is equipped with LabVIEW package guided.



Figure 9. Eddie mobile robot for the experiments

In the experiments, back-stepping technique control methodology with PSO is applied on a real Eddie mobile robot in order to validate the controller adaptation, robustness and effectiveness in terms of minimum tracking error and in generating best velocity control action despite the presence of bounded external disturbances. The control data is transmitted to the Eddie mobile robot model, which admits right wheel velocity and left wheel velocity as input reference signals by using wire communication. This is done after data conversion from MATLAB M-file format to LabVIEW package version 2010 format in the Eddie mobile robot.

Velocities commands sent by the computer are represented as coded messages which are recognized by the microcontroller. Based on received characters, the microcontroller creates control actions for servo motors. The output voltages of the two encoder sensors are converted to coded messages by the microcontroller and sent to the personal computer in order to calculate the tracking error of the mobile robot during the motion.

The initial pose for the Eddie mobile robot starts at position (0.1 and 0) meter and orientation 0.5π radian and should follow the desired continuous trajectory, as show in Fig. 10. where the desired trajectory starts at position (0, 0, 0.5π).

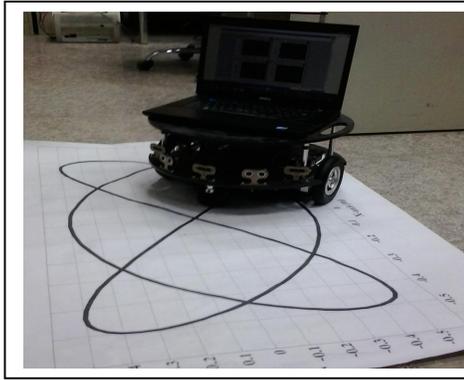


Figure 10 Real set-up experiment of Eddie mobile robot for continuous trajectory tracking.

After 122 samples, the Eddie mobile robot has followed and finished the tracking of the desired path, as shown in Fig. 11a with small drifting from the desired trajectory and the distance of the trajectory did not exceed 6.936m. Figure 11b shows the actual orientation of the mobile robot with small error.

The tracking error in x-coordinate and y-coordinate were reasonably accurate, as shown in Fig. 11c, because the velocity control action, shown in Fig. 11d demonstrates how the Eddie mobile robot tries to correct the pose and orientation errors.

From the simulation results and lab experiments, the on-line nonlinear kinematic control methodology based on back-stepping and PSO gives the best control result expected because of the precious ability of PSO in finding the optimal parameters of the proposed controller which depends on the previous values with smooth step changes.

The mean-square error for each component of the state error $MSE(ex, ey, e\theta)$ for simulation results and experimental work are calculated, as shown in Table 1.

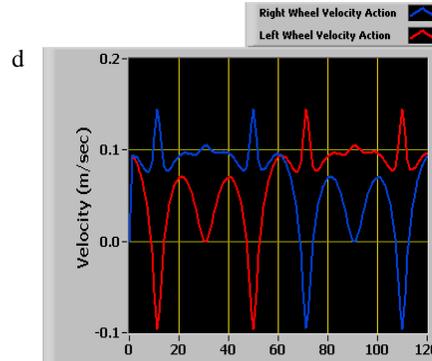
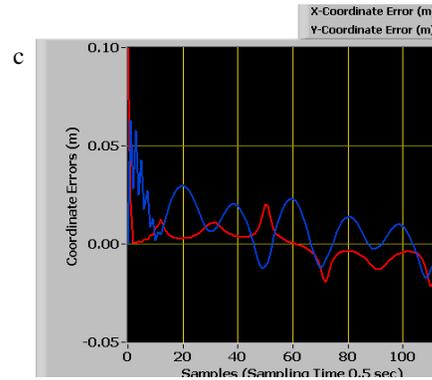
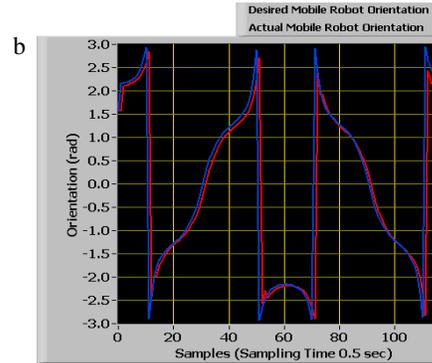
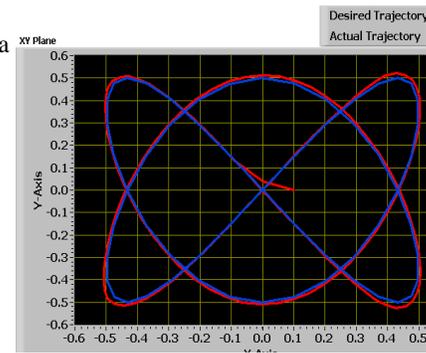


Figure 11 Practical results (a) desired and actual mobile robot trajectories; (b) desired and actual mobile robot orientations; (c) desired and actual mobile robot coordinate errors; (d) desired and actual mobile robot velocity actions.

Table 2 The percentage of MSE between simulation results and experimental work

(MSE of X-coordinate) 100%	26.5%
(MSE of Y-coordinate) 100%	24.6%
(MSE of Orientation) 100%	17.3%

The difference between simulations results and experimental results is caused by the residual errors in the experimental results due to the inherent friction present in the real system, especially during tracking the continuous gradient path and modelling errors, due to the difficulty of estimating or measuring the geometric, kinematics or inertial parameters, or from incomplete knowledge of the system components.

5. Conclusions

An optimal on-line nonlinear kinematic trajectory tracking control methodology for differential wheeled mobile robot has been presented in this paper. The proposed controller consists of a back-stepping technique with Lyapunov criterion and on-line PSO to tune the parameters of the controller that has been designed and tested using Matlab package and carried out on real Eddie mobile robot using LabVIEW package.

Simulation results and experimental work show evidently that the proposed nonlinear kinematic controller model has demonstrated the capability of tracking continuous gradients desired trajectories and effectively minimizing the tracking errors of the differential wheeled mobile robot model and has the capability of generating smooth and optimum suitable velocity (v_r and v_L) commands, without sharp spikes.

References

- [1] A. Filipescu, V. Minzu, B. Dumitrascu and E. Minca, Trajectory-tracking and discrete-time sliding-model control of wheeled mobile robot, IEEE International Conference on Information and Automation, Shenzhen, China, June 2011, pp. 27-32.
- [2] R-J Wai and C-M Liu, Design of dynamic petri recurrent fuzzy neural network and its application to path-tracking control of nonholonomic mobile robot, IEEE Transactions on Industrial Electronics. 56 (7) (2009) 2667-2683.
- [3] T. Sahin and E. Zergeroglu, A computationally efficient path planner for a collection of wheeled mobile robots with limited sensing zones, IEEE International Conference on Robotics and Automation, Roma, Italy, 10-14 April 2007, pp. 1074-1079.
- [4] T. Hague, N. Tillet, Navigation and control of an autonomous horticultural robot, Mechatronics, 6 (2) (1996) 165-180.
- [5] F. Mnif and F. Touati, An adaptive control scheme for nonholonomic mobile robot with parametric uncertainty, International Journal of Advanced Robotic Systems, 2 (1) (2005) 59-63.
- [6] T. Das and n. Kar, Design and implementation of an adaptive fuzzy logic-based controller for wheeled mobile robots, IEEE Transactions on Control System Technology, 14 (3) (2006) 501-510.
- [7] R. Fierro and F. L. Lewis, Control of a nonholonomic mobile robot using neural networks, IEEE Transactions on Neural Networks, 9 (4) (1998) 589-600.
- [8] G. Klancar and I. Skrjanc, Tracking error model-based predictive control for mobile robots in real time, Robotics and Autonomous Systems, 55 (2007) 460-469.
- [9] F. Martins, W. Celesta, R. Carelli, M. Sarcinelli-Filho, and T.F. Bastos-Filho, An adaptive dynamic controller for autonomous mobile robot trajectory tracking, Control Engineering Practice. 16 (2008) 1354-1363.
- [10] Z-G. Hou, A-M. Zou, L. Cheng and M. Tan, Adaptive control of an electrically driven nonholonomic mobile robot via back-stepping and fuzzy approach, IEEE Transactions on Control Systems Technology, 17 (4) (2009) 803-815.
- [11] B. Park, S. Yoo, J. Park and Y. Choi, A simple adaptive control approach for trajectory tracking of electrically driven nonholonomic mobile robots, IEEE Transactions on Control Systems Technology, 18 (5) (2010) 1199-1206.
- [12] H-X. Zhang, G-J. Dal and H. Zeng, A trajectory tracking control method for nonholonomic mobile robots, Proceedings of the International Conference on Wavelet

-
- Analysis and Pattern Recognition, Beijing, China, 2-4 Nov. 2007, pp. 7-11.
- [13] J. Ye, Tracking control for nonholonomic mobile robots: Integrating the analog neural network into the backstepping technique, *Neurocomputing*, 71 (2008) 3373-3378.
- [14] J. Ye, Adaptive control of nonlinear PID-based analogue neural network for a nonholonomic mobile robot, *Neurocomputing*, 71 (2008) 1561-1565.
- [15] R. Siegwart and I. R. Nourbakhah, *Introduction to autonomous mobile robots*, MIT Press, 2004.
- [16] T. Wang, C. Tsai, Adaptive trajectory tracking control of a wheeled mobile robot via Lyapunov techniques. The 30th Annual Conference of the IEEE Industrial Electronics Society, Busan, Korea, November 2-5, 2004, pp. 389-394.
- [17] S. Sastry, *Nonlinear Systems Analysis, Stability and Control*, Springer – Verlag New York Inc, 1999.
- [18] M. Vidyasagar, *Nonlinear Systems Analysis*, 2nd Edition, Prentice Hall, Inc., Englewood Cliffs, NJ, 1993.
- [19] Fang, L. Chen and Z. Shen, Application of an improved PSO algorithm to optimal tuning of PID gains for water turbine governor. *Energy Conversion and Management*. Vol. 52, (2011), pp.1763-1770.
- [20] J-S. Chiou, S-H. Tsai and M-T. Liu, A PSO-based adaptive fuzzy PID-controllers. *Simulation Modelling Practice and Theory*. Vol. 26, (2012), pp. 49-59.
- [21] J. Derrac, S. Garc, D. Molina and F. Herrera ,a Practical Tutorial on the use of Nonparametric Statistical Tests as a Methodology for Comparing Evolutionary and Swarm Intelligence Algorithms, *Journal of Swarm and Evolutionary Computation*, Vol.1, (2011), pp. 3–18.
- [22] Internet website <http://parallax.com/>. Robotics with the Eddie Mobile Robot text manual. Accessed Jun 2014.
- [23] A. Al-Araji, M. Abbod and H. Al-Raweshidy, Applying Posture Identifier in Designing an Adaptive Nonlinear Predictive Controller for Nonholonomic Mobile Robot. *Neurocomputing*, Vol. 99, (2013), pp. 543-554.