On 3-prime ideal with respect to an element of a near ring

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Abstract

In this paper ,we introduce the notions 3- prime ideal with respect to an element x denoted by (x-3-prime ideal) of a near ring and the 3-prime ideal near ring with respect to an element x denoted by (x-3-prime ideal near ring) ,and we studied the image and inverse image of x-3-prime ideal under epimomorphism and the direct product of x-3-prime ideal of near ring .

Key word

Near ring ,ideal of near ring ,3-prime ideal, completely prime ideal ,completely semi prime ideal with respect to an element x of N , direct near ring the union and intersection ideals .

Introduction

We will refer that all near rings and ideals in this paper are left . In 1989 the notion of completely semi prime ideal (C.S.P.I) was introduced by P.DHeena [8].In 1988 the notion of near ring N was introduced by N.J Groenewald [7]and in this year he introduced the notion 3-prime ideal of a near ring .In 2011 H.Hadi and Showq M. they introduced notions of completely semi prime ideal with respect to an element x of a near ring and the completely semi prime ideals with respect to an element near ring (x-C.S.P.I near ring) [4] . They established many results and obtained many correspondents between (C.S.P.I) and (x-C.S.P.I) of a near ring . The purpose of this paper is as mention in the abstract .

1.Preliminaries

In this section we give some concepts that we need in second section .

Definition (1.1) [2]

A left near ring is a set N together with two binary operations "+" and "." such that

a. (N,+) is a group (not necessarily abelian

b. (N, .) is a semigroup.

c. $(n_1 + n_2) . n_3 = n_1 . n_3 + n_2 . n_3$

For all $n_1, n_2, n_3, \in N$,

Definition (1.2) [2]:

Let N be a near ring. A normal subgroup I of (N,+) is called a left ideal of N if

(1) $I.N \subseteq I$.

(2). $\forall$ n, $n_1 \in N$ and for all $i \in I$,

Definition (1.3) [6]:

An ideal I of near ring N is called a 3-prime ideal for all a,b$\in$ N , $a.N.b \subseteq I$ implies $a \in I \vee b \in I$.

Definition *(1.4)[7]*

Let I be an ideal of a near ring N. Then I is called a completely prime ideal of N if $\forall x, y \in N$, $x . y \in I$ implies $x \in I$ or $y \in I$, denoted by C.P.I of N .

Definition *(1.5) [5]*

Let(N_1,+,.) and (N_2 ,+′,.′) be two near rings. The mapping f : $N_1 \rightarrow N_2$ is called a near ring homomorphism if for all m, n $\in N_1$

f(m+ n) = f(m) +′ f(n) and f(m. n) = f(m) .′ f(n).

Theorem *(1.6) [4]*

Let $f : (N_1,+,.) \rightarrow (N_2,+',.')$ be a near ring homomorphism

(1) If I is an ideal of a near ring N_1 , then f(I) is an ideal of a near ring N_2.

(2) If J is an ideal of a near ring N_2 , then $f^{-1}(J)$ is an ideal of the near ring N_1

theorem (1.7)[6]

Let $\{I_j\}_{j\in J}$ be a family of ideals of a near ring N ,then

(1) $\bigcap_{j\in J} I_j$ is an ideal of N.

(2) if $\{I_j\}_{j\in J}$ is a chain of ideals of anear ring N, then $\bigcup_{j\in J} I_j$ is an ideal of N.

Definition (1.8) [8]

Let $\{Nj\}_{j\in J}$ be a family of near rings , J is an index set and

$$\prod_{j\in J} N_j = \{(x_j) : x_j \in N_j, for \text{ all } j \in J\}$$

be the directed product of N_j with the component wise defined operations '+' and '.', is called the direct product near ring of the near rings N_j .

Definition (1. 9) [1]

If I_1 and I_2 are ideals of a near ring N ,then

$$I_1 \cdot I_2 = \{i_1 \cdot i_2 : i_1 \in I_1, i_2 \in I_2\}.$$

Definition (1. 10) [8]

A near ring N is called Integral domain if N has no zero divisions.

Definition (1. 11) [8]

The factor near ring N/I is defined as in case of ring .

Definition (1. 12) [3]

let N be a near ring and $x \in N$, I is called completely semi prime ideal with respect to the element x denoted by (x-C.S.P.I) or(x- completely semi prime ideal)of N if for all $y \in N$, if $x \cdot y^2 \in I$ implies $y \in I$

2. The main Results

This section is devoted to study 3-prime ideal with respect to an element of a near ring ,and x-3-prime ideal near ring .

Definition (2.1)

An ideal I of near ring N is called 3-prime ideal with respect to an element of near ring denoted by (x-3-prime ideal) if for all $a,b \in N$ $x.(a.N.b) \subseteq I \to x.a \in I \vee x.b \in I$.

Example(2.2)

Considers the set N={ 0,a,b,c} be a near ring with addition and multiplication defined by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	a	0	a
a	0	a	a	0
b	0	a	b	c
3	0	a	c	b

Let I={0,a} be a c-3-prime ideal since $c.(a.N.b) \subseteq I \to c.a \in I \vee c.b \in I$.

Proposition (2.3)

Let $\{I_j\}_{j\in J}$ be a family of x- 3-prime ideal of a near ring N for all $j \in J$, $x \in N$ .Then $\bigcap_{j\in J} I_j$ is a x- 3-prime ideal of N .

Proof

Let $x,a,b \in N$ $\bigcap_{j\in J} I_j$ be an ideal [1.7] since I_j is x- 3-prime ideal of N, $I_j \neq \phi$, $I_j \subseteq N$ .let $x.a.N.b \subseteq \bigcap_{j \in J} I_j$

$x.a.N.b \subseteq I_j$,$\forall$j $\in$ J ,since I_j is x- 3-prime ideal $x.a \in I_j \vee x.b \in I_j$

$x.a \in \bigcap_{j\in J} I_j \vee x.b \in \bigcap_{j\in J} I_j$ implies $\bigcap_{j\in J} I_j$ is an x- 3-prime ideal of N.

Proposition (2.3)

Let $\{I_j\}_{j\in J}$ be a chain of a x-3-Prime ideals of a near ring N ,then $\bigcup_{j\in J} I_j$ is x-3-Prime ideal of N where $j \in J$.

Proof

Let $\{I_j\}_{j\in J}$ be a chain of x-3-Prime ideal of near ring . $\rightarrow \bigcup_{j\in J} I_j$ is an ideal of N [1.7] Now let $x.a.N.b \subseteq \bigcup_{j\in J} I_j$, $x.a.N.b \subseteq I_j$ $\forall j \in J$ since I_j is x-3-prime ideal $x.a \in I_j \vee x.b \in I_j$

$$x.a \in \bigcup_{j\in J} I_j \vee x.b \in \bigcup_{j\in J} I_j$$

$\rightarrow \bigcup_{j\in J} I_j$ is an x- 3-prime ideal of N.

Remark (2.4)

If I_1 and I_2 be two x- 3-prime ideals of near ring such that $I_1 \not\subset I_2$ and $I_2 \not\subset I_1$, then $I_1 \bigcup I_2$ of N may be not x- 3-prime ideal .

Example (2.5)

considers the set N={ 0,a,b,c} be a near ring with addition and multiplication defined by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
3	0	a	b	c

Let I_1={0,a} and I_2={0,b} are b-3-prime ideal since

$$b.a.N.b \subseteq I_1 \rightarrow$$
$$b.a \in I_1 \vee b.b \in I_1$$

$$b.a.N.b \subseteq I_2 \rightarrow$$
$$b.a \in I_2 \vee b.b \in I_2$$

but $I_1 \bigcup I_2$ ={0,a,b} is not an ideal of near ring N.

Remark (2.6)

If I_1 and I_2 be two x-3-prime ideal of near ring N, then I_1 . I_2 of N may be not x-3-prime ideal of N.

Example (2.7)

considers the near ring N={ 0,a,b,c} with addition and multiplication defined by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Let I_1={0,a} and I_2={0,b} are c-3-prime ideal of N ,but $I_1.I_2$={0} is not c -3-prime ideal of N since $c.b.N.a = \{0\} \subseteq I_1.I_2$ but $c.b \notin I_1.I_2 \vee c.a \notin I_1.I_2$

Definition (2.8)

The near ring N is called 3- prime ideal near ring with respect to an element x denoted by (x-3-Prime ideal near ring), if every ideal of a near ring N is an x- 3-prime ideal of N ,where $x \in N$.

Example (2.9)

Consider the near ring N={ 0,a,b,c} with addition and multiplication defined by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
3	0	a	b	c

N is c-3- prime ideal near ring since all ideals of N ,I_1={0} and I_2=N are c-3- prime ideal .

Lemma (2.10)

If N is a near integral domain , then {0} is x -3-prime ideal of N,for all $x \in N - \{0\}$.

Proposition (2.11)

Let N be a near ring with multiplicative identity e', then I is e'- 3-prime ideal of near ring N if and only if I is a 3-prime ideal of N.

Proof : $\leftarrow$

Let $y,z \in N$, e' is the identity $e'.y.N.z \subseteq I$ implies that $y.N.z \subseteq I$. since I is 3-prime ideal of N then $y \in I \vee z \in I$ implies $e'.y \in I \vee e'.z \in I$ therefor I is e'- 3-prime ideal of N.

$\rightarrow$

Let $y,z \in N$ and I is e'-3-prime ideal of N.Let $y.N.z \subseteq I$, $e'.y.N.z \subseteq I$ since I is e' - 3-prime ideal of N $\rightarrow e'.y \in I \vee e'.z \in I$ implies $y \in I \vee z \in I$,since e' the identity of N $\rightarrow$ I is 3-prime ideal of N.

Remark (2.12)

Not all x- I is 3-prime ideal of a near ring are C.P.I of N.

Example (2.13)

considers the near ring N={ 0,1,2,3} be a near ring with addition and multiplication defined by the following tables :

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	0	0	0
3	0	1	2	3

Let I_1 ={0,1} be 2-3-prime ideal of N .I is not C.P.I of N, since $2.3 = 0 \in \mathrm{I}$ but $2 \notin \mathrm{I} \wedge 3 \notin \mathrm{I}$.

Remark (2. 14)

In general not all x-3-prime ideal are 3-prime ideal .

Example (2.15)

Consider the near ring N in example (2.13) let I ={0,1} be 2-3- prime ideal but I is not 3-prime ideal of N , since $2.N.3 \subseteq I$ but $2 \notin \mathrm{I} \wedge 3 \notin \mathrm{I}$.

Remark (2. 16)

In general not all x-3-prime ideal are x.C.S.P.I of N .

Example (2.17)

Consider the near ring N in example (2.5) let I ={0,a} be c-3- prime ideal but I is not a- C.S.P.I of N , since $c.b^2 = o \in I$ but $b \notin \mathrm{I}$.

Theorem (2.18)

Let (N_1,+,.) and $(\mathrm{N}_2, +', .')$ be two near rings , $f : N_1 \to N_2$ be an epimomorphism and I be x- 3-prime ideal of N_1 .Then f(I) is f(x)- 3-prime ideal of N_2 .

Proof

Let I be - 3-prime ideal of N_1 f(I) be an ideal of N_2 by using theorem [1.6]

Let $c, y, z \in N_2$, $\exists$ $a, b \in N_1$ such that $f(a) = y, f(x) = c, f(b) = z \in N_2$,hence

$f(x).' f(a).N_2.f(b) \subseteq f(I)$ since f be an epimomorphism $f(x.a.N_1.b) \subseteq f(I)$ since I is x- 3-prime ideal of N_1

$\to x.a.N_1.b \subseteq I$

$$x.a \in I \ \vee \ x.b \in I$$
$$f(x.a) \in f(I) \vee f(x.b) \in f(I)$$
$$f(x).' f(a) \in f(I) \vee f(x).' f(b) \in f(I)$$

f(x) is f(x)- 3-prime ideal of N_2 .

Theorem (2.19)

Let (N_1,+,.) and $(N2, +', .')$ be two near rings, and $f : N_1 \to N_2$ be epiomomorphism and J be a f(x)- 3-prime ideal of N_2 . Then f^{-1}(J) is a x-3-prime ideal of N_1 , where y= f(x) , $\ker f \subseteq f^{-1}(I)$.

Proof

Let $x, a, b \in N_1$, f^{-1} (J) is an ideal by using theorem [1.6] ,

$$x.a.N_1.b \subseteq f^{-1}(J)$$
$$f(x.a.N_1.b) \subseteq J$$
$$f(x).' f(a).' N_2.' f(b) \subseteq J$$

since J is f(x)- 3-prime ideal of N_2

$$f(x).'f(a) \in J \lor f(x).'f(b) \in J$$
$$f(x.a) \in J \lor f(x.b) \in J$$

$$x.a \in f^{-1}(J) \lor x.b \in f^{-1}(J)$$

$\rightarrow f^{-1}(J)$ is a x-3-prime ideal of N_1.

Proposition (2.20)

If N is non zero near ring then I is 0-3-prime ideal of N.

Theorem (2.21)

Let $\{N_j\}_{j\in J}$ be a family of a near rings, $x_j \in N_j$ and I_j be x_j- 3-prime ideal of N_j for all $j \in J$ .Then $\prod_{i\in J} I_j$ is (x_j)- 3-prime ideal of the direct product near ring $\prod_{i\in J} N_j$.

Proof

Let $(a_j),(b_j),(x_j) \in \prod_{j\in J} N_j$,and $\prod_{j\in J} I_j$ is an ideal of $\prod_{j\in J} N_j$ by using definition (1.9) such that

$(x_j).(a_j).\prod_{j\in J} N_j.(b_j) \subseteq .\prod_{j\in J} I_j$ for all $j \in J$

$x_j.a_j.N_j.b_j \subseteq .I_j$ since I_j is x_j - 3-prime ideal of N_j for all $j \in J$,

$\rightarrow x_j.a_j \in I_j \lor x_j.b_j \in I_j$, $\forall$ $j \in J$

$(x_j.a_j) \in \prod_{j\in J} I_j \lor (x_j.b_j) \in \prod_{j\in J} I_j$

$(x_j).(a_j) \in \prod_{j\in J} I_j \lor (x_j).(b_j) \in \prod_{j\in J} I_j$

$\rightarrow \prod_{i\in J} I_j$ is (x_j)- 3-prime ideal of the direct product near ring $\prod_{i\in J} N_j$.

Theorem (2.22)

Let I be an ideal of the x- 3-prime ideal near ring N .Then the factor near ring N/I is x+I - 3-prime ideal near ring .

proof

The natural homomorphism $\mathrm{nat}_I : N \rightarrow N/I$ which is defined by nat_I (x) =x+I , for all $x \in N$,is an epimomorphism .Now let J be an ideal of the factor near ring N/I .Then by theorem (1.6) we have $nat_I^{-1}(J)$ is an ideal of the near ring N.$\Rightarrow nat_I^{-1}(J)$ is a x-3-prime ideal of N [since N is x-3-prime ideal near ring .By theorem (2-18) we have $nat(nat_I^{-1}(J)) = J$ is

nat_I (x) -3-prime ideal of $N/I \Rightarrow$ J is x+I – 3-prime ideal of factor near ring .Then N/I is x+I - 3-prime ideal near ring .

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الخلاصة:

قدمنا في هذا البحث مفهومي المثالية 3-prime ideal بالنسبة لعنصر ما في الحلقة القريبة N والتي يرمـــز لها بالرمز (x-3-prime ideal)، وايــضا حـــلقـــة المثاليات القريبة 3-prime ideal بالنسبة لعنصر ما في N والتي يرمز لها بالرمز (x-3-prime ideal near ring). كما ودرسنا الصــور المباشـــرة ومعكـــوس الصـــورة للمثاليـــة تحت التشاكل الشامل كما وأعطينا بعض الخواص التي تتعلق بهذه المثالية .